Effects of radiation and chemical reaction on MHD mixed convective visco-elastic fluid from a vertical surface with Ohmic heating

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Abstract
The effects of radiation and chemical reaction with simultaneous heat and mass transfer on MHD mixed convective visco-elastic fluid from a vertical surface with ohmic heating has been discussed. A uniform magnetic field of strength $B_0$ is applied normal to the plate. Approximate solutions have been derived for the velocity, temperature, concentration, shearing stress, rate of heat transfer and rate of mass transfer. For solving the governing equations of motion, multi-parameter perturbation technique has been used. The fluid velocity and shearing stress are discussed graphically with the help of various flow parameters involved in the equation.

Keywords: MHD, Ohmic heating, radiation, chemical reaction.
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INTRODUCTION
The study of hydromagnetic flow with heat and mass transfer has gained significant interest owing to its applications in several biological and engineering systems when the flow is considered visco-elastic over a permeable boundary. The simultaneous heat and mass transfer problems with chemical reaction are of importance in many processes and have therefore, received a considerable amount of interest in recent years. In processes such as drying, evaporation at the surface of water body, energy transfer in wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. A reaction is said to be of the order $n$, if the reaction rate is proportional to the $n$-power of concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself. In well mixed system, the reaction is heterogeneous if it takes place at an interface and homogeneous if takes place in solution. Somess[1], Soudalgekar and Ganesan[2], Khair and Bejan[3] and Lin and Wu [4] have studied heat and mass transfer on flow past a vertical plate. Elbashbeshy[5] has discussed heat and mass transfer along a vertical plate under the combined buoyancy force effects of thermal and species diffusion in the presence of magnetic field. The problem of combined heat and mass transfer in MHD free convective flow from a vertical surface with ohmic heating and viscous dissipation has been studied by Chen [6]. Raptis et.al [7] has discussed the viscous flow over a linearly stretching sheet in...
the presence of chemical and magnetic field. The effect of the chemical reaction and radiation absorption on the unsteady free convective flow past a semi-infinite vertical permeable moving plate with heat surface and suction investigated by Ibrahim et.al[8], Hossein and Alim [9] have studied the radiation effect on free and forced convection flows past a vertical plate, including various physical aspects. Ghaly [10] has analyzed the radiation effects on a certain MHD free convection flow. Muthucumarswamy and Ganeshan [11], Muthucumarswamy [12] have studied first order homogeneous chemical reaction flow past infinite vertical plate. Sharma et al. [13, 14] have discussed radiation effect on free convective flow along a uniform moving porous vertical plate in the presence of heat source/sink and transverse magnetic field. Choudhury et al. [15, 16] have studied in this field for visco-elastic fluid. In this paper, an attempt has been made to study the effect of radiation and chemical reaction with simultaneous heat and mass transfer in MHD mixed convective visco-elastic fluid from a vertical surface with ohmic heating. The visco-elastic fluid is characterized by Walters’s liquid (Model B’ which possess both viscosity and elasticity.

The constitutive equation for Walters liquid (Model B) is

\[ \sigma_{ik} = -p \delta_{ik} + \sigma_{ik}, \]

\[ \sigma_{ik} = 2 \eta_0 \epsilon_{ik} - 2k_0 \epsilon_{ik} \]  \hspace{1cm} (1.1)

where \( \sigma_{ik} \) is the stress tensor, \( p \) is isotropic pressure, \( \delta_{ik} \) is the metric tensor of a fixed co-ordinate system \( x^i \), \( \nu_1 \) is the velocity vector, the contravariant form of \( \epsilon_{ik} \) given by

\[ \epsilon_{ik} = \frac{\partial \delta_{ik}}{\partial t} + \nu_1 \epsilon_{im} \epsilon_{jm} - \nu_1 \epsilon_{im} \epsilon_{jm} \]  \hspace{1cm} (1.2)

It is the convected derivative of the deformation rate tensor \( \epsilon_{ik} \) defined by

\[ 2\epsilon_{ik} = \nu_1 \epsilon_{ik} + k_0 \epsilon_{ik} \]

Here \( \eta_0 \) is the limiting viscosity at the small rate of shear which is given by

\[ \eta_0 = \int_0^\infty N(\tau) d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau) d\tau \]  \hspace{1cm} (1.3)

\( N(\tau) \) being the relaxation spectrum as introduced by Walters [17, 18]. This idealized model is a valid approximation of Walters liquid (Model B) taking very short memories into account so that terms involving

\[ \int_0^\infty \tau^0 N(\tau) d\tau \leq 2 \]

have been neglected.

**MATHEMATICAL FORMULATION**

We consider two dimensional mixed convection flow of an incompressible and electrically conducting visco-elastic fluid past an infinite vertical porous plate. The \( x^1 \)-axis is taken along the plate in upward direction and \( y^1 \)-axis is normal to the plate. A uniform magnetic field of strength \( B_0 \) is applied along the normal to the plate. Let \( u^1 \) and \( v^1 \) be the components of the velocity in \( x^1 \) and \( y^1 \) direction respectively, taken along and perpendicular to the plate. The equations governing the fluid flow and heat and mass transfer are as follows:

\[ \frac{\partial u^1}{\partial t} + v^1 \frac{\partial u^1}{\partial y^1} = \nu \frac{\partial^2 u^1}{\partial y^1^2} + \frac{k_0}{\rho} \left( \nu \frac{\partial^4 u^1}{\partial y^1^2} \right) - \frac{k_0}{\rho} \left( \nu^1 \frac{\partial^4 u^1}{\partial y^1^2} \right) - \frac{\sigma B_0^2}{\rho} u^1 - \nu \frac{\partial^2 u^1}{\partial y^1^2} \]

\[ \phi = C' - C_\infty, \quad Pr = \frac{\mu c_p}{k}, \quad Sc = \frac{v}{D'} \]

\[ Gr = \frac{g \beta (T_w - T_\infty)v}{v_0^2}, \quad E = \frac{v_0^2}{c_p(T_w - T_\infty)} \]

subject to boundary conditions:

\[ \begin{align*}
 y' & = 0; \quad u' = 0, T' = T_w, C' = C_w \\
 y' & \to \infty; \quad u' = 0, T' \to T_\infty, C' \to C_\infty \\
 \end{align*} \]  \hspace{1cm} (2.5)

we introduce the non-dimensional parameters as

\[ \gamma = \frac{v_0 y'}{v}, \quad u = \frac{u'}{v_0}, \quad \theta = \frac{t' - T_\infty}{T_w - T_\infty} \]

\[ \phi = \frac{C' - C_\infty}{C_w - C_\infty}, \quad Pr = \frac{\mu c_p}{k}, \quad Sc = \frac{v}{D'} \]

\[ Gr = \frac{g \beta (T_w - T_\infty)v}{v_0^2}, \quad E = \frac{v_0^2}{c_p(T_w - T_\infty)} \]

\[ F = \frac{4\gamma v'}{\rho c_p v_0^2}, \quad k = \frac{k_0 \nu_0^2}{\rho v_0^2}, \quad M = \frac{\sigma B_0^2 v'}{\rho v_0^2} \]  \hspace{1cm} (2.6)
\[ K_r = \frac{K_i y}{y^3} \]

where \( Pr \) is the Prandtl number, \( Sc \) is the Schmidt number, \( Gr \) is the Grashof number for heat transfer, \( Gm \) is the Grashof number for mass transfer, \( K \) is the permeability parameter, \( E \) is the Eckert number, \( F \) is the radiation, \( k \) is the visco-elastic parameter, \( M \) is the magnetic parameter, \( K_r \) is the chemical reaction parameter.

From (2.1) we get,
\[ \nu' = -V_0 = \text{constant} \]

Introducing the non-dimensional parameters from (2.6) in the equations (2.2) - (2.4) we get,
\[ \frac{d^2 u}{dy^2} + \frac{du}{dy} - \left( M + \frac{1}{K} \right) u + k \frac{d^3 u}{dy^3} = -(Gr \theta + Gm \phi) \]  
\[ \frac{d^2 \theta}{dy^2} + \frac{d \theta}{dy} - PrF \theta = -kPrE \frac{du}{dy} - PrE \left( \frac{du}{dy} \right)^2 - PrME u^2 \]  
\[ \frac{d^2 \phi}{dy^2} + Sc \frac{d \phi}{dy} - KrSc \phi = 0 \]

The transformed boundary conditions are
\[ y = 0 : u = 0, \theta = 1, \phi = 1 \]
\[ y = \infty: u = 0, \theta = 0, \phi = 0 \]

The physical variables \( u, \theta \) and \( \phi \) can expand in the power of the Eckert number. This can be possible physically as \( E \) for the flow of an incompressible fluid is always less than unity.

**METHOD OF SOLUTION**

We represent the velocity, temperature and concentration as
\[ u(y) = u_0(y) + E u_1(y) \]
\[ \theta(y) = \theta_0(y) + E \theta_1(y) \]
\[ \phi(y) = \phi_0(y) + E \phi_1(y) \]

Using (3.1) in equations (2.7) to (2.9) and equating the co-efficient of like powers of \( E \), neglecting the higher order terms we get,
\[ k u_0'' + u_0' + u_0' = \left( M + \frac{1}{K} \right) u_0 = -(Gr \theta_0 + Gm \phi_0) \]  
\[ k u_1'' + u_1' + u_1' = \left( M + \frac{1}{K} \right) u_1 = -(Gr \theta_1 + Gm \phi_1) \]  
\[ \theta_0' + Pr \theta_0' - PrF \theta_0 = 0 \]  
\[ \theta_1' + Pr \theta_1' - PrF \theta_1 = -(kPr u_0'' + Pr u_0'^2 + PrMu_0^2) \]  
\[ \phi_0'' + Sc \phi_0' - KrSc \phi_0 = 0 \]  
\[ \phi_1'' + Sc \phi_1' - KrSc \phi_1 = 0 \]

with relevant boundary conditions:
\[ y = 0, u_0 = 0, u_1 = 0, \theta_0 = 1, \theta_1 = 0, \phi_0 = 0, \phi_1 = 0 \]
\[ y = \infty, u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0, \phi_0 = 0, \phi_1 = 0 \]

Solving equations (3.4), (3.6) and (3.7) with the help of boundary condition (3.8), we get
\[ \theta_0 = e^{-a_3 y}, \phi_0 = e^{-a_2 y} \]

\[ \phi_1 = 0 \]

Thus, \( \phi = e^{-a_2 y} \)

Now, to solve equations (3.2), (3.3) and (3.5), we use perturbation technique
\[ u_0 = u_{00} + ku_{01} \]
\[ u_1 = u_{10} + ku_{11} \]
\[ \theta_1 = \theta_{10} + k\theta_{11} \]

as \( k \ll 1 \) for small shear rate as given by Nowinski and Ismail [19].

Solving (3.2), (3.3) and (3.5), subject to boundary conditions (3.9) and equating the co-efficient of like powers of \( k \), neglecting higher order terms we get
\[ u_{00}'' + u_{00}' = \left( M + \frac{1}{K} \right) u_{00} = -Gr e^{-a_3 y} - Gm e^{-a_2 y} \]  
\[ u_{01}'' + u_{01}' = \left( M + \frac{1}{K} \right) u_{01} = -u_{00}'' \]
The purpose of this study is to bring out the effects of radiation and chemical reaction on mixed MHD convective visco-elastic fluid flow. The non-dimensional rate of heat transfer in the form of Nusselt number is given by

\[ \text{Nu} = \frac{h}{\kappa} = \frac{h}{\kappa} \]

The non-dimensional rate of mass transfer in terms of Sherwood number is given by

\[ \text{Sh} = \frac{\frac{d\phi}{dy}}{\phi} = \frac{\frac{d\phi}{dy}}{\phi} \]

The modified boundary conditions are:

\[ y = 0: u_0 = 0, \theta_0 = 0 \]
\[ y \to \infty: u_0 = 0, \theta_0 = 0 \]
\[ u_1 = 0, \theta_1 = 0, \theta_1 = 0 \]

Solving equations (3.10) to (3.15) with boundary condition (3.16) we get,

\[ u_0 = A_2 e^{-\alpha_3y} + A_3 e^{-\alpha_4y} - A_2 e^{-\alpha_2y} + A_1 e^{-\alpha_1y} - A_1 e^{-\alpha_2y} + k(A_2 e^{-\alpha_2y} - A_3 e^{-\alpha_1y} - A_1 e^{-\alpha_2y}) \]
\[ u_1 = B_2 e^{-\alpha_1y} + B_3 e^{-\alpha_2y} + B_4 e^{-\alpha_3y} + B_5 e^{-\alpha_4y} + B_6 e^{-\alpha_5y} + B_7 e^{-\alpha_6y} + B_8 e^{-\alpha_7y} + B_9 e^{-\alpha_8y} + B_{10} e^{-\alpha_9y} + B_{11} e^{-\alpha_{10}y} \]

The velocity profile is given by

\[ u = u_0 + E u_1 \]

Knowing the velocity field, the shearing stress can be obtained, which in non-dimensional form is given by

\[ \sigma = -A_2 \alpha_6 - A_1 \alpha_5 + A_2 \alpha_4 + A_1 \alpha_3 + A_1 \alpha_2 + A_3 \alpha_2 + A_2 \alpha_1 + A_3 \alpha_1 + \alpha_2 \alpha_2 \]

The non-dimensional rate of heat transfer in the form of Nusselt number is given by

\[ N_u = \frac{d\theta}{dy} = \theta_0 + E \theta_1 \]

The non-dimensional rate of mass transfer in terms of Sherwood number is given by

\[ S_h = \frac{d\phi}{dy} = \phi_0 + E \phi_1 \]

The constants are not given due to sake of brevity.

**RESULTS AND DISCUSSION**

The purpose of this study is to bring out the effects of radiation and chemical reaction on mixed MHD convective visco-elastic fluid flow. The effects of visco-elastic parameter is exhibited through the non-dimensional parameter. The non-zero values of the parameter \( k \) characterize the visco-elastic fluid and \( k = 0 \) represent the Newtonian fluid flow phenomenon. Figures 1 - 8 depict the fluid velocity against \( y \) for various flow parameters viz. Prandtl number \( Pr \), Grashof number for heat transfer \( Gr \), Grashof number for mass transfer \( Gm \), radiation parameter \( F \), chemical reaction parameter \( Kr \), Schmidt number \( Sc \), permeability parameter \( K \) of physical interest Eckert number \( E \), Magnetic parameter \( M \) as the Prandtl number \( Pr \) helps to study the simultaneous effects of momentum and thermal diffusion in the fluid flow, the Grashof number \( Gr \) characterizes the free convection parameter of heat transfer and \( Gm \) characterizes the free convection parameter for mass transfer. The Schmidt number \( Sc \) studies the combined effect of momentum and mass diffusion, Eckert number \( E \) helps to study the dissipation of mechanical energy into...
thermal energy due to the presence of viscosity, Magnetic parameter $M$ estimates the Lorentz force in the fluid flow region etc. In our study the results are discussed for the flow past an externally cooled plate ($Gr > 0$). From the figures, it is observed that the fluid velocity accelerates rapidly near the plate and then decelerates in both Newtonian and non-Newtonian fluid flow mechanism. Also, the growth of visco-elasticity depict the enhancement of fluid velocity at all points in the fluid flow regions in comparison with the Newtonian fluid flow phenomenon for variation of other flow parameters. From the practical point of view, it is very important to know the shearing stress and consequently, the viscous drag in non-dimensional form.

**Figure 1**: Fluid velocity $u$ against $y$ for $Pr=8$, $Gm=2$, $Gr=3$, $Sc=5$, $K=1$, $F=4$, $M=2$, $Kr=1$

**Figure 2**: Fluid velocity $u$ against $y$ for $Pr=5$, $Gm=2$, $Gr=3$, $Sc=5$, $K=1$, $F=4$, $M=2$, $Kr=1$

**Figure 3**: Fluid velocity $u$ against $y$ for $Pr=8$, $Gm=5$, $Gr=3$, $Sc=5$, $K=1$, $F=4$, $M=2$, $Kr=1$

**Figure 4**: Fluid velocity $u$ against $y$ for $Pr=8$, $Gm=2$, $Gr=5$, $Sc=5$, $K=1$, $F=4$, $M=2$, $Kr=1$

**Figure 5**: Fluid velocity $u$ against $y$ for $Pr=8$, $Gm=2$, $Gr=3$, $Sc=5$, $K=3$, $F=4$, $M=2$, $Kr=1$

**Figure 6**: Fluid velocity $u$ against $y$ for $Pr=8$, $Gm=2$, $Gr=3$, $Sc=5$, $K=1$, $F=3$, $M=2$, $Kr=1$

**Figure 7**: Fluid velocity $u$ against $y$ for $Pr=8$, $Gm=2$, $Gr=3$, $Sc=5$, $K=1$, $F=4$, $M=2$, $Kr=0.6$

**Figure 8**: Fluid velocity $u$ against $y$ for $Pr=8$, $Gm=2$, $Gr=3$, $Sc=5$, $K=1$, $F=4$, $M=2$, $Kr=1$
Figures 9 -16 reveal the variation of shearing stress against different flow parameters. From the graphical illustrations it can be remarked that the shearing stress exhibits a diminishing nature with the increasing values of Pr and tI number Pr (figure 9), radiation parameter F (figure 12), Magnetic parameter M (figure 13), chemical reaction parameter Kr (figure 14), Schmidt number Sc (figure 15) but reverse patterns have been demonstrated for rising values of thermal Grashof number Gr (figure 10), mass Grashof number Gm (figure 11) and permeability parameter K (figure 16) for variations of other flow parameters. Again, the figures demonstrate that the growth of visco-elasticity accelerates the shearing stress or viscous drag with the rising values of the flow parameters r, Gr, Gm in the fluid flow region but decelerates for permeability parameter K, Schmidt number Sc, magnetic parameter M, chemical reaction parameter Kr in compared to simple Newtonian fluid. The rate of heat transfer in the form of Nusselt number and the rate of mass transfer in the form of Sherwood number are not significantly affected by the variation of visco-elastic parameter.

CONCLUSIONS
The visco-elastic effects on radiation and chemical reaction with simultaneous heat and mass transfer in MHD mixed convective fluid from a vertical surface with ohmic heating has been presented. The conclusions of the study are as follows:

- The fluid velocity accelerates rapidly near the plate but then decelerates in both Newtonian and non-Newtonian fluid flow phenomenon.
• The growth of visco-elasticity shows an enhancement of the shearing stress at the plate with the rising values of Grash of number for heat transfer, Grash of number for mass transfer, radiation parameter, Pr and tl number.
• The enhancements of visco-elastic parameter with rising values of permeability parameter, Schmidt number, and magnetic parameter depict a diminishing nature of the shearing stress at the plate.
• The temperature and concentration fields are not significantly affected by the visco-elastic parameter.

REFERENCES

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