Research Article

Profit-function of two-dissimilar cold standby units under the influence of electrical fluctuations and EM vibrations

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Abstract

Introduction: Reliability is a measure of how well a system performs or meets its design requirements. It is hence the prime concern of all scientists and engineers engaged in developing such a system. It a large number of papers on a two unit standby redundant systems have appeared because of the large number of parameters governing system behaviour and the complex environment with uncertainties. In this paper we have taken failure which is caused due to **EM Vibrations (Electro Magnetic vibrations)** and other failure due to **electrical fluctuations** such as **voltage fluctuations**. When the main unit fails then cold standby system becomes operative. **Voltage fluctuations or electromagnetic vibrations** cannot occur simultaneously in both the units and after failure the unit undergoes very costly repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters of interest and profit analysis have been evaluated.
Keywords: Profit-function, electrical fluctuations, EM vibrations.

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INTRODUCTION

The mathematical theory of reliability has grown out of the demands of modern technology and particularly out of experiences in World War II (1939-1945). In this paper we have taken failure which is caused due to **EM Vibrations (Electro Magnetic vibrations)**. and other failure due to **electrical fluctuations** such as **voltage fluctuations**. When the main operative unit fails then cold standby system becomes operative. **Electrical fluctuations** cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of very high cost in case of failure due to **electrical fluctuations** immediately. Failure due to EM Vibrations may be destructive. The repair is done on the basis of first fail first repaired.

ASSUMPTIONS

1. $F_1(t)$ and $F_2(t)$ are the failure time dirtributions due to **EM Vibrations** and **electrical fluctuations** and repair is of two types -Type -I ,Type-II with repair time distributions as $G_1(t)$ and $G_2(t)$ respectively .

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2. The **electrical fluctuations and EM Vibrations** are non-instantaneous and they cannot occur simultaneously in both the units.
3. Whenever the **electrical fluctuations** or **EM Vibrations** occur within specified limit of the unit, it works as normal as before . But as soon as there occur electrical fluctuations or EM Vibrations of magnitude beyond specified limit of the unit the operation of the unit stops automatically.
4. The repair starts immediately after the **electrical fluctuations** or **EM Vibrations** of beyond specified limit of the unit are over and works on the principle of first come first served basis.
5. The repair facility does no damage to the units and after repair units are as good as new.
6. The switches are perfect and instantaneous.
7. All random variables are mutually independent.
8. When both the units fail, we give priority to operative unit for repair.
9. The other failure mode Is due to some EM Vibrations.
10. Repairs are perfect and failure of a unit is detected immediately and perfectly.
11. The system is down when both the units are non-operative.

SYMBOLS FOR STATES OF THE SYSTEM

$F_1(t)$ and $F_2(t)$ are the failure rates due to **EM Vibrations** and **electrical fluctuation**s

$G_1(t)$, $G_2(t)$ – repair time distribution due to Type -I, Type-II respectively

Superscripts O, CS, ELF, EMVF

Operative, Cold Standby, failure due to Electrical fluctuations, failure due to EM Vibrations respectively

Subscripts: nelf, elf, emv, ur, wr, uR

No electrical fluctuations, electrical fluctuations, EM Vibrations, under repair, waiting for repair, under repair continued from previous state respectively

Up states: 0, 1, 2;

Down states: 3, 4

Regeneration point: 0, 1, 2

Notations

$M_i(t)$ System having started from state I is up at time t without visiting any other regenerative state

$A_i(t)$ state is up state as instant t

$R_i(t)$ System having started from state I is busy for repair at time t without visiting any other regenerative state.

$B_i(t)$ The server is busy for repair at time t.

$H_i(t)$ Expected number of visits by the server for repairing given that the system initially starts from regenerative state i

By electrical fluctuations or EM Vibrations we mean electrical fluctuations or EM Vibrations beyond the specified limit

STATES OF THE SYSTEM

0(O_{nelf} , CS_{nelf})

One unit is operative and the other unit is cold standby and there are no Electrical fluctuations in both the units.

1($ELF_{elf, ur}$, O_{nelf})

The operating unit fails due to Electrical fluctuations and is under repair immediately of very costly Type- I and standby unit starts operating with no Electrical fluctuations.

2($EMVF_{nelf,ev, ur}$, O_{nelf})

The operative unit fails due to EM Vibrations and undergoes repair of type II and the standby unit becomes operative with no electrical fluctuations.

3($ELF_{elf,uR}$, $EMVF_{nelf ,ev,wr}$)

The first unit fails due to Electrical fluctuations and under very costly Type-! Repair is continued from state 1 and the other unit fails due to EM Vibrations and is waiting for repair of Type -II.

4($ELF_{elf,uR}$, $ELF_{elf,wr}$)

The one unit fails due to Electrical fluctuations is continues under repair of very costly Type - I from state 1 and the other unit also fails due to Electrical fluctuations. is waiting for repair of very costly Type- I.

5($EMVF_{nelf, ev, uR}$, $ELF_{elf, wr}$)

The operating unit fails due to EM Vibrations and under repair of Type - II continues from the state 2 and the other unit fails due to Electrical fluctuations is waiting for repair of very costly Type- I.

6($EMVF_{nelf,ev,uR}$, $EMVF_{nelf,ev,wr}$)

The operative unit fails due to EM Vibrations and under repair continues from state 2 of Type –II and the other unit is also failed due to EM Vibrations and is waiting for repair of Type-II and there is no Electrical fluctuations.

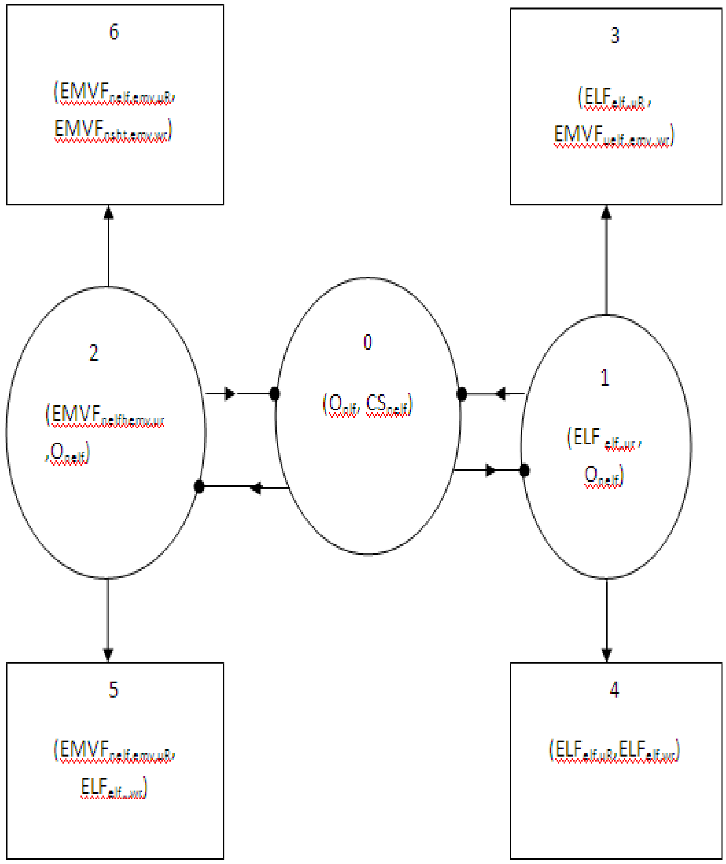

Figure 1: The State Transition Diagram

● regeneration point -states 0,1,2

○ Up State

□ Down State

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$p_{01} = \int_0^\infty \overline{F}_2(t)dF_1(t)$, $p_{02} = \int_0^\infty \overline{F}_1(t)dF_2(t)$

$p_{10} = \int_0^\infty \overline{F}_2(t)dG_1(t)$, $p_{13} = p_{11}^{(3)} = p_{11}^{(4)} = \int_0^\infty \overline{G}_1(t)dF_2(t)$

$p_{25} = p_{22}^{(5)} = p_{22}^{(6)} = \int_0^\infty \overline{G}_2(t)dF_1(t)$

clearly

$p_{01} + p_{02} = 1$,

$p_{10} + p_{13} = (p_{11}^{(3)}) + p_{14} = (p_{11}^{(4)}) = 1$,

$p_{20} + p_{25} = (p_{22}^{(5)}) + p_{26} = (p_{22}^{(6)}) = 1$ (1)

And mean sojourn time are

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt$ (2)

Mean Time To System Failure

$\varnothing_0(t) = Q_{01}(t)[s]\ \varnothing_1(t) + Q_{02}(t)[s]\ \varnothing_2(t)$

$\varnothing_1(t) = Q_{10}(t)[s]\ \varnothing_0(t) + Q_{13}(t) + Q_{14}(t)$

$\varnothing_2(t) = Q_{20}(t)[s]\ \varnothing_0(t) + Q_{25}(t) + Q_{26}(t)$ (3-5)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-5) and solving for

$\o_0^*(s) = N_1(s) / D_1(s)$ (6)

Where

$N_1(s) = Q_{01}^*[Q_{13}^*(s) + Q_{14}^*(s)] + Q_{02}^*[Q_{25}^*(s) + Q_{26}^*(s)]$

$D_1(s) = 1 - Q_{01}^* Q_{10}^* - Q_{02}^* Q_{20}^*$

Making use of relations (1) and (2) it can be shown that $\o_0^*(0) = 1$, which implies that $\o_0^*(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \frac{d}{ds} \O_0^{*}(s) \Big|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$$= (\mu_0 + p_{01} \mu_1 + p_{02} \mu_2) / (1 - p_{01} p_{10} - p_{02} p_{20})$$

where

$\mu_0 = \mu_{01} + \mu_{02}$, $\mu_1 = \mu_{10} + \mu_{13} + \mu_{14}$

$\mu_2 = \mu_{20} + \mu_{25} + \mu_{26}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state I is up at time t without making any other regenerative state belonging to E. By probabilistic arguments, we have

The value of $M_0(t)$, $M_1(t)$, $M_2(t)$ can be found easily.

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{11}^{(3)}(t)[c]A_1(t) + q_{11}^{(4)}(t)[c]A_1(t)$,

$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + [q_{22}^{(5)}(t)[c] + q_{22}^{(6)}(t)] [c]A_2(t)$ (7-9)

Taking Laplace Transform of eq. (7-9) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (10)

Where

$N_2(s) = \hat{M}_0(s)(1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s)) (1- \hat{q}_{22}^{(5)}(s) - \hat{q}_{22}^{(6)}(s)) + \bar{q}_{01}(s)\hat{M}_1(s)$

$[1- \hat{q}_{22}^{(5)}(s) - \hat{q}_{22}^{(6)}(s)] + \hat{q}_{02}(s) \hat{M}_2(s)(1- \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))$

$D_2(s) = (1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s)) \{ 1 - \hat{q}_{22}^{(5)}(s) - \hat{q}_{22}^{(6)}(s))[1-(\hat{q}_{01}(s) \hat{q}_{10}(s))(1-$

$\hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))]$

The steady state availability

$$A_0 = \lim_{t\to\infty} [A_0(t)] = \lim_{s\to 0} [s\, \hat{A}_0(s)] = \lim_{s\to 0} \frac{s\, N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0} \frac{N_2(s) + s\, N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)}$$ (11)

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^{\infty} A_0(z)\,dz$ So that $\bar{\lambda}_u(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{S D_2(S)}$ (12)

The expected down time of the system in (0,t] is

$\lambda_d(t) = t - \lambda_u(t)$ So that $\bar{\lambda}_d(s) = \frac{1}{s^2} - \bar{\lambda}_u(s)$ (13)

The expected busy period of the server when there is failure due to Electrical Fluctuations in (0,t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$

$R_1(t) = S_1(t) + q_{01}(t)[c]R_1(t) + [q_{11}^{(3)}(t) + q_{11}^{(4)}(t)[c]R_1(t)$,

$R_2(t) = q_{20}(t)[c]R_0(t) + [q_{22}^{(6)}(t)+q_{22}^{(5)}(t)][c]R_2(t)$ (14-16)

Taking Laplace Transform of eq. (14-16) and solving for $\bar{R}_0(s)$

$\bar{R}_0(s) = N_3(s) / D_3(s)$ (17)

Where

$N_3(s) = \hat{q}_{01}(s) \hat{S}_1(s)$ and

$D_3(s) = (1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s)) - \hat{q}_{01}(s)$ is already defined.

In the long run, $R_0 = \frac{N_3(0)}{D_3'(0)}$ (18)

The expected period of the system under Eectrical Fluctuations. in (0,t] is

$\lambda_{rv}(t) = \int_0^{\infty} R_0(z)dz$ So that $\bar{\lambda}_{rv}(s) = \frac{\hat{R}_0(s)}{s}$

The expected Busy period of the server when there is EM Vibrations in (0,t]

$B_0(t) = q_{01}(t)[c]B_1(t) + q_{02}(t)[c]B_2(t)$
$B_1(t) = q_{01}(t)[c]B_1(t) + [q_{11}^{(3)}(t) + q_{11}^{(4)}(t)]\,[c]B_1(t)$,
$B_2(t) = T_2(t) + q_{02}(t)[c]\,B_2(t) + [q_{22}^{(5)}(t) + q_{22}^{(6)}(t)]\,[c]B_2(t)$
$T_2(t) = e^{-\lambda_1 t} G_2(t)$ (19- 21)

Taking Laplace Transform of eq. (19-21) and solving for $\bar{B}_0(s)$

$\bar{B}_0(s) = N_4(s) / D_2(s)$ (22)

Where

$N_4(s) = \hat{q}_{02}(s)\,\hat{T}_2(s))$

And $D_2(s)$ is already defined.

In steady state, $B_0 = \frac{N_4(0)}{D_2'(0)}$ (23)

The expected busy period of the server for repair in (0,t] is

$\lambda_{ru}(t) = \int_0^{\infty} B_0(z)dz$ So that $\bar{\lambda}_{ru}(s) = \frac{\hat{B}_0(s)}{s}$ (24)

The expected number of visits by the repairman for repairing the different units in (0,t]

$H_0(t) = Q_{01}(t)[s][1 + H_1(t)] + Q_{02}(t)[s][1 + H_2(t)]$
$H_1(t) = Q_{10}(t)[s]H_0(t)] + [Q_{11}^{(3)}(t) + Q_{11}^{(4)}(t)]\,[s]H_1(t)$,
$H_2(t) = Q_{20}(t)[s]H_0(t) + [Q_{22}^{(5)}(t) + Q_{22}^{(6)}(t)]\,[c]H_2(t)$ (25-27)

Taking Laplace Transform of eq. (25-27) and solving for $H_0^*(s)$

$H_0^*(s) = N_6(s) / D_3(s)$ (28)

In the long run, $H_0 = \frac{N_6(0)}{D_3'(0)}$ (29)

COST BENEFIT ANALYSIS

The cost-benefit function of the system considering mean up-time, expected busy period of the system under electrical fluctuations when the units stops automatically, expected busy period of the system under EM vibrations , expected number of visits by the repairman for unit failure.

The expected total cost-benefit incurred in (0,t] is

C(t) = Expected total revenue in (0,t] - expected total repair cost in (0,t] failure due to Electrical fluctuations

- expected total repair cost due to EM Vibrations for repairing the units in (0,t]
- expected busy period of the system under Electrical fluctuations when the units automatically stop in (0,t]
- expected number of visits by the repairman for repairing of the units in (0,t]

The expected total cost per unit time in steady state is

$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$

$= K_1A_0 - K_2R_0 - K_3B_0 - K_4H_0$

Where

K_1: revenue per unit up-time,
K_2: cost per unit time for which the system is under repair of type- I
K_3: cost per unit time for which the system is under repair of type-II
K_4: cost per visit by the repairman for units repair.

CONCLUSION

After studying the system, we have analysed graphically that when the failure rate due to Electrical fluctuations or failure rate due to EM vibrations increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

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