

# Gain - function of two non-identical warm standby system subject to failure due to voltage fluctuations and steep high acoustics with switch failure

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## Abstract

**Introduction:** Two-unit standby system subject to environmental conditions such as shocks, change of weather conditions etc. have been discussed in reliability literature by several authors due to significant importance in defence, industry etc. In the present paper we have taken two-non-identical warm standby system with failure time distribution as exponential and repair time distribution as general. The Role of voltage fluctuations and steep high acoustics under which the system operates plays significant role on its working. We are considering system under (i) voltage fluctuations and (ii) Steep high acoustics causing different types of failure requiring different types of repair facilities. Using semi Markov regenerative point technique we have calculated different reliability characteristics such as MTSF, reliability of the system, availability analysis in steady state, busy period analysis of the system under repair, expected number of visits by the repairman in the long run and gain-function and graphs are drawn.

**Keyword:** warm standby, voltage fluctuations, steep high acoustics, switches failure.

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Received Date: 08/09/2014 Accepted Date: 18/09/2014

## Access this article online

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DOI: 18 September  
2014

## INTRODUCTION

The mathematical theory of reliability has grown out of the demands of modern technology and particularly out of experiences in World War II (1939-1945) with complex military systems although the concept of reliability is as old as man himself.

## Assumptions

1. The failure time distribution is exponential whereas the repair time distribution is arbitrary of two non-identical units.
2. The repair facility is of four types :
  - Type I, II repair facility
    - when failure due to voltage fluctuations and steep high acoustics of first unit occurs respectively and
  - Type III, IV repair facility
    - when failure due to voltage fluctuations and steep high acoustics of the second unit occurs respectively.

3. The repair starts immediately upon failure of units and the repair discipline is FCFS.
4. The repairs are perfect and start immediately as soon as the voltage fluctuations and steep high acoustics of the system becomes normal. The voltage fluctuations and acoustics in both the units do not go steep high simultaneously.
5. The failure of a unit is detected immediately and perfectly.
6. The switches are perfect and instantaneous.
7. All random variables are mutually independent.

## SYMBOLS FOR STATES OF THE SYSTEM

**Superscripts:** O, WS, SO, FVF, FHA SFO

Operative, Warm Standby, Stops the operation, failure due to voltage fluctuations, failure due to steep High acoustics, Switch failed but operable respectively

**Subscripts:** nvf, vf, sha, ur, wr, uR

No voltage fluctuations, voltage fluctuations, steep high acoustics, under repair, waiting for repair, under repair continued respectively

Up states: 0, 1, 2, 9;

Down states: 3, 4, 5, 6, 7, 8, 10, 11

Regeneration point: 0, 1, 2, 4, 7, 10

### States of the System

**0( $O_{nvf}$ ,  $WS_{nvf}$ )**

One unit is operative and the other unit is warm standby and there are no voltage fluctuations and no steep high acoustics in both the units.

**1( $SO_{nvf}$ ,  $O_{nvf}$ )**

The operation of the first unit stops automatically due to voltage fluctuations and warm standby unit's starts operating with no voltage fluctuations.

**2( $FVF_{ur}$ ,  $O_{nvf}$ )**

The first unit fails and undergoes repair after the voltage fluctuations are over and the other unit continues to be operative with no voltage fluctuations.

**3( $FVF_{uR}$ ,  $SO_{uvf}$ )**

The repair of the first unit is continued from state 2 and the operation of second unit stops automatically due to voltage fluctuations.

**4( $FVF_{ur}$ ,  $SO_{sha}$ )** The first unit fails and undergoes repair after the voltage fluctuations are over and the other unit also stops automatically due to steep high acoustics.

**5( $FVF_{uR}$ ,  $FHA_{sha,wr}$ )** The repair of the first unit is continued from state 4 and the other unit is failed due to steep high acoustics in it and is waiting for repair.

**6( $O_{nvf}$ ,  $FHA_{ur}$ )** The first unit becomes operative with no voltage fluctuations and the second unit is failed due to high acoustics is under repair.

**7( $SO_{nsha}$ ,  $SFO_{nvf,ur}$ )**

The operation of the first unit stops automatically due to steep high acoustics and during switchover to the second unit switch fails and undergoes repair and there is no voltage fluctuations.

**8( $FHA_{sha,wr}$ ,  $SFO_{nvf,uR}$ )**

The repair of failed switch is continued from state 7 and the first unit is failed after steep high acoustics is waiting for repair.

**9( $O_{nvf}$ ,  $SO_{sha}$ )**

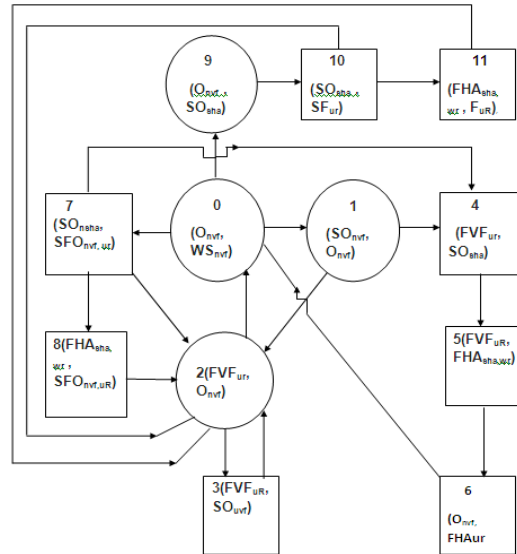
The first unit is operative with no voltage fluctuations and the operation of warm standby second unit is stopped automatically due to steep high acoustics.

**10( $SO_{sha}$ ,  $SF_{ur}$ )**

The operation of the first unit stops automatically due to steep high acoustics and the second unit switch fails and undergoes repair after the steep high acoustics is over.

**11( $FHA_{sha,wr}$ ,  $F_{uR}$ )**

The repair of the second unit is continued from state 10 and the first unit is failed due to steep high acoustics is waiting for repair.



☐ Up State

☐ Down State

$$s=0$$

$$= (\mu_0 + p_{01} \mu_1 + p_{01} p_{12} \mu_2 + p_{09} \mu_9) / (1 - p_{01} p_{12} p_{20})$$

where

$$\mu_0 = \mu_{01} + \mu_{07} + \mu_{09}, \mu_1 = \mu_{12} + \mu_{14}, \mu_2 = \mu_{20} + \mu_{22}^{(3)}, \mu_9 = \mu_{9,10}$$

## AVAILABILITY ANALYSIS

Let  $M_i(t)$  be the probability of the system having started from state  $I$  is up at time  $t$  without making any other regenerative state belonging to  $E$ . By probabilistic arguments, we have

The value of  $M_0(t)$ ,  $M_1(t)$ ,  $M_2(t)$ ,  $M_4(t)$  can be found easily.

The point wise availability  $A_i(t)$  have the following recursive relations

$$\begin{aligned} A_0(t) &= M_0(t) + q_{01}(t)[c]A_1(t) + q_{07}(t)[c]A_7(t) + q_{09}(t)[c]A_9(t) \\ A_1(t) &= M_1(t) + q_{12}(t)[c]A_2(t) + q_{14}(t)[c]A_4(t), A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + q_{22}^{(3)}(t)[c]A_2(t) \\ A_4(t) &= q_{46}^{(3)}(t)[c]A_6(t), A_6(t) = q_{60}(t)[c]A_0(t) \\ A_7(t) &= (q_{72}(t) + q_{72}^{(8)}(t)) [c]A_2(t) + q_{74}(t) [c]A_4(t) \\ A_9(t) &= M_9(t) + q_{9,10}(t)[c]A_{10}(t), A_{10}(t) = q_{10,2}(t)[c]A_2(t) + q_{10,2}^{(11)}(t)[c]A_2(t) \end{aligned} \quad (7-14)$$

Taking Laplace Transform of eq. (7-14) and solving for  $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \quad (15)$$

Where

$$N_2(s) = (1 - \hat{q}_{22}^{(3)}(s)) \{ \hat{M}_0(s) + \hat{q}_{01}(s) \hat{M}_1(s) + \hat{q}_{09}(s) \hat{M}_9(s) \} + \hat{M}_2(s) \{ \hat{q}_{01}(s) \hat{q}_{42}(s) + \hat{q}_{07}(s) (\hat{q}_{72}(s) + \hat{q}_{73}^{(8)}(s)) + \hat{q}_{09}(s) \hat{q}_{9,10}(s) \} + \hat{q}_{10,2}^{(11)}(s) \}$$

$$\begin{aligned} D_2(s) &= (1 - \hat{q}_{22}^{(3)}(s)) \{ 1 - \hat{q}_{46}^{(5)}(s) \hat{q}_{60}(s) (\hat{q}_{01}(s) \hat{q}_{44}(s) + \hat{q}_{07}(s) \hat{q}_{74}(s)) \\ &\quad - \hat{q}_{20}(s) \{ \hat{q}_{01}(s) \hat{q}_{12}(s) + \hat{q}_{07}(s) (\hat{q}_{72}(s) + \hat{q}_{72}^{(8)}(s) + \hat{q}_{09}(s) \hat{q}_{9,10}(s) \\ &\quad + \hat{q}_{10,2}(s) + \hat{q}_{10,2}^{(11)}(s) \} \} \end{aligned}$$

The steady state availability

$$A_0 = \lim_{t \rightarrow \infty} [A_0(t)] = \lim_{s \rightarrow 0} [s \hat{A}_0(s)] = \lim_{s \rightarrow 0} \frac{s N_2(s)}{D_2(s)}$$

Using L' Hospital's rule, we get

$$A_0 = \lim_{s \rightarrow 0} \frac{N_2(s) + s N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \quad (16)$$

Where

$$\begin{aligned} N_2(0) &= p_{20}(\hat{M}_0(0) + p_{01}\hat{M}_1(0) + p_{09}\hat{M}_9(0)) + \hat{M}_2(0)(p_{01}p_{12} + p_{07}(p_{72} \\ &\quad + p_{72}^{(8)} + p_{09})) \\ D_2(0) &= p_{20} \{ \mu_0 + p_{01} \mu_1 + (p_{01} p_{14} + p_{07} p_{74}) \mu_4 + p_{07} \mu_7 + p_{07} \mu_7 + p_{09}(\mu_9 + \mu_{10}) \\ &\quad + \mu_2 \{ 1 - ((p_{01} p_{14} + p_{07} p_{74})) \} \} \\ \mu_4 &= \mu_{46}^{(5)}, \mu_7 = \mu_{72} + \mu_{72}^{(8)} + \mu_{74}, \mu_{10} = \mu_{10,2} + \mu_{10,2}^{(11)} \end{aligned}$$

The expected up time of the system in  $(0, t]$  is

$$\lambda_u(t) = \int_0^t A_0(z) dz \text{ So that } \hat{\lambda}_u(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(s)}{s D_2(s)} \quad (17)$$

The expected down time of the system in  $(0, t]$  is

$$\lambda_d(t) = t - \lambda_u(t) \text{ So that } \hat{\lambda}_d(s) = \frac{1}{s^2} - \hat{\lambda}_u(s) \quad (18)$$

## The expected busy period of the server for repairing the failed unit under steep high acoustics in $(0, t]$

$$\begin{aligned} R_0(t) &= S_0(t) + q_{01}(t)[c]R_1(t) + q_{07}(t)[c]R_7(t) + q_{09}(t)[c]R_9(t) \\ R_1(t) &= S_1(t) + q_{12}(t)[c]R_2(t) + q_{14}(t)[c]R_4(t), \\ R_2(t) &= q_{20}(t)[c]R_0(t) + q_{22}^{(3)}(t)[c]R_2(t) \\ R_4(t) &= q_{46}^{(3)}(t)[c]R_6(t), R_6(t) = q_{60}(t)[c]R_0(t) \\ R_7(t) &= (q_{72}(t) + q_{72}^{(8)}(t)) [c]R_2(t) + q_{74}(t) [c]R_4(t) \\ R_9(t) &= S_9(t) + q_{9,10}(t)[c]R_{10}(t), R_{10}(t) = q_{10,2}(t) + q_{10,2}^{(11)}(t)[c]R_2(t) \end{aligned} \quad (19-26)$$

Taking Laplace Transform of eq. (19-26) and solving for  $\hat{R}_0(s)$

$$\hat{R}_0(s) = N_3(s) / D_2(s) \quad (27)$$

Where

$$N_2(s) = (1 - \hat{q}_{22}^{(3)}(s)) \{ \hat{S}_0(s) + \hat{q}_{01}(s) \hat{S}_1(s) + \hat{q}_{09}(s) \hat{S}_9(s) \} \text{ and } D_2(s) \text{ is already defined.}$$

$$\text{In the long run, } R_0 = \frac{N_3(0)}{D_2'(0)} \quad (28)$$

where  $N_3(0) = p_{20}(\hat{S}_0(0) + p_{01}\hat{S}_1(0) + p_{09}\hat{S}_9(0))$  and  $D_2'(0)$  is already defined.

The expected period of the system under steep high acoustics in  $(0, t]$  is

$$\lambda_{rv}(t) = \int_0^\infty R_0(z) dz \text{ So that } \widehat{\lambda_{rv}}(s) = \frac{\widehat{R_0}(s)}{s}$$

**The expected Busy period of the server for repairing the failed units under voltage fluctuations by the repairman in  $(0, t]$**

$$\begin{aligned} B_0(t) &= q_{01}(t)[c]B_1(t) + q_{07}(t)[c]B_7(t) + q_{09}(t)[c]B_9(t) \\ B_1(t) &= q_{12}(t)[c]B_2(t) + q_{14}(t)[c]B_4(t), B_2(t) = q_{20}(t)[c]B_0(t) + q_{22}^{(3)}(t)[c]B_2(t) \\ B_4(t) &= T_4(t) + q_{46}^{(3)}(t)[c]B_6(t), B_6(t) = T_6(t) + q_{60}(t)[c]B_0(t) \\ B_7(t) &= (q_{72}(t) + q_{72}^{(8)}(t)) [c]B_2(t) + q_{74}(t) [c]B_4(t) \\ B_9(t) &= q_{9,10}(t)[c]B_{10}(t), B_{10}(t) = T_{10}(t) + (q_{10,2}(t) + q_{10,2}^{(11)}(t)) [c]B_2(t) \end{aligned} \quad (29-36)$$

Taking Laplace Transform of eq. (29-36) and solving for  $\widehat{B_0}(s)$

$$\widehat{B_0}(s) = N_4(s) / D_2(s) \quad (37)$$

Where

$$\begin{aligned} N_4(s) &= (1 - \widehat{q}_{22}^{(3)}(s)) \{ \widehat{q}_{01}(s) \widehat{q}_{14}(s) ( \widehat{T}_4(s) + \widehat{q}_{46}^{(5)}(s) \widehat{T}_6(s) ) + \widehat{q}_{07}^{(3)}(s) \widehat{q}_{74}(s) ( \widehat{T}_4(s) \\ &+ \widehat{q}_{46}^{(5)}(s) \widehat{T}_6(s) ) + \widehat{q}_{09}(s) \widehat{q}_{9,10}(s) \widehat{T}_{10}(s) \} \end{aligned}$$

And  $D_2(s)$  is already defined.

$$\text{In steady state, } B_0 = \frac{N_4(0)}{D_2'(0)} \quad (38)$$

where  $N_4(0) = p_{20} \{ (p_{01} p_{14} + p_{07} p_{74}) (\widehat{T}_4(0) + \widehat{T}_6(0)) + p_{09} \widehat{T}_{10}(0) \}$  and  $D_2'(0)$  is already defined.

The expected busy period of the server for repair in  $(0, t]$  is

$$\lambda_{ru}(t) = \int_0^\infty B_0(z) dz \text{ So that } \widehat{\lambda_{ru}}(s) = \frac{\widehat{B_0}(s)}{s} \quad (39)$$

**The expected Busy period of the server for repair of switch in  $(0, t]$**

$$\begin{aligned} P_0(t) &= q_{01}(t)[c]P_1(t) + q_{07}(t)[c]P_7(t) + q_{09}(t)[c]P_9(t) \\ P_1(t) &= q_{12}(t)[c]P_2(t) + q_{14}(t)[c]P_4(t), P_2(t) = q_{20}(t)[c]P_0(t) + q_{22}^{(3)}(t)[c]P_2(t) \\ P_4(t) &= q_{46}^{(3)}(t)[c]P_6(t), P_6(t) = q_{60}(t)[c]P_0(t) \\ P_7(t) &= L_7(t) + (q_{72}(t) + q_{72}^{(8)}(t)) [c]P_2(t) + q_{74}(t) [c]P_4(t) \\ P_9(t) &= q_{9,10}(t)[c]P_{10}(t), P_{10}(t) = (q_{10,2}(t) + q_{10,2}^{(11)}(t)) [c]P_2(t) \end{aligned} \quad (40-47)$$

Taking Laplace Transform of eq. (40-47) and solving for

$$\widehat{P_0}(s) = N_5(s) / D_2(s) \quad (48)$$

where  $N_5(s) = \widehat{q}_{07}(s) \widehat{L}_7(s) (1 - \widehat{q}_{22}^{(3)}(s))$  and  $D_2(s)$  is defined earlier.

$$\text{In the long run, } P_0 = \frac{N_5(0)}{D_2'(0)} \quad (49)$$

where  $N_5(0) = p_{20} p_{07} \widehat{L}_4(0)$  and  $D_2'(0)$  is already defined.

The expected busy period of the server for repair of the switch in  $(0, t]$  is

$$\lambda_{rs}(t) = \int_0^\infty P_0(z) dz \text{ So that } \widehat{\lambda_{rs}}(s) = \frac{\widehat{P_0}(s)}{s} \quad (50)$$

**The expected number of visits by the repairman for repairing the non-identical units in  $(0, t]$**

$$\begin{aligned} H_0(t) &= Q_{01}(t)[c]H_1(t) + Q_{07}(t)[c]H_7(t) + Q_{09}(t)[c]H_9(t) \\ H_1(t) &= Q_{12}(t)[c][1 + H_2(t)] + Q_{14}(t)[c][1 + H_4(t)], H_2(t) = Q_{20}(t)[c]H_0(t) + Q_{22}^{(3)}(t)[c]H_2(t) \\ H_4(t) &= Q_{46}^{(3)}(t)[c]H_6(t), H_6(t) = Q_{60}(t)[c]H_0(t) \\ H_7(t) &= (Q_{72}(t) + Q_{72}^{(8)}(t)) [c]H_2(t) + Q_{74}(t) [c]H_4(t) \\ H_9(t) &= Q_{9,10}(t)[c][1 + H_{10}(t)], H_{10}(t) = (Q_{10,2}(t)[c] + Q_{10,2}^{(11)}(t)) [c]H_2(t) \end{aligned} \quad (51-58)$$

Taking Laplace Transform of eq. (51-58) and solving for  $H_0^*(s)$

$$H_0^*(s) = N_6(s) / D_3(s) \quad (59)$$

Where

$$\begin{aligned} N_6(s) &= (1 - Q_{22}^{(3)*}(s)) \{ Q_{01}^*(s) ( Q_{12}^*(s) + Q_{14}^*(s) ) + Q_{09}^*(s) Q_{9,10}^*(s) \} \\ D_3(s) &= (1 - Q_{22}^{(3)*}(s)) \{ 1 - ( Q_{01}^*(s) Q_{14}^*(s) + Q_{07}^*(s) Q_{74}^*(s) ) Q_{46}^{(5)*}(s) Q_{60}^*(s) \} \\ &- Q_{20}^*(s) \{ Q_{01}^*(s) Q_{12}^*(s) + Q_{07}^*(s) ( Q_{72}^*(s) ) + Q_{72}^{(8)*}(s) + \\ &Q_{09}^*(s) Q_{9,10}^*(s) ( Q_{10,2}^*(s) + Q_{10,2}^{(11)*}(s) ) \} \end{aligned}$$

$$\text{In the long run, } H_0 = \frac{N_6(0)}{D_3'(0)} \quad (60)$$

where  $N_6(0) = p_{20}(p_{01} + p_{09})$  and  $D_3'(0)$  is already defined.

#### The expected number of visits by the repairman for repairing the switch in (0,t]

$$\begin{aligned} V_0(t) &= Q_{01}(t)[c]V_1(t) + Q_{07}(t)[c]V_7(t) + Q_{09}(t)[c]V_9(t) \\ V_1(t) &= Q_{12}(t)[c]V_2(t) + Q_{14}(t)[c]V_4(t), V_2(t) = Q_{20}(t)[c]V_0(t) + Q_{22}^{(3)}(t)[c]V_2(t) \\ V_4(t) &= Q_{46}^{(3)}(t)[c]V_6(t), V_6(t) = Q_{60}(t)[c]V_0(t) \\ V_7(t) &= (Q_{72}(t)[1+V_2(t)] + Q_{72}^{(8)}(t)[c]V_2(t) + Q_{74}(t)[c]V_4(t) \\ V_9(t) &= Q_{9,10}(t)[c]V_{10}(t), V_{10}(t) = (Q_{10,2}(t) + Q_{10,2}^{(11)}(t))[c]V_2(t) \end{aligned} \quad (61-68)$$

Taking Laplace-Stieltjes transform of eq. (61-68) and solving for  $V_0^*(s)$

$$V_0^*(s) = N_7(s) / D_4(s) \quad (69)$$

where  $N_7(s) = Q_{07}^*(s) Q_{72}^*(s) (1 - Q_{22}^{(3)*}(s))$  and  $D_4(s)$  is the same as  $D_3(s)$

$$\text{In the long run, } V_0 = \frac{N_7(0)}{D_4'(0)} \quad (70)$$

where  $N_7(0) = p_{20} p_{07} p_{72}$  and  $D_3'(0)$  is already defined.

### GAIN- FUNCTION ANALYSIS

The gain- function of the system considering mean up-time, expected busy period of the system under voltage fluctuations when the units stops automatically, expected busy period of the server for repair of unit and switch, expected number of visits by the repairman for unit failure, expected number of visits by the repairman for switch failure.

The expected total cost-benefit incurred in (0,t] is

$C(t)$  = Expected total revenue in (0,t] - expected total repair cost for switch in (0,t]

- expected total repair cost for repairing the units due to voltage fluctuations in (0, t ] when the units automatically stop in (0,t]
- expected busy period of the system under steep high acoustics
- expected number of visits by the repairman for repairing the switch in (0,t]
- expected number of visits by the repairman for repairing of the non-identical units in (0,t]

The expected total cost per unit time in steady state is

$$\begin{aligned} C &= \lim_{t \rightarrow \infty} (C(t)/t) = \lim_{s \rightarrow 0} (s^2 C(s)) \\ &= K_1 A_0 - K_2 P_0 - K_3 B_0 - K_4 R_0 - K_5 V_0 - K_6 H_0 \end{aligned}$$

Where

$K_1$ : revenue per unit up-time,

$K_2$ : cost per unit time for which the system is under switch repair,

$K_3$ : cost per unit time for which the system is under repair due to voltage fluctuations when units automatically stop,

$K_4$ : cost per unit time for which the system is under repair due to steep high acoustics,

$K_5$ : cost per visit by the repairman for which switch repair,

$K_6$ : cost per visit by the repairman for units repair.

### CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to voltage fluctuations, failure rate due to steep high acoustics increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

### REFERENCES

1. Dhillon, B.S. and Natesen, J, Stochastic Anaysis of outdoor Power Systems in fluctuating environment, Microelectron. Reliab., 1983; 23, 867-881.
2. Goel, L.R., Sharma, G.C. and Gupta, Rakesh Cost Analysis of a Two-Unit standby system with different weather conditions, Microelectron. Reliab., 1985; 25, 665-659.
3. Goel, L.R., Sharma G.C. and Gupta Parveen, Stochastic Behavior and Profit Analysis of a redundant system with slow switching device, Microelectron Reliab., 1986; 26, 215-219.
4. Cao, Jinhua, Stochastic Behavior of a Man Machine System Operating under Changing Environment Subject to a Markov Process with two States, Microelectron. Reliab. 1989, 28, pp. 373-378.
5. Barlow, R.E. and Proschan, F., Mathematical theory of Reliability, 1965; John Wiley, New York.
6. Gnedenko, B.V., Belyayar, Yu.K. and Soloyer, A.D., Mathematical Methods of Reliability Theory, 1969 ; Academic Press, New York.

Source of Support: None Declared  
Conflict of Interest: None Declared