Reliability Analysis of Time – Dependent Stress-Strength System When the Number of Cycles Follow Binomial Distribution

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Research Article

Abstract: Reliability analysis of time dependent stress strength system is carried out by using stress / strength models considering each of the stress and strength variables as deterministic or random fixed or random independent. The number of cycles in any period of time t is assumed to be random. Reliability is found for the nine models when the number of cycles follows binomial distribution where stress and strength follow exponential distribution.

Keywords: Binomial distribution, Exponential distribution, deterministic, random- fixed, random- independent and Reliability.

Introduction

Time dependent stress strength system is defined by [1]. A component fails if the stress on it is exceeding than its strength. In the present paper, the uncertainty about the stress and strength variables is classified into three categories:

1. Deterministic: the variable assumes values that are exactly known a priory.
2. Random fixed: the variable is random at any particular instant of time; the word fixed in this classification refers to the behaviour of the random variable with respect to time and/or cycles; it means that the random variable changes or varies with time in a known manner.
3. Random independent: the variable is not only random but unlike the random fixed case, the successive values assumed by the variables are statistically independent, in accordance with Kapur and Lamberson and Schatz, *et al*. Here, in this paper, the components are assumed to be identical and the number of cycles for any time period t is assumed to be random. Expression for system reliability have been attained when number of cycles can be follow binomial distribution and stress and strength both follow exponential distribution.

Notation

Reliability Evaluation:

Number of cycles occurring in a given time interval is follows **Binomial distribution**

$$P(X=x)=\binom{n}{x}p^x q^{n-x}\ x=0,1,2,\dots..n.$$

Case 1: Deterministic Stress and Deterministic Strength

Let the stress x_i be known and non decreasing and strength y_i be known and non increasing. Let n^* be such that $R_{n^*}\ and\ R_{n^*+1}=0$. Then we have $R_i=1\ for\ i=0,1,2,\dots.n^*$ and $R_i=0\ for\ i=n^*+1,n^*+2,\dots.$

$$R(t)=\sum_{i=0}^{\infty}\pi_i(t)R_i\ =\sum_{i=0}^{n}\binom{n}{i}p^i q^{n-i}R_i$$

$$R(t)=\pi_0(t)R_0\ =\binom{n}{0}p^0 q^n=q^n$$

$$R(t)=\sum_{i=0}^{n}\pi_i(t)R_i$$

$$=1.\sum_{i=0}^{n}\pi_i(t)=\sum_{i=0}^{n}\binom{n}{i}p^i q^{n-i}$$

$$= q^n \left[1 + \frac{p}{q}\right]^n = 1$$

Case 2: Deterministic Stress and random- fixed Strength

Let the stress be a known constant x_0. Let $y_i = y_0$, $i = 1,2,\ldots$.be the strength random variable with a known probability density function $f_0(y_0)$. Then

$$R_i = P[E_i] = \int_{x_0}^{\infty} f_0(y_0)dy_0 \triangleq R$$

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t)R_i \; = \pi_0(t)R_0 + \sum_{i=1}^{\infty} \pi_i(t)R_i$$

$$= q^n(1) + R\sum_{i=1}^{n} \pi_i(t) \; = q^n + R\big(1 - \pi_0(t)\big)$$

$$R(t) \; = q^n + R(1 - q^n) \; or \; R + (1 - R)q^n$$

Case 3: Deterministic Stress and Random independent Strength

Let the stress be a known constant x_0. Let $g(y)$ be the probability density function of random- independent strength y. Then

$$R_i = R^i \,, i = 0,1,2, \ldots\ldots n$$

$$where \; R = \int_{x_0}^{\infty} g(y)dy$$

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t)R_i = \sum_{i=0}^{n} \binom{n}{i} p^i q^{n-i} R^i$$

$$= q^n \sum_{i=0}^{n} \left(\frac{pR}{q}\right)^i \binom{n}{i} = q^n \left(1 + \frac{pR}{q}\right)^n \; or \; (q + pR)^n$$

Case 4: Random - fixed Stress and deterministic Strength

Let x_0 be the random fixed stress with a known probability density function $f_0(x_0)$ that does not vary with time and y_0 is the deterministically known strength i.e. constant in time. Then

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t)R_i \; = \pi_0(t)R_0 + \sum_{i=1}^{\infty} \pi_i(t)R_i$$

$$= q^n(1) + R\sum_{i=1}^{n} \pi_i(t) \; = q^n + R\big(1 - \pi_0(t)\big)$$

$$R(t) \; = q^n + R(1 - q^n) \; or \; R + (1 - R)q^n$$

$$where \; R = \int_{0}^{y_0} f_0(x_0)dx_0$$

Case 5: Random - fixed Stress and random- fixed Strength

Let x_0 and y_0 be the random fixed stress and strength with known probability density functions $f_0(x_0)$ and $g_0(y_0)$ respectively. x_0 and y_0 will be assumed not vary with time that is $a_i = b_i = 0, i = 1,2, \ldots$.. Hence

$$R_i = \int_{0}^{\infty} g_0(y_0) \int_{0}^{y_0} f_0(x_0)dx_0 dy_0 = R, i = 1,2, \ldots.$$

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t)R_i \; = \pi_0(t)R_0 + \sum_{i=1}^{\infty} \pi_i(t)R_i$$

$$= q^n(1) + R\sum_{i=1}^{n}\pi_i(t) \ = q^n + R\big(1-\pi_0(t)\big)$$

$R(t) \ = q^n + R(1-q^n) \ or \ R + (1-R)q^n \ where \ R \ is \ given \ by \ above$

Case 6: Random - fixed Stress and Random- independent Strength

Let $f(x)$ is the probability density function of random fixed stress X and $g(y)$ is the probability density function of random independent strength . We have

$$R_i = \int_0^\infty f(x)\left(\int_x^\infty g(y)dy\right)^i dx \ \ i = 0,1,2,\ldots..n$$

$$R(t) = \sum_{i=0}^{n}\pi_i(t)R_i$$

$$= \sum_{i=o}^{n}\binom{n}{i}p^i q^{n-i}\int_0^\infty f(x)\left(\int_x^\infty g(y)dy\right)^i dx$$

$$= q^n\int_0^\infty f(x)\sum_{i=0}^{n}\binom{n}{i}\left(\frac{p}{q}\int_x^\infty g(y)dy\right)^i dx$$

$$= q^n\int_0^\infty f(x)\left(1+\frac{p}{q}\int_x^\infty g(y)dy\right)^n dx$$

$$R(t) = q^n\int_0^\infty f(x)\left(1+\frac{p}{q}G(x)\right)^n dx \ where \ G(x) = \int_x^\infty g(y)dy$$

Case 7: Random - independent Stress and deterministic Strength

The random independent stress is denoted by X and the deterministic strength by y_0 which is constant in time. Then

$$R = \int_0^{y_0} f(x)dx$$

$$R(t) = \sum_{i=0}^{\infty}\pi_i(t)R_i = \sum_{i=0}^{n}\binom{n}{i}p^i q^{n-i}R^i$$

$$= q^n\sum_{i=0}^{n}\left(\frac{pR}{q}\right)^i\binom{n}{i} = q^n\left(1+\frac{pR}{q}\right)^n or \ (q+pR)^n$$

$$R(t) = (q+pR)^n \ where \ R = \int_0^{y_0} f(x)dx$$

Case 8: Random - independent Stress and Random- fixed Strength

Let g(y) is the probability density function of random fixed strength X and f(x) is the probability density function of random independent stress X. We have

$$R(t) = \sum_{i=o}^{n}\binom{n}{i}p^i q^{n-i}\int_0^\infty g(y)\left(\int_0^y f(x)dx\right)^i dy$$

$$= q^n\int_0^\infty g(y)\sum_{i=0}^{n}\binom{n}{i}\left(\frac{p}{q}\int_0^y f(x)dx\right)^i dy$$

$$= q^n \int_0^\infty g(y) \left(1 + \frac{p}{q} \int_0^y f(x)dx \right)^n dy$$

$$R(t) = q^n \int_0^\infty g(y) \left(1 + \frac{p}{q} F(y) \right)^n dx \; where \; F(y) = \int_0^y f(x)dx$$

Case 9: Random- independent Stress and random- independent Strength

Let $f(x)$ and $g(y)$ represent the probability density functions for stress X and strength Y respectively. Further the random variables be independent on each cycle. Then

$$R_n = R^n \,, n = 0,1,2, \dots \dots n$$

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t) R_i = \sum_{i=0}^{n} \binom{n}{i} p^i q^{n-i} R^i$$

$$= q^n \sum_{i=0}^{n} \left(\frac{pR}{q}\right)^i \binom{n}{i} = q^n \left(1 + \frac{pR}{q}\right)^n$$

$$R(t) = (q + pR)^n$$

$$where \; R = \int_0^\infty f(x) \int_x^\infty g(y)dydx$$

In Binomial Distribution, the Stress and Strength Follow Exponential Distribution

Case 2: Deterministic Stress and random- fixed Strength

$$R(t) = q^n + R(1 - q^n)$$

$$where \; R = \int_{x_0}^\infty f_0(y_0)dy_0$$

$$= \int_{x_0}^\infty \mu e^{-\mu y_0} dy_0 = e^{-\mu x_0}$$

$$R(t) = q^n + e^{-\mu x_0}(1 - q^n)$$

Case 3: Deterministic Stress and random- independent Strength

$$R(t) = (q + pR)^n$$

$$where \; R = \int_{x_0}^\infty g(y)dy$$

$$= \int_{x_0}^\infty \mu e^{-\mu y} dy = e^{-\mu x_0}$$

$$R(t) = (q + pe^{-\mu x_0})^n$$

Case 4: Random- fixed stress and deterministic Strength

$$R(t) = q^n + R(1 - q^n), where \; R = \int_0^{y_0} f_0(x_0)dx_0$$

$$R = \int_0^{y_0} \lambda e^{-\lambda x_0} \, dx_0 = \left(1 - e^{-\lambda y_0}\right)$$

$$R(t) = q^n + \left(1 - e^{-\lambda y_0}\right)(1 - q^n)$$

$$R(t) = 1 - e^{-\lambda y_0}(1 - q^n)$$

Case 5: Random- fixed Stress and random- fixed Strength

$$R(t) = q^n + R(1 - q^n)$$

$$where\ R = \int_0^\infty g_0(y_0) \int_0^{y_0} f_0(x_0) dx_0 dy_0$$

$$= \int_0^\infty \mu e^{-\mu y_0} \int_0^{y_0} \lambda e^{-\lambda x_0} dx_0\, dy_0$$

$$R = \frac{\lambda}{(\lambda + \mu)}$$

$$R(t) = q^n + \frac{\lambda}{(\lambda + \mu)}(1 - q^n)$$

$$R(t) = \frac{1}{(\lambda + \mu)}[\lambda + q^n \mu]$$

Case 6: Random- fixed Stress and random- independent Strength

$$R(t) = \int_o^\infty f(x)\,(q + pG(x))^n dx\ where\ G(x) = \int_x^\infty g(y)dy$$

$$G(x) = \int_x^\infty \mu e^{-\mu y} dy = e^{-\mu x}$$

$$R(t) = \int_0^\infty \lambda e^{-\lambda x}(q + pe^{-\mu x})^n\, dx$$

Case 7: Random- independent Stress and deterministic Strength

$$R(t) = (q + pR)^n\ where\ R = \int_0^{y_0} f(x)dx$$

$$R = \int_0^{y_0} \lambda e^{-\lambda x} dx = \left(1 - e^{-\lambda y_0}\right)$$

$$R(t) = \left(q + p\left(1 - e^{-\lambda y_0}\right)\right)^n = \left(1 - pe^{-\lambda y_0}\right)^n$$

Case 8: Random- independent Stress and random- fixed Strength

$$R(t) = \int_0^\infty g(y)\left(q + pF(y)\right)^n dy\ where\ F(y) = \int_0^y f(x)dx$$

$$F(y) = \int_0^y \lambda e^{-\lambda x} dx = \left(1 - e^{-\lambda y}\right)$$

$$R(t) = \int_0^\infty \mu e^{-\mu y}\left(q + p\left(1 - e^{-\lambda y}\right)\right)^n dy = \int_0^\infty \mu e^{-\mu y}\left(1 - pe^{-\lambda y}\right)^n dy$$

Case 9: Random- independent Stress and random- independent Strength

$$R(t) = (q + pR)^n$$

$$where\ R = \int_0^\infty f(x) \int_x^\infty g(y)dydx$$

$$R = \int_{0}^{\infty} \lambda e^{-\lambda x} \int_{x}^{\infty} \mu e^{-\mu y} dx\, dy = \frac{\lambda}{(\lambda + \mu)}$$

$$R(t) = \left(q + p\left(\frac{\lambda}{(\lambda + \mu)}\right)\right)^{n}$$

Numerical Results

Case 2: Deterministic Stress and random- fixed Strength:

Table 1

n	μ	Q	x_0	R
1	0.2	0.5	2	0.83516
2	0.2	0.5	2	0.75274
3	0.2	0.5	2	0.71153
4	0.2	0.5	2	0.690925
5	0.2	0.5	2	0.680623
6	0.2	0.5	2	0.675471
7	0.2	0.5	2	0.672896
8	0.2	0.5	2	0.671608
9	0.2	0.5	2	0.670964
10	0.2	0.5	2	0.670642

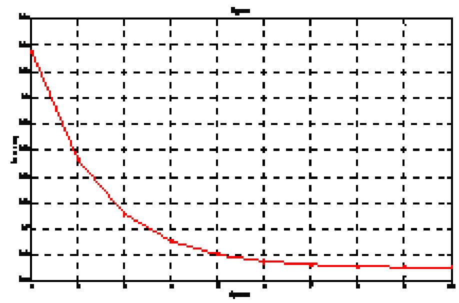


Table 2

n	μ	Q	x_0	R
1	0.1	0.5	2	0.909365
2	0.2	0.5	2	0.75274
3	0.3	0.5	2	0.60521
4	0.4	0.5	2	0.483746
5	0.5	0.5	2	0.387633
6	0.6	0.5	2	0.312113
7	0.7	0.5	2	0.252483
8	0.8	0.5	2	0.205014
9	0.9	0.5	2	0.166929
10	1	0.5	2	0.13618

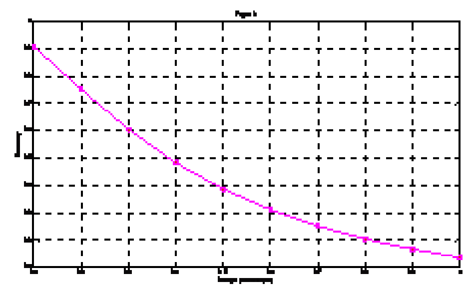


Table 3

n	μ	q	x_0	R
1	0.1	0.5	2	0.909365
1	0.2	0.5	2	0.83516
1	0.3	0.5	2	0.774406
1	0.4	0.5	2	0.724664
1	0.5	0.5	2	0.68394
1	0.6	0.5	2	0.650597
1	0.7	0.5	2	0.623298
1	0.8	0.5	2	0.600948
1	0.9	0.5	2	0.582649
1	1	0.5	2	0.567668

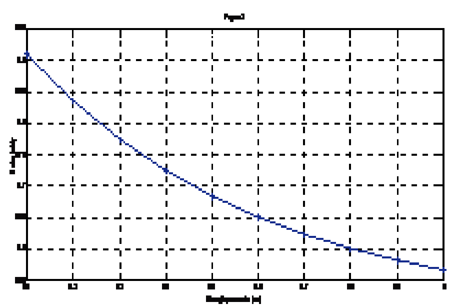


Table 4

n	μ	q	x_0	R
1	0.5	0.1	2	0.431091
2	0.5	0.2	2	0.393164
3	0.5	0.3	2	0.384947
4	0.5	0.4	2	0.384062
5	0.5	0.5	2	0.387633
6	0.5	0.6	2	0.397372
7	0.5	0.7	2	0.419937
8	0.5	0.8	2	0.473932
9	0.5	0.9	2	0.612776
10	0.5	1	2	1

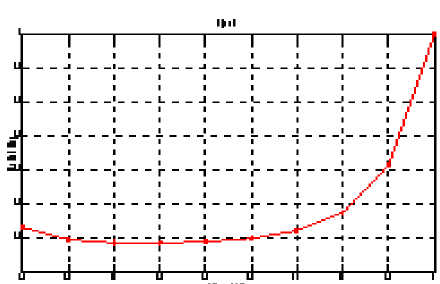


Table 5

n	μ	q	x_0	R
1	0.5	0.1	2	0.431091
1	0.5	0.2	2	0.494304
1	0.5	0.3	2	0.557516
1	0.5	0.4	2	0.620728
1	0.5	0.5	2	0.68394
1	0.5	0.6	2	0.747152
1	0.5	0.7	2	0.810364
1	0.5	0.8	2	0.873576
1	0.5	0.9	2	0.936788
1	0.5	1	2	1

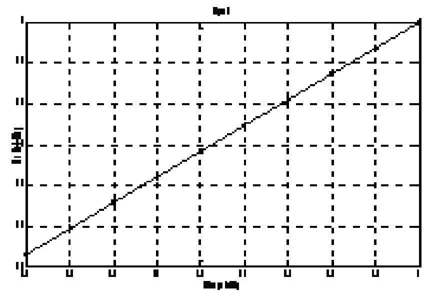
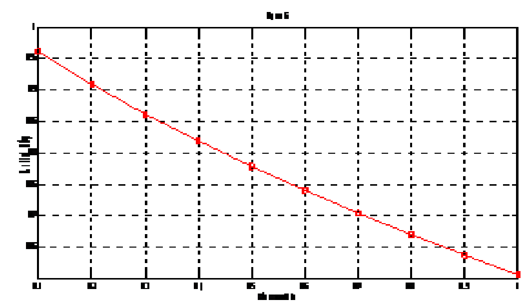


Table 6:

n	μ	q	x_0	R
1	0.5	0.9	2	0.936788
1	0.5	0.8	2	0.873576
1	0.5	0.7	2	0.810364
1	0.5	0.6	2	0.747152
1	0.5	0.5	2	0.68394
1	0.5	0.4	2	0.620728
1	0.5	0.3	2	0.557516
1	0.5	0.2	2	0.494304
1	0.5	0.1	2	0.431091
1	0.5	0	2	0.367879

Table 7:

n	μ	q	x_0	R
1	0.5	0.2	0.1	0.960984
2	0.5	0.2	0.2	0.908644
3	0.5	0.2	0.3	0.861822
4	0.5	0.2	0.4	0.819021
5	0.5	0.2	0.5	0.778872
6	0.5	0.2	0.6	0.740835
7	0.5	0.2	0.7	0.704692
8	0.5	0.2	0.8	0.670321
9	0.5	0.2	0.9	0.637628
10	0.5	0.2	1	0.606531



Case 3: Deterministic Stress and Random- independent Strength:

Table 1:

n	μ	p	q	x_0	R
1	0.1	0.1	0.9	0.2	0.99802
2	0.1	0.2	0.8	0.2	0.992095
3	0.1	0.3	0.7	0.2	0.982284
4	0.1	0.4	0.6	0.2	0.968692
5	0.1	0.5	0.5	0.2	0.951467
6	0.1	0.6	0.4	0.2	0.930799
7	0.1	0.7	0.3	0.2	0.906916
8	0.1	0.8	0.2	0.2	0.88008
9	0.1	0.9	0.1	0.2	0.85058
10	0.1	1	0	0.2	0.818731

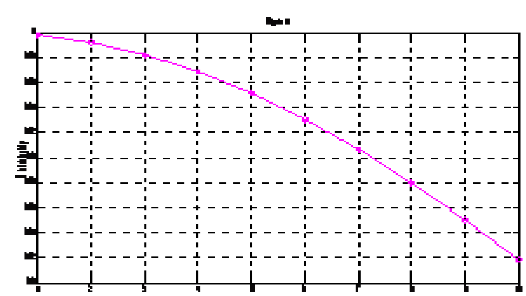


Table 2:

n	μ	P	q	x_0	R
1	0.5	0.9	0.1	0.2	0.914354
2	0.5	0.8	0.2	0.2	0.853536
3	0.5	0.7	0.3	0.2	0.813175
4	0.5	0.6	0.4	0.2	0.790437
5	0.5	0.5	0.5	0.2	0.783681
6	0.5	0.4	0.6	0.2	0.792272
7	0.5	0.3	0.7	0.2	0.816483
8	0.5	0.2	0.8	0.2	0.857505
9	0.5	0.1	0.9	0.2	0.917542
10	0.5	0	1	0.2	1

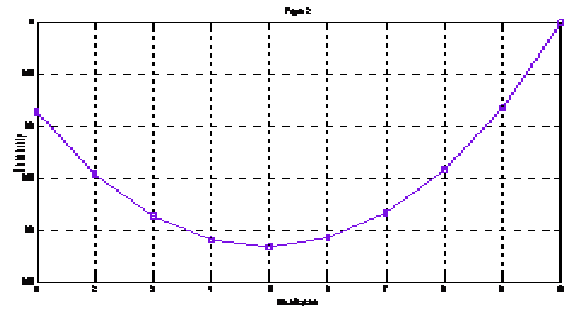


Table 3:

n	μ	P	q	x_0	R
2	0.1	0.1	0.9	0.5	0.99027
2	0.2	0.2	0.8	0.5	0.962297
2	0.3	0.3	0.7	0.5	0.918171
2	0.4	0.4	0.6	0.5	0.860242
2	0.5	0.5	0.5	0.5	0.791033
2	0.6	0.6	0.4	0.5	0.713165
2	0.7	0.7	0.3	0.5	0.629296
2	0.8	0.8	0.2	0.5	0.542073
2	0.9	0.9	0.1	0.5	0.454094
2	1	1	0	0.5	0.367879

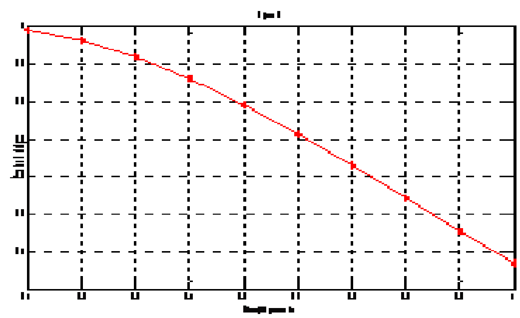


Table 4

n	μ	P	q	x_0	R
2	0.1	0.9	0.1	0.5	0.91414
2	0.2	0.8	0.2	0.5	0.853536
2	0.3	0.7	0.3	0.5	0.814498
2	0.4	0.6	0.4	0.5	0.794306
2	0.5	0.5	0.5	0.5	0.791033
2	0.6	0.4	0.6	0.5	0.803403
2	0.7	0.3	0.7	0.5	0.830662
2	0.8	0.2	0.8	0.5	0.872476
2	0.9	0.1	0.9	0.5	0.928839
2	1	0	1	0.5	1

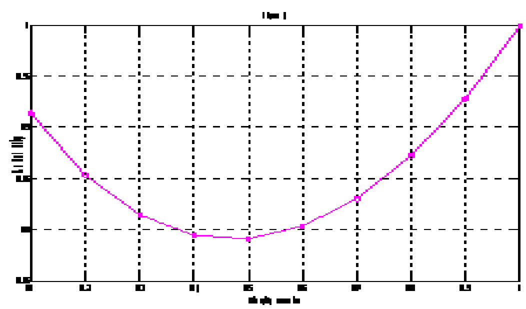


Table 5

n	μ	P	q	x_0	R
2	0.5	0.1	0.9	0.1	0.99027
2	0.5	0.2	0.8	0.2	0.962297
2	0.5	0.3	0.7	0.3	0.918171
2	0.5	0.4	0.6	0.4	0.860242
2	0.5	0.5	0.5	0.5	0.791033
2	0.5	0.6	0.4	0.6	0.713165
2	0.5	0.7	0.3	0.7	0.629296
2	0.5	0.8	0.2	0.8	0.542073
2	0.5	0.9	0.1	0.9	0.454094
2	0.5	1	0	1	0.367879

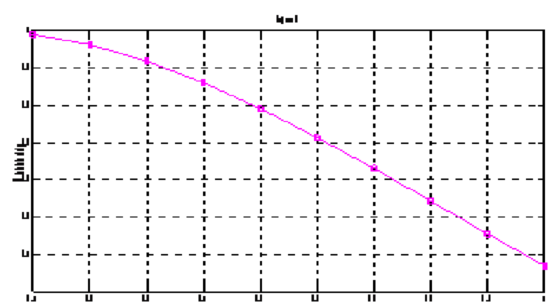


Table 6

n	µ	P	q	x_0	R
2	0.5	0.9	0.1	0.1	0.91414
2	0.5	0.8	0.2	0.2	0.853536
2	0.5	0.7	0.3	0.3	0.814498
2	0.5	0.6	0.4	0.4	0.794306
2	0.5	0.5	0.5	0.5	0.791033
2	0.5	0.4	0.6	0.6	0.803403
2	0.5	0.3	0.7	0.7	0.830662
2	0.5	0.2	0.8	0.8	0.872476
2	0.5	0.1	0.9	0.9	0.928839
2	0.5	0	1	1	1

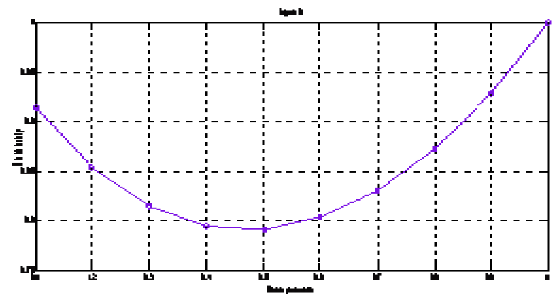


Table 7

n	µ	P	q	x_0	R
1	0.5	0.9	0.1	0.1	0.956106
2	0.5	0.8	0.2	0.2	0.853536
3	0.5	0.7	0.3	0.3	0.735081
4	0.5	0.6	0.4	0.4	0.630922
5	0.5	0.5	0.5	0.5	0.556527
6	0.5	0.4	0.6	0.6	0.518561
7	0.5	0.3	0.7	0.7	0.522378
8	0.5	0.2	0.8	0.8	0.579446
9	0.5	0.1	0.9	0.9	0.717351
10	0.5	0	1	1	1

Case 4: Random- fixed stress and deterministic Strength

Table 1

n	λ	q	y_0	R
1	0.1	0.5	2	0.590635
2	0.2	0.5	2	0.49726
3	0.3	0.5	2	0.51979
4	0.4	0.5	2	0.578754
5	0.5	0.5	2	0.643617
6	0.6	0.5	2	0.703512
7	0.7	0.5	2	0.75533
8	0.8	0.5	2	0.798892
9	0.9	0.5	2	0.835024
10	1	0.5	2	0.864797

Table 2

n	λ	q	y_0	R
2	0.1	0.5	2	0.385952
2	0.2	0.5	2	0.49726
2	0.3	0.5	2	0.588391
2	0.4	0.5	2	0.663003
2	0.5	0.5	2	0.72409
2	0.6	0.5	2	0.774104
2	0.7	0.5	2	0.815052
2	0.8	0.5	2	0.848578
2	0.9	0.5	2	0.876026
2	1	0.5	2	0.898499

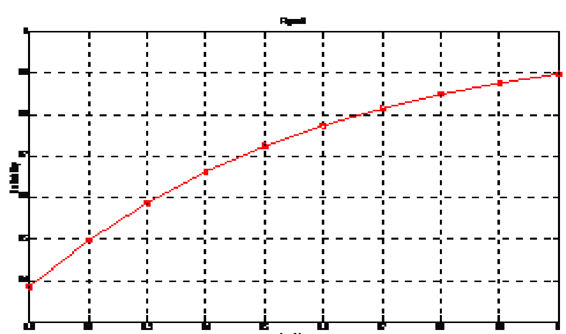


Table 3

n	λ	q	y_0	R
2	0.5	0.1	2	0.635799
2	0.5	0.2	2	0.646836
2	0.5	0.3	2	0.66523
2	0.5	0.4	2	0.690981
2	0.5	0.5	2	0.72409
2	0.5	0.6	2	0.764557
2	0.5	0.7	2	0.812381
2	0.5	0.8	2	0.867563
2	0.5	0.9	2	0.930103
2	0.5	1	2	1

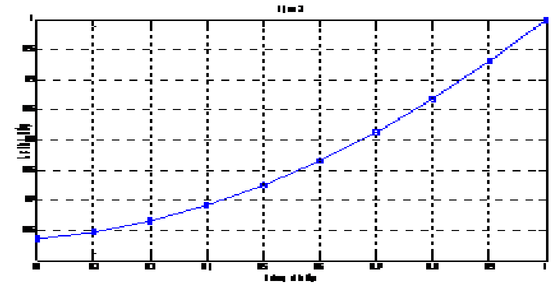


Table 4

n	λ	Q	y_0	R
2	0.5	0.9	2	0.930103
2	0.5	0.8	2	0.867563
2	0.5	0.7	2	0.812381
2	0.5	0.6	2	0.764557
2	0.5	0.5	2	0.72409
2	0.5	0.4	2	0.690981
2	0.5	0.3	2	0.66523
2	0.5	0.2	2	0.646836
2	0.5	0.1	2	0.635799
2	0.5	0	2	0.632121

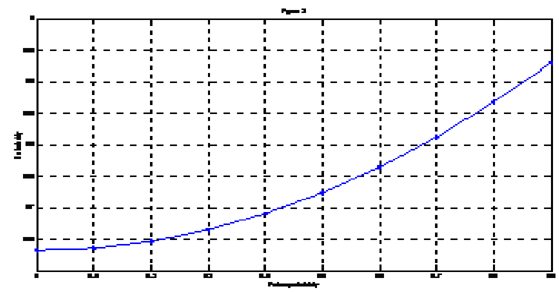
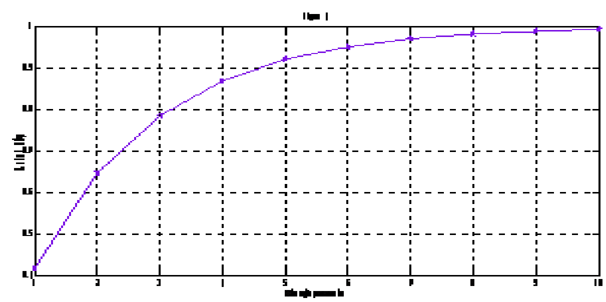


Table 5

n	λ	q	y_0	R
2	0.5	0.2	1	0.417731
2	0.5	0.2	2	0.646836
2	0.5	0.2	3	0.785795
2	0.5	0.2	4	0.870078
2	0.5	0.2	5	0.921198
2	0.5	0.2	6	0.952204
2	0.5	0.2	7	0.971011
2	0.5	0.2	8	0.982417
2	0.5	0.2	9	0.989335
2	0.5	0.2	10	0.993532



Case 5: random – fixed Stress and random- fixed Strength

Table 1

n	λ	q	μ	R
1	3	0.5	2	0.8
2	3	0.5	2	0.7
3	3	0.5	2	0.65
4	3	0.5	2	0.625
5	3	0.5	2	0.6125
6	3	0.5	2	0.60625
7	3	0.5	2	0.603125
8	3	0.5	2	0.601563
9	3	0.5	2	0.600781
10	3	0.5	2	0.600391

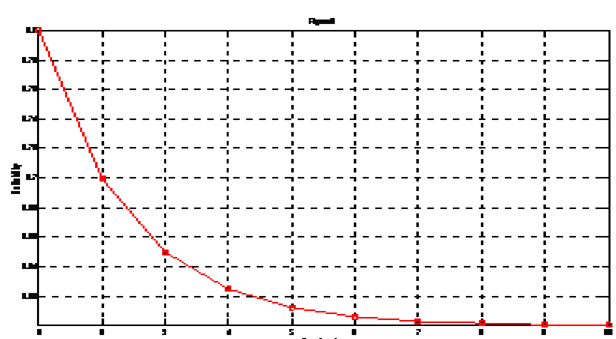


Table 2

n	λ	q	μ	R
2	1	0.5	0.5	0.75
2	2	0.5	0.5	0.85
2	3	0.5	0.5	0.892857
2	4	0.5	0.5	0.916667
2	5	0.5	0.5	0.931818
2	6	0.5	0.5	0.942308
2	7	0.5	0.5	0.95
2	8	0.5	0.5	0.955882
2	9	0.5	0.5	0.960526
2	10	0.5	0.5	0.964286

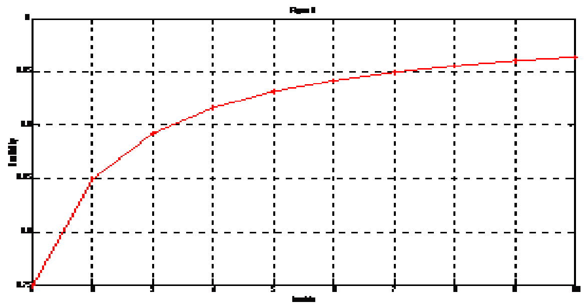


Table 3

n	λ	q	μ	R
2	0.5	0.1	0.5	0.505
2	0.5	0.2	0.5	0.52
2	0.5	0.3	0.5	0.545
2	0.5	0.4	0.5	0.58
2	0.5	0.5	0.5	0.625
2	0.5	0.6	0.5	0.68
2	0.5	0.7	0.5	0.745
2	0.5	0.8	0.5	0.82
2	0.5	0.9	0.5	0.905
2	0.5	1	0.5	1

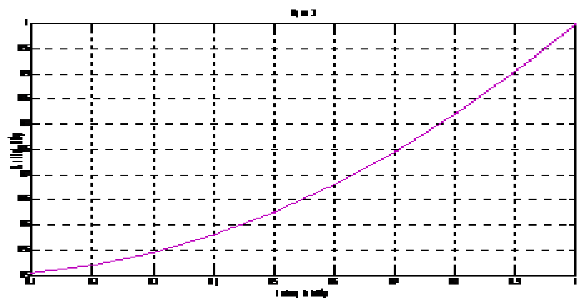
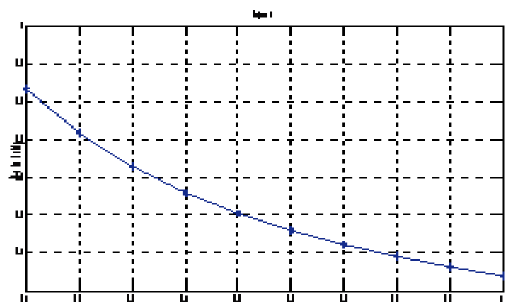


Table 4

n	λ	q	μ	R
2	0.5	0.1	0.1	0.835
2	0.5	0.1	0.2	0.717143
2	0.5	0.1	0.3	0.62875
2	0.5	0.1	0.4	0.56
2	0.5	0.1	0.5	0.505
2	0.5	0.1	0.6	0.46
2	0.5	0.1	0.7	0.4225
2	0.5	0.1	0.8	0.390769
2	0.5	0.1	0.9	0.363571
2	0.5	0.1	1	0.34



Case 6: Random- fixed Stress and random- independent Strength

Table 1

n	p	q	λ	μ	R
1	0.5	0.5	0.2	0.1	0.83333
1	0.5	0.5	0.2	0.2	0.75
1	0.5	0.5	0.2	0.3	0.7
1	0.5	0.5	0.2	0.4	0.6666
1	0.5	0.5	0.2	0.5	0.6428
1	0.5	0.5	0.2	0.6	0.625
1	0.5	0.5	0.2	0.7	0.6111
1	0.5	0.5	0.2	0.8	0.6
1	0.5	0.5	0.2	0.9	0.5909
1	0.5	0.5	0.2	1	0.5833

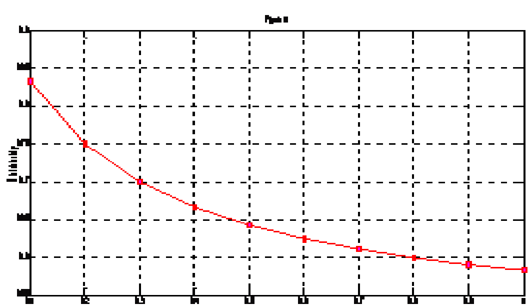


Table 2

p	q	λ	μ	n	R
0.5	0.5	0.2	0.1	1	0.8333
0.5	0.5	0.2	0.1	2	0.7083
0.5	0.5	0.2	0.1	3	0.6125
0.5	0.5	0.2	0.1	4	0.5375
0.5	0.5	0.2	0.1	5	0.4776
0.5	0.5	0.2	0.1	6	0.4291
0.5	0.5	0.2	0.1	7	0.3891
0.5	0.5	0.2	0.1	8	0.3556
0.5	0.5	0.2	0.1	9	0.3241
0.5	0.5	0.2	0.1	10	0.3030

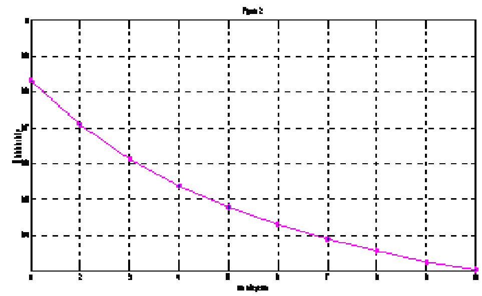
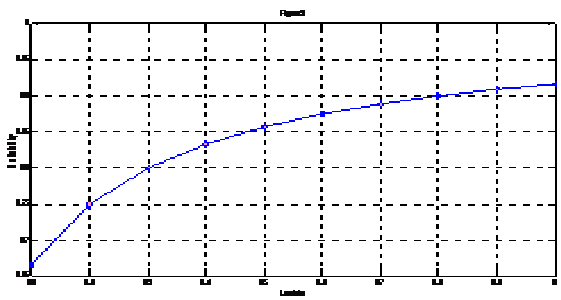


Table 3

p	q	μ	n	λ	R
0.5	0.5	0.2	1	0.1	0.6666
0.5	0.5	0.2	1	0.2	0.75
0.5	0.5	0.2	1	0.3	0.8
0.5	0.5	0.2	1	0.4	0.8333
0.5	0.5	0.2	1	0.5	0.8571
0.5	0.5	0.2	1	0.6	0.875
0.5	0.5	0.2	1	0.7	0.8889
0.5	0.5	0.2	1	0.8	0.9
0.5	0.5	0.2	1	0.9	0.9091
0.5	0.5	0.2	1	1	0.9167



Case 7: Random- independent Stress and deterministic Strength

Table 1

n	λ	p	y_0	R
1	0.2	0.2	0.5	0.819033
2	0.2	0.2	0.5	0.670814
3	0.2	0.2	0.5	0.549419
4	0.2	0.2	0.5	0.449992
5	0.2	0.2	0.5	0.368558
6	0.2	0.2	0.5	0.301861
7	0.2	0.2	0.5	0.247234
8	0.2	0.2	0.5	0.202493
9	0.2	0.2	0.5	0.165848
10	0.2	0.2	0.5	0.135835

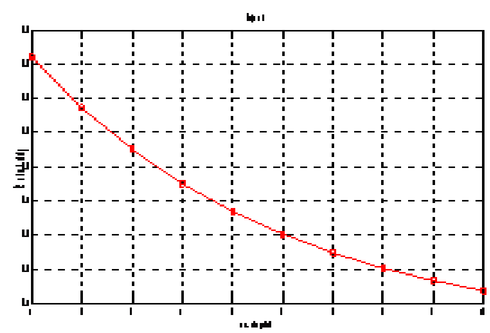


Table 2

n	λ	p	y_0	R
3	1	0.2	0.5	0.678442
3	2	0.2	0.5	0.795114
3	3	0.2	0.5	0.872007
3	4	0.2	0.5	0.920977
3	5	0.2	0.5	0.951553
3	6	0.2	0.5	0.970424
3	7	0.2	0.5	0.981991
3	8	0.2	0.5	0.989051
3	9	0.2	0.5	0.993349
3	10	0.2	0.5	0.995963

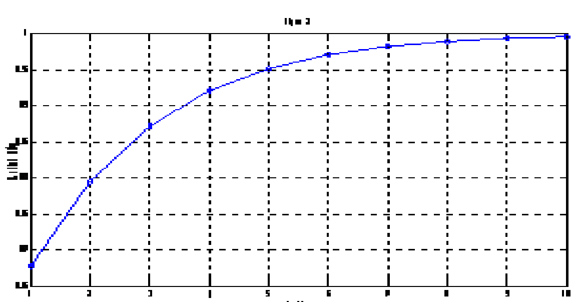


Table 3

n	λ	p	y_0	R
3	0.5	0.1	0.5	0.784083
3	0.5	0.2	0.5	0.601724
3	0.5	0.3	0.5	0.450089
3	0.5	0.4	0.5	0.326342
3	0.5	0.5	0.5	0.227651
3	0.5	0.6	0.5	0.151181
3	0.5	0.7	0.5	0.094097
3	0.5	0.8	0.5	0.053565
3	0.5	0.9	0.5	0.026752
3	0.5	1	0.5	0.010823

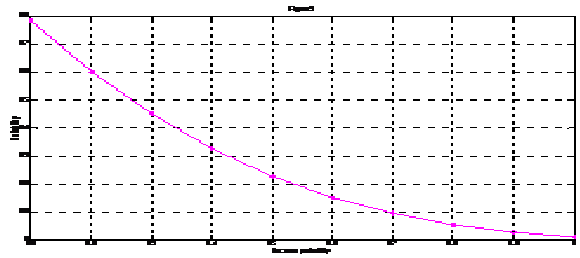
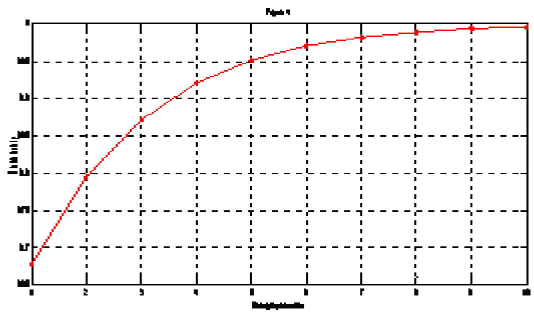


Table 4

n	λ	p	x_0	R
3	0.5	0.2	1	0.678442
3	0.5	0.2	2	0.795114
3	0.5	0.2	3	0.872007
3	0.5	0.2	4	0.920977
3	0.5	0.2	5	0.951553
3	0.5	0.2	6	0.970424
3	0.5	0.2	7	0.981991
3	0.5	0.2	8	0.989051
3	0.5	0.2	9	0.993349
3	0.5	0.2	10	0.995963



Case 8: Random- independent Stress and random- fixed Strength

Table 1

n	μ	p	λ	R
1	0.5	0.5	0.1	0.6666
1	0.5	0.5	0.2	0.75
1	0.5	0.5	0.3	0.8
1	0.5	0.5	0.4	0.8333
1	0.5	0.5	0.5	0.8571
1	0.5	0.5	0.6	0.875
1	0.5	0.5	0.7	0.8889
1	0.5	0.5	0.8	0.9
1	0.5	0.5	0.9	0.9091
1	0.5	0.5	1	0.9167

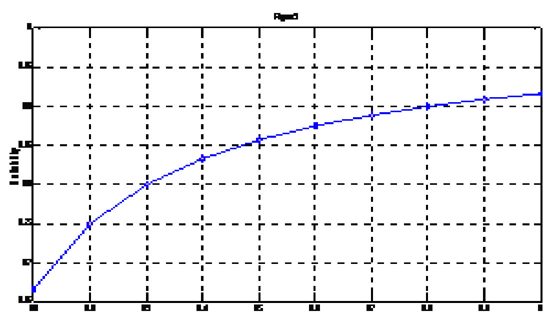


Table 2

n	λ	p	μ	R
1	0.2	0.5	0.1	0.83333
1	0.2	0.5	0.2	0.75
1	0.2	0.5	0.3	0.7
1	0.2	0.5	0.4	0.6666
1	0.2	0.5	0.5	0.6428
1	0.2	0.5	0.6	0.625
1	0.2	0.5	0.7	0.6111
1	0.2	0.5	0.8	0.6
1	0.2	0.5	0.9	0.5909
1	0.2	0.5	1	0.5833

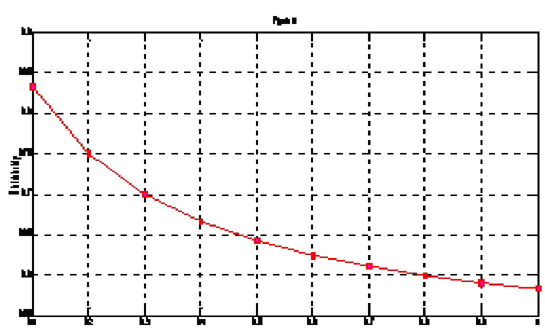
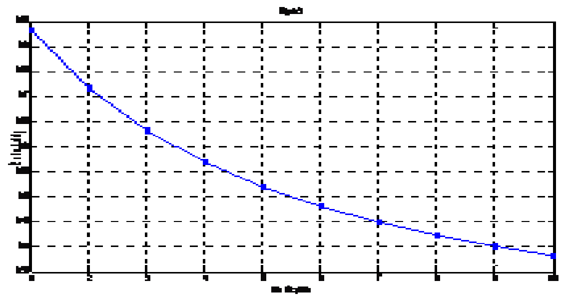


Table 3

n	λ	p	μ	R
1	0.2	0.5	0.1	0.8333
2	0.2	0.5	0.1	0.7166
3	0.2	0.5	0.1	0.6321
4	0.2	0.5	0.1	0.5688
5	0.2	0.5	0.1	0.5199
6	0.2	0.5	0.1	0.4811
7	0.2	0.5	0.1	0.4496
8	0.2	0.5	0.1	0.4234
9	0.2	0.5	0.1	0.4012
10	0.2	0.5	0.1	0.3821



Case 9: Random- independent Stress and random- independent Strength

Table 1

n	λ	μ	p	q	R
1	0.2	0.5	0.9	0.1	0.357143
2	0.2	0.5	0.8	0.2	0.183673
3	0.2	0.5	0.7	0.3	0.125
4	0.2	0.5	0.6	0.4	0.106622
5	0.2	0.5	0.5	0.5	0.109792
6	0.2	0.5	0.4	0.6	0.13281
7	0.2	0.5	0.3	0.7	0.184864
8	0.2	0.5	0.2	0.8	0.291357
9	0.2	0.5	0.1	0.9	0.513261
10	0.2	0.5	0	1	1

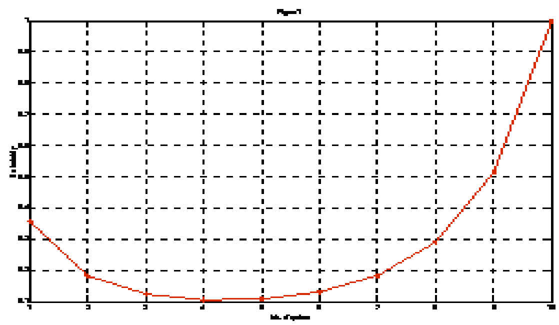


Table 2

n	λ	μ	p	q	R
1	0.5	0.1	0.1	0.9	0.983333
2	0.5	0.1	0.2	0.8	0.934444
3	0.5	0.1	0.3	0.7	0.857375
4	0.5	0.1	0.4	0.6	0.758835
5	0.5	0.1	0.5	0.5	0.647228
6	0.5	0.1	0.6	0.4	0.531441
7	0.5	0.1	0.7	0.3	0.419636
8	0.5	0.1	0.8	0.2	0.318285
9	0.5	0.1	0.9	0.1	0.231617
10	0.5	0.1	1	0	0.161506

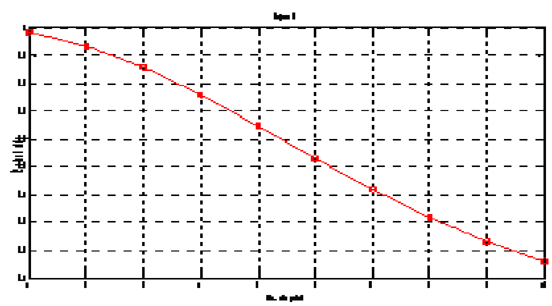


Table 3

n	λ	μ	p	q	R
2	0.1	0.5	0.9	0.1	0.0625
2	0.2	0.5	0.8	0.2	0.183673
2	0.3	0.5	0.7	0.3	0.316406
2	0.4	0.5	0.6	0.4	0.444444
2	0.5	0.5	0.5	0.5	0.5625
2	0.6	0.5	0.4	0.6	0.669421
2	0.7	0.5	0.3	0.7	0.765625
2	0.8	0.5	0.2	0.8	0.852071
2	0.9	0.5	0.1	0.9	0.929847
2	1	0.5	0	1	1

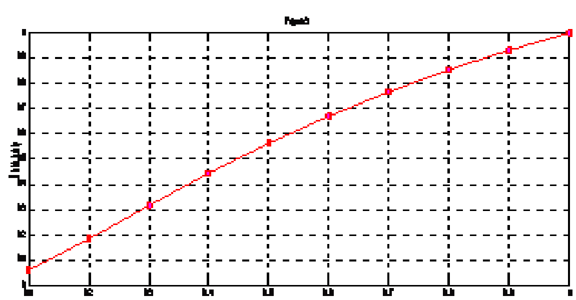


Table 4

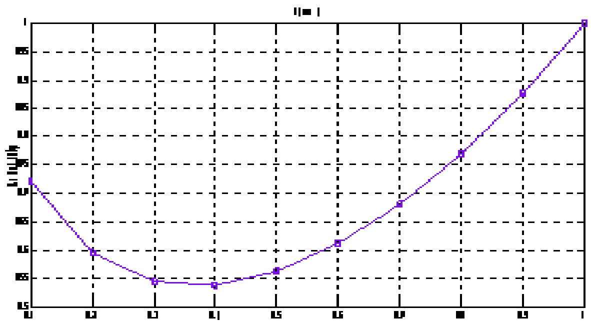
n	λ	μ	p	q	R
2	0.1	0.5	0.1	0.9	0.840278
2	0.2	0.5	0.2	0.8	0.734694
2	0.3	0.5	0.3	0.7	0.660156
2	0.4	0.5	0.4	0.6	0.604938
2	0.5	0.5	0.5	0.5	0.5625
2	0.6	0.5	0.6	0.4	0.528926
2	0.7	0.5	0.7	0.3	0.501736
2	0.8	0.5	0.8	0.2	0.47929
2	0.9	0.5	0.9	0.1	0.460459
2	1	0.5	1	0	0.444444

Table 5

n	λ	μ	p	q	R
2	0.5	0.1	0.1	0.9	0.966944
2	0.5	0.2	0.2	0.8	0.88898
2	0.5	0.3	0.3	0.7	0.787656
2	0.5	0.4	0.4	0.6	0.676049
2	0.5	0.5	0.5	0.5	0.5625
2	0.5	0.6	0.6	0.4	0.452562
2	0.5	0.7	0.7	0.3	0.350069
2	0.5	0.8	0.8	0.2	0.257751
2	0.5	0.9	0.9	0.1	0.177602
2	0.5	1	1	0	0.111111

Table 6

n	λ	μ	p	q	R
2	0.5	0.1	0.9	0.1	0.7225
2	0.5	0.2	0.8	0.2	0.595102
2	0.5	0.3	0.7	0.3	0.543906
2	0.5	0.4	0.6	0.4	0.537778
2	0.5	0.5	0.5	0.5	0.5625
2	0.5	0.6	0.4	0.6	0.61124
2	0.5	0.7	0.3	0.7	0.680625
2	0.5	0.8	0.2	0.8	0.768994
2	0.5	0.9	0.1	0.9	0.875561
2	0.5	1	0	1	1



Conclusion

Reliability is found when the number of cycles follows Binomial distribution for the following nine models:1. Deterministic stress *and* deterministic Strength, 2.Deterministic stress *and* random- fied strength, 3. Deterministic Stress *and* random-independent strength, 4. Random- fixed Stress *and* deterministic strength, 5.Random- fixed stress *and* random – fixed strength, 6. Random- fixed stress *and* random- independent Strength, 7.Random- independent Stress and deterministic Strength, 8. Random- independent Stress *and* Random- fixed Strength, 9. Random- independent Stress and random-independent Strength for random cycles Reliability computed for all the nine models when stress and strength follow exponential distribution.

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