Cost-benefit analysis of two-unit cold standby system under the influence of steep high temperatures and human error

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Abstract

Introduction: A large number of papers on a two unit standby redundant systems have appeared because of the large number of parameters governing system behavior and the complex environment with uncertainties. In this paper we have taken open failure mode which is caused due to steep high temperatures (SH temp.) and other failure mode due to human error. When the main unit fails then cold standby system becomes operative. SH temp. cannot occur simultaneously in both the units and after failure the unit undergoes very costly repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters of interest and profit analysis have been evaluated.

Keyword: Cost-benefit, steep high temperature.

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INTRODUCTION

In this paper we have taken open failure mode which is caused due to SH temp. and other failure due to human errors. When the main operative unit fails then cold standby system becomes operative. SH temp. Cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of very high cost in case of open failure mode immediately. Failure due to human error may be destructive. It may be due to repairman's carelessness or he may be unskilled. The repair is done on the basis of first fail first repaired.

ASSUMPTIONS

1. The failure time distributions are general $F_1(t)$ and $F_2(t)$ and repair is of two types -Type -I, Type-II with repair time distributions as $G_1(t)$ and $G_2(t)$ respectively.
2. The repair starts immediately upon failure of units.
3. The switches are perfect and instantaneous.
4. All random variables are mutually independent.
5. When both the units fail, we give priority to operative unit for repair.

6. The failure of open mode Is due to some human errors.
7. Repairs are perfect and failure of a unit is detected immediately and perfectly.
8. The system is down when both the units are non-operative.
9. After repair, the unit's works like a new one.

SYMBOLS FOR STATES OF THE SYSTEM

$F_1(t), F_2(t)$ - failure time distribution due to SH temp. Mode and open failure mode

$G_1(t), G_2(t)$ – repair time distribution due to Type -I, Type-II respectively

Superscripts O, CS, SHTF, OF

Operative, Cold Standby, Steep high temp. failure mode, open failure mode due to human error respectively

Subscripts nsht, sht, he, ur, wr, uR

No steep high temp., Steep high temp., human error, under repair, waiting for repair, under repair continued from previous state respectively

Up states: 0, 1, 2;

Down states: 3,4,5,6

Regeneration point: 0, 1, 2

States of the System

0(O_{nsht}, CS_{nsht})

One unit is operative and the other unit is cold standby and there are no SH temp. in both the units.

1($SHTF_{sht, ur}$, O_{nsht})

The operating unit fails due to SH temp. and is under repair immediately of very costly Type- I and standby unit starts operating with no SH temp.

2($OF_{nsht,he, ur}$, O_{nsht})

The operative unit fails due to human error (open failure mode) and undergoes repair of type II and the standby unit becomes operative with no SH temp..

3($SHTF_{sht,uR}$, $OF_{nsht, he,wr}$)

The first unit fails due to SH temp. and under very costly Type-! Repair is continued from state 1 and the other unit fails due to human errors and is waiting for repair of Type -II.

4($SHTF_{sht,uR}$, $SHTF_{sht,wr}$)

The one unit fails due to SH temp. is continues under repair of very costly Type - I from state 1 and the other unit also fails due to SH temp. is waiting for repair of very costly Type- I.

5($OF_{nsht, he, uR}$, $SHTF_{sht, wr}$)

The operating unit fails due to human error (open failure mode) and under repair of Type - II continues from the state 2 and the other unit fails due to SH temp. is waiting for repair of very costly Type- I.

6($OF_{nsht,he,uR}$, $OF_{nsht,he,wr}$)

The operative unit fails due to human error and under repair continues from state 2 of Type –II and the other unit is also failed due to human error and is waiting for repair of Type-II and there is no SH temp.

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions :

$p_{01} = \int_0^\infty \overline{F_2}(t) dF_1(t)$, $p_{02} = \int_0^\infty \overline{F_1}(t) dF_2(t)$

$p_{10} = \int_0^\infty \overline{F_2}(t) dG_1(t)$, $p_{13} = p_{11}^{(3)} = p_{11}^{(4)} = \int_0^\infty \overline{G_1}(t) dF_2(t)$

$p_{25} = p_{22}^5 = p_{22}^6 = \int_0^\infty \overline{G_2}(t) dF_1(t)$

clearly

$p_{01} + p_{02} = 1$,

$p_{10} + p_{13} = (p_{11}^3) + p_{14} = (p_{11}^4) = 1$,

$p_{20} + p_{25} = (p_{22}^5) + p_{26} = (p_{22}^6) = 1$ (1)

And mean sojourn time are

$\mu_0 = E(T) = \int_0^\infty P[T > t] dt$ (2)

Mean Time To System Failure

$Ø_0(t) = Q_{01}(t)[s] \, Ø_1(t) + Q_{02}(t)[s] \, Ø_2(t)$

$\emptyset_1(t) = Q_{10}(t)[s] \emptyset_0(t) + Q_{13}(t) + Q_{14}(t)$

$\emptyset_2(t) = Q_{20}(t)[s] \emptyset_0(t) + Q_{25}(t) + Q_{26}(t)$ (3-5)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-5)

and solving for

$\emptyset_0^*(s) = N_1(s) / D_1(s)$ (6)

Where

$N_1(s) = Q_{01}^*[Q_{13}^*(s) + Q_{14}^*(s)] + Q_{02}^*[Q_{25}^*(s) + Q_{26}^*(s)]$

$D_1(s) = 1 - Q_{01}^* Q_{10}^* - Q_{02}^* Q_{20}^*$

Making use of relations (1) and (2) it can be shown that $\emptyset_0^*(0) = 1$, which implies that $\emptyset_0^*(t)$ is a proper distribution.

$\text{MTSF} = E[T] = \dfrac{d}{ds} \emptyset_0^{*(s)} \Big| = (D_1'(0) - N_1'(0)) / D_1(0)$

$\qquad s=0$

$= (\mu_0 + p_{01} \mu_1 + p_{02} \mu_2) / (1 - p_{01} p_{10} - p_{02} p_{20})$

where

$\mu_0 = \mu_{01} + \mu_{02}, \mu_1 = \mu_{10} + \mu_{13} + \mu_{14}$

$\mu_2 = \mu_{20} + \mu_{25} + \mu_{26}$

Availability analysis

Let $M_i(t)$ be the probability of the system having started from state I is up at time t without making any other regenerative state belonging to E. By probabilistic arguments, we have The value of $M_0(t)$, $M_1(t)$, $M_2(t)$ can be found easily.

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{11}^3(t)[c]A_1(t) + q_{11}^4(t)[c]A_1(t),$

$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + [q_{22}^5(t)[c] + q_{22}^6(t)][c]A_2(t)$ (7-9)

Taking Laplace Transform of eq. (7-9) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (10)

Where

$N_2(s) = \hat{M}_0(s)(1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))(1 - \hat{q}_{22}^5(s) - \hat{q}_{22}^6(s)) + \overline{q}_{01}(s)\hat{M}_1(s)$

$[1 - \hat{q}_{22}^5(s) - \hat{q}_{22}^6(s)] + \hat{q}_{02}(s) \hat{M}_2(s)(1 - \hat{q}_{11}^3(s) - \hat{q}_{11}^4(s))$

$D_2(s) = (1 - \hat{q}_{11}^3(s) - \hat{q}_{11}^4(s)) \{ 1 - \hat{q}_{22}^5(s) - \hat{q}_{22}^6(s)) [1 - (\hat{q}_{01}(s) \hat{q}_{10}(s))(1 - \hat{q}_{11}^3(s) - \hat{q}_{11}^4(s))]$

The steady state availability

$A_0 = \lim_{t \to \infty}[A_0(t)] = \lim_{s \to 0}[s \hat{A}_0(s)] = \lim_{s \to 0} \dfrac{s N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$A_0 = \lim_{s \to 0} \dfrac{N_2(s) + s N_2'(s)}{D_2'(s)} = \dfrac{N_2(0)}{D_2'(0)}$ (11)

The expected up time of the system in (0,t] is

$\lambda_{u}(t) = \int_0^\infty A_0(z)dz$ So that $\overline{\lambda_u}(s) = \dfrac{\hat{A}_0(s)}{s} = \dfrac{N_2(s)}{sD_2(s)}$ (12)

The expected down time of the system in (0,t] is

$\lambda_{d}(t) = t - \lambda_{u}(t)$ So that $\overline{\lambda_d}(s) = \dfrac{1}{s^2} - \overline{\lambda_u}(s)$ (13)

The expected busy period of the server when there is open failure mode failure in (0,t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$

$R_1(t) = S_1(t) + q_{01}(t)[c]R_1(t) + [q_{11}^3(t) + q_{11}^4(t)][c]R_1(t),$

$R_2(t) = q_{20}(t)[c]R_0(t) + [q_{22}^6(t) + q_{22}^5(t)][c]R_2(t)$ (14-16)

Taking Laplace Transform of eq. (14-16) and solving for $\overline{R_0}(s)$

$\overline{R_0}(s) = N_3(s) / D_3(s)$ (17)

Where

$N_3(s) = \hat{q}_{01}(s) \acute{S}_1(s)$ and

$D_3(s)= (1 - \hat{q}_{11}^3(s)- \hat{q}_{11}^4(s)) - \hat{q}_{01}(s)$ is already defined.

In the long run, $R_0 = \dfrac{N_5(0)}{D_2'(0)}$ (18)

The expected period of the system under SH temp. in (0,t] is

$\lambda_{rv}(t) = \int_0^\infty R_0(z)dz$ So that $\overline{\lambda_{rv}}(s) = \dfrac{\hat{R}_0(s)}{s}$

The expected Busy period of the server when there is human error in (0,t]

$B_0(t) = q_{01}(t)[c]B_1(t) + q_{02}(t)[c]B_2(t)$

$B_1(t) = q_{01}(t)[c]B_1(t) + [q_{11}^3(t)+ q_{11}^4(t)] [c]B_1(t),$

$B_2(t) = T_2(t) + q_{02}(t)[c] B_2(t) + [q_{22}^5(t)+ q_{22}^6(t)] [c]B_2(t)$

$T_2(t) = e^{-\lambda_1't}G_2(t)$ (19- 21)

Taking Laplace Transform of eq. (19-21) and solving for $\overline{B_0}(s)$

$\overline{B_0}(s) = N_4(s) / D_2(s)$ (22)

Where

$N_4(s) = \hat{q}_{02}(s) \hat{T}_2(s))$

And $D_2(s)$ is already defined.

In steady state, $B_0 = \dfrac{N_4(0)}{D_2'(0)}$ (23)

The expected busy period of the server for repair in (0,t] is

$\lambda_{ru}(t) = \int_0^\infty B_0(z)dz$ So that $\overline{\lambda_{ru}}(s) = \dfrac{\hat{B}_0(s)}{s}$ (24)

The expected number of visits by the repairman for repairing the different units in (0,t]

$H_0(t) = Q_{01}(t)[s][1+ H_1(t)] + Q_{02}(t)[s][1+ H_2(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + [Q_{11}^3(t)+ Q_{11}^4(t)] [s]H_1(t),$

$H_2(t) = Q_{20}(t)[s]H_0(t) + [Q_{22}^5(t) +Q_{22}^6(t)] [c]H_2(t)$ (25-27)

Taking Laplace Transform of eq. (25-27) and solving for $H_0^s(s)$

$H_0^s(s) = N_6(s) / D_3(s)$ (28)

In the long run, $H_0 = \dfrac{N_6(0)}{D_3'(0)}$ (29)

COST BENEFIT ANALYSIS

The cost-benefit function of the system considering mean up-time, expected busy period of the system under SH temp. when the units stops automatically, expected busy period of the server for repair of unit, expected number of visits by the repairman for unit failure. The expected total cost-benefit incurred in (0,t] is C(t) = Expected total revenue in (0,t] - expected total repair cost in (0,t] due to OF mode

- expected total repair cost due to human error for repairing the units in (0,t]
- expected busy period of the system under SH temp. when the units automatically stop in (0,t]
- expected number of visits by the repairman for repairing of the units in (0,t]

The expected total cost per unit time in steady state is

$C = \lim_{t \to \infty}(C(t)/t) = \lim_{s \to 0}(s^2 C(s))$

$= K_1A_0 - K_2R_0 - K_3B_0 - K_4H_0$

Where

K_1: revenue per unit up-time,

K_2: cost per unit time for which the system is under repair of type- I

K_3: cost per unit time for which the system is under repair of type-II

K_4: cost per visit by the repairman for units repair.

CONCLUSION

After studying the system, we have analysed graphically that when the failure rate, SH temp. Rate increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

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