Research Article

Benefit-function of two-identical cold standby system subject to failure due to greenhouse effect and failure due to thermal radiation

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Abstract

Introduction: The **greenhouse effect** is a process by which thermal radiation from a planetary surface is absorbed by atmospheric greenhouse gases, and is re-radiated in all directions. Since part of this re-radiation is back towards the surface and the lower atmosphere, it results in an elevation of the average surface temperature above what it would be in the absence of the gases. Solar radiation at the frequencies of visible light largely passes through the atmosphere to warm the planetary surface, which then emits this energy at the lower frequencies of infrared thermal radiation. Infrared radiation is absorbed by greenhouse gases, which in turn re-radiate much of the energy to the surface and lower atmosphere. The mechanism is named after the effect of solar radiation passing through glass and warming a greenhouse, but the way it retains heat is fundamentally different as a greenhouse works by reducing airflow, isolating the warm air inside the structure so that heat is not lost by convection. If an ideal thermally conductive blackbody were the same distance from the Sun as the Earth is, it would have a temperature of about 5.3 °C. However, since the Earth reflects about 30% of the incoming sunlight, this idealized planet's effective temperature (the temperature of a blackbody that would emit the same amount of radiation) would be about −18 °C. The surface temperature of this hypothetical planet is 33 °C below Earth's actual surface temperature of approximately 14 °C. The mechanism that produces this difference between the actual surface temperature and the effective temperature is due to the atmosphere and is known as the greenhouse effect. Earth's natural greenhouse effect makes life as we know it possible. However, human activities, primarily the burning of fossil fuels and clearing of forests, have intensified the natural greenhouse effect, causing global warming. **Thermal radiation** is electromagnetic radiation generated by the thermal motion of charged particles in matter. All matter with a temperature greater than absolute zero emits thermal radiation. When the temperature of the body is greater than absolute zero, inter atomic collisions cause the kinetic energy of the atoms or molecules to change. This results in charge-acceleration and/or dipole oscillation which produces electromagnetic radiation, and the wide spectrum of radiation reflects the wide spectrum of energies and accelerations that occur even at a single temperature. Examples of thermal radiation include the visible light and infrared light emitted by an incandescent light bulb, the infrared radiation emitted by animals and detectable with an infrared camera, and the cosmic microwave background radiation. Thermal radiation is different from thermal convection and thermal conduction—a person near a raging bonfire feels radiant heating from the fire, even if the surrounding air is very cold. Sunlight is part of thermal radiation generated by the hot plasma of the Sun. The Earth also emits thermal radiation, but at a much lower intensity and different spectral distribution (infrared rather than visible) because it is cooler. The Earth's absorption of solar radiation, followed by its outgoing thermal radiation are the two most important processes that determine the temperature and climate of the Earth. Reliability is a measure of how well a system performs or meets its design requirements. It is hence the prime concern of all scientists and engineers engaged in developing such a system.. In this paper we have taken **FGHE- failure due to greenhouse effect** and FTR-**failure due to thermal radiation.** When the main unit fails due to Thermal radiation then cold standby system becomes operative. Thermal radiation cannot occur simultaneously in both the units and after failure the unit undergoes very costly repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Cold Standby, FGHE - failure due to Greenhouse effect, FTR - failure due to Thermal radiation, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

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INTRODUCTION

The characteristics of thermal radiation depend on various properties of the surface it is emanating from, including its temperature, its spectral absorptivity and spectral emissive power, as expressed by Kirchhoff's law The radiation is not monochromatic, i.e., it does not consist of just a single frequency, but comprises a continuous dispersion of photon energies, its characteristic spectrum. If the radiating body and its surface are in thermodynamic equilibrium and the surface has perfect absorptivity at all wavelengths, it is characterized as a black body. A black body is also a perfect emitter. The radiation of such perfect emitters is called black-body radiation. The ratio of any body's emission relative to that of a black body is the body's emissivity, so that a black body has an emissivity of unity. Absorptivity, reflectivity, and emissivity of all bodies are dependent on the wavelength of the radiation. The temperature determines the wavelength distribution of the electromagnetic radiation. For example, fresh snow, which is highly reflective to visible light (reflectivity about 0.90), appears white due to reflecting sunlight with a peak wavelength of about 0.5 micrometers. Its emissivity, however, at a temperature of about -5 °C, peak wavelength of about 12 micrometers, is 0.99. A **greenhouse gas** (sometimes abbreviated **GHG**) is a gas in an atmosphere that absorbs and emits radiation within the thermal infrared range. This process is the fundamental cause of the greenhouse effect. The primary greenhouse gases in the Earth's atmosphere are water vapor, carbon dioxide, methane, nitrous oxide, and ozone. Greenhouse gases greatly affect the temperature of the Earth; without them, Earth's surface would average about 33 °C colder, which is about 59 °F below the present average of 14 °C (57 °F). Since the beginning of the Industrial Revolution (taken as the year 1750), the burning of fossil fuels and extensive clearing of native forests has contributed to a 40% increase in the atmospheric concentration of carbon dioxide, from 280 to 392.6parts per million (ppm) in 2012 and has now reached 400 ppm in the northern hemisphere. This increase has occurred despite the uptake of a large portion of the emissions by various natural "sinks" involved in the carbon cycle. Anthropogenic carbon dioxide(CO_2) emissions (i.e., emissions produced by human activities) come from combustion of carbon-based fuels, principally wood, coal, oil, and natural gas. Under ongoing greenhouse gas emissions, available Earth System Models project that the Earth's surface temperature could exceed historical analogs as early as 2047 affecting most ecosystems on Earth and the livelihoods of over 3 billion people worldwide. Greenhouse gases also trigger ocean bio-geochemical changes with broad ramifications in marine systems. Stochastic behaviour of systems operating under changing environments has widely been studied. Dhillon, B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment. Kan Cheng 1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken **FGHE - failure due to Greenhouse effect** and other **FTR- failure due to Thermal radiation**. When the main operative unit **fails due to Thermal radiation-FTR** then cold standby system becomes operative. **Thermal radiation** cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of very high cost in case of **FTR-failure due to Thermal radiation** immediately. The repair is done on the basis of first fail first repaired.

Assumptions

1. $F_1(t)$ and $F_2(t)$ are general failure time distributions due to **Thermal radiation** and **Greenhouse effect.** The repair is of two types -Type -I, Type-II with repair time distributions as $G_1(t)$ and $G_2(t)$ respectively.
2. The **Thermal radiation** is non-instantaneous and it cannot come simultaneously in both the units.

3. Whenever the **thermal radiation** occur within specified limit of the unit, it works as normal as before. But as soon as there occur **Thermal radiation** of magnitude beyond specified limit of the unit the operation of the unit stops automatically.
4. The repair starts immediately after the **Thermal radiation** of beyond specified limit of the unit are over and works on the principle of first fail first repaired basis.
5. The repair facility does no damage to the units and after repair units are as good as new.
6. The switches are perfect and instantaneous.
7. All random variables are mutually independent.
8. When both the units fail, we give priority to operative unit for repair.
9. **FTR -** Failure due to thermal radiation when it is beyond specified limit.
10. Repairs are perfect and failure of a unit is detected immediately and perfectly.
11. The system is down when both the units are non-operative.

SYMBOLS FOR STATES OF THE SYSTEM

$F_1(t)$ and $F_2(t)$ –the failure time distribution due to **Thermal radiation** and **Greenhouse effect** respectively
$G_1(t)$, $G_2(t)$ – repair time distribution Type -I, Type-II due to **Thermal radiation and Greenhouse effect** respectively
Superscripts: O, CS, FGHE, FTR
Operative, Cold Standby, failure due to greenhouse effect, Failure due to thermal radiation respectively
Subscripts: ntrf, trf, ghe, ur, wr, uR
No Thermal radiation failure, Thermal radiation failure, green house effect, under repair, waiting for repair, under repair continued from previous state respectively
Up states: 0, 1, 2;
Down states: 3, 4
Regeneration point: 0, 1, 2
Notations
$M_i(t)$ System having started from state I is up at time t without visiting any other regenerative state
A_i (t) state is up state as instant t
R_i (t) System having started from state I is busy for repair at time t without visiting any other regenerative state.
B_i (t) the server is busy for repair at time t.
$H_i(t)$ Expected number of visits by the server for repairing given that the system initially starts from regenerative state i
By Thermal radiation we mean Thermal radiation beyond the specified limit
States of the System
0(O_{ntrf}, CS_{ntrf})
One unit is operative and the other unit is cold standby and there is no Thermal radiation in both the units.
1(SOFTR $_{trf, ur}$, O_{ntrf})
The operating unit fails due to Thermal radiation and is under repair immediately of very costly Type- I and standby unit starts operating with no Thermal radiation.
2(FGHE $_{ntrf, ghe, ur}$, O_{ntrf})
The operative unit fails due to FGHE resulting from Greenhouse effect and undergoes repair of type II and the standby unit becomes operative with no thermal radiation.
3($FTR_{trf,uR}$, FGHE $_{ntrf, ghe, wr}$)
The first unit fails due to Thermal radiation and under very costly Type-I repair is continued from state 1 and the other unit fails due to FGHE resulting from Greenhouse effect and is waiting for repair of Type -II.
4(FTR $_{trf, uR}$, $FTR_{trf,wr}$)
The one unit fails due to Thermal radiation is continues under repair of very costly Type - I from state 1 and the other unit also fails due to Thermal radiation is waiting for repair of very costly Type- I.
5(FGHE $_{ntrf, ghe, uR}$, FTR $_{trf, wr}$)
The operating unit fails due to Greenhouse effect (FGHE mode) and under repair of Type - II continues from the state 2 and the other unit fails due to Thermal radiation is waiting for repair of very costly Type- I.
6(FGHE $_{ntrf,ghe,uR}$, FGHE $_{ntrf,ghe,wr}$)
The operative unit fails due to FGHE resulting from Greenhouse effect and under repair continues from state 2 of Type – II and the other unit is also failed due to FGHE resulting from Greenhouse effect and is waiting for repair of Type-II and there is no Thermal radiation.

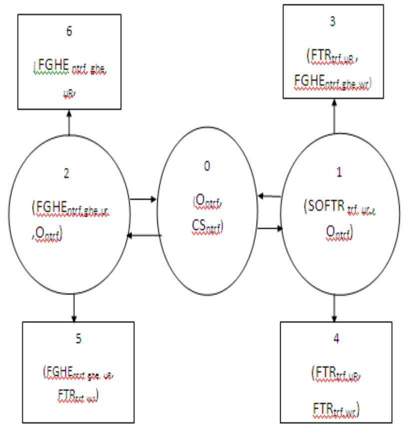

Figure 1: The State Transition Diagram

• Regeneration point - states 0, 1, 2. ○ Up State □ Down State

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$p_{01} = \int_0^\infty \overline{F}_2(t)\,dF_1(t)$, $p_{02} = \int_0^\infty \overline{F}_1(t)\,dF_2(t)$

$p_{10} = \int_0^\infty \overline{F}_2(t)\,dG_1(t)$, $p_{13} = p_{11}^{(3)} = p_{11}^{(4)} = \int_0^\infty \overline{G}_1(t)\,dF_2(t)$

$p_{25} = p_{22}^{(5)} = p_{22}^{(6)} = \int_0^\infty \overline{G}_2(t)\,dF_1(t)$

Clearly

$p_{01} + p_{02} = 1,$

$p_{10} + p_{13} = (p_{11}^{(3)}) + p_{14} = (p_{11}^{(4)}) = 1,$

$p_{20} + p_{25} = (p_{22}^{(5)}) + p_{26} = (p_{22}^{(6)}) = 1$ (1)

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P|T > t|\,dt$ (2)

Mean Time To System Failure

$\text{Ø}_0(t) = Q_{01}(t)[s]\,\text{Ø}_1(t) + Q_{02}(t)[s]\,\text{Ø}_2(t)$

$\text{Ø}_1(t) = Q_{10}(t)[s]\,\text{Ø}_0(t) + Q_{13}(t) + Q_{14}(t)$

$\text{Ø}_2(t) = Q_{20}(t)[s]\,\text{Ø}_0(t) + Q_{25}(t) + Q_{26}(t)$ (3-5)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-5) and solving for

$ø_0^*(s) = N_1(s) / D_1(s)$ (6)

Where

$N_1(s) = Q_{01}^*[Q_{13}^*(s) + Q_{14}^*(s)] + Q_{02}^*[Q_{25}^*(s) + Q_{26}^*(s)]$

$D_1(s) = 1 - Q_{01}^* Q_{10}^* - Q_{02}^* Q_{20}^*$

Making use of relations (1) and (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0^*(t)$ is a proper distribution.

$$MTSF = E[T] = \frac{d}{ds}\, ø_0^*(s)\Big|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$= (\mu_0 + p_{01}\,\mu_1 + p_{02}\,\mu_2) / (1 - p_{01}\,p_{10} - p_{02}\,p_{20})$

Where

$\mu_0 = \mu_{0\,1} + \mu_{0\,2},\ \mu_1 = \mu_{1\,0} + \mu_{1\,3} + \mu_{1\,4}$

$$\mu_2 = \mu_{20} + \mu_{25} + \mu_{26}$$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state I is up at time t without making any other regenerative state. By probabilistic arguments, we have

The value of $M_0(t)$, $M_1(t)$, $M_2(t)$ can be found easily.

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{11}^{(3)}(t)[c]A_1(t) + q_{11}^{(4)}(t)[c]A_1(t),$

$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + [q_{22}^{(5)}(t)[c] + q_{22}^{(6)}(t)]\,[c]A_2(t)$ (7-9)

Taking Laplace Transform of eq. (7-9) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (10)

Where

$N_2(s) = \hat{M}_0(s)(1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))\,(1 - \hat{q}_{22}^{(5)}(s) - \hat{q}_{22}^{(6)}(s)) + \overline{q}_{01}(s)\,\hat{M}_1(s)$
$[1 - \hat{q}_{22}^{(5)}(s) - \hat{q}_{22}^{(6)}(s)] + \hat{q}_{02}(s)\,\hat{M}_2(s)(1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))$

$D_2(s) = (1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))\,\{1 - \hat{q}_{22}^{(5)}(s) - \hat{q}_{22}^{(6)}(s)\,)[1 - (\hat{q}_{01}(s)\,\hat{q}_{10}(s))(1 -$
$\hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))]$

The steady state availability

$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\,\hat{A}_0(s)] = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$A_0 = \lim_{s\to 0}\frac{N_2(s) + s\,N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)}$ (11)

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^{\infty} A_0(z)\,dz$ So that $\overline{\lambda_u}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (12)

The expected down time of the system in (0, t] is

$\lambda_d(t) = t - \lambda_u(t)$ So that $\overline{\lambda_d}(s) = \frac{1}{s^2} - \overline{\lambda_u}(s)$ (13)

The expected busy period of the server when there is FGHE-failure resulting from Greenhouse effect in (0, t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$

$R_1(t) = S_1(t) + q_{01}(t)[c]R_1(t) + [q_{11}^{(3)}(t) + q_{11}^{(4)}(t)[c]R_1(t),$

$R_2(t) = q_{20}(t)[c]R_0(t) + [q_{22}^{(6)}(t) + q_{22}^{(5)}(t)][c]R_2(t)$ (14-16)

Taking Laplace Transform of eq. (14-16) and solving for $\overline{R_0}(s)$

$\overline{R_0}(s) = N_3(s) / D_3(s)$ (17)

Where

$N_3(s) = \hat{q}_{01}(s)\,\hat{S}_1(s)$ and

$D_3(s) = (1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s)) - \hat{q}_{01}(s)$ is already defined.

In the long run, $R_0 = \frac{N_3(0)}{D_3'(0)}$ (18)

The expected period of the system under FGHE-failure resulting from Green House effect in (0, t] is

$\lambda_{rv}(t) = \int_0^{\infty} R_0(z)\,dz$ So that $\overline{\lambda_{rv}}(s) = \frac{\hat{R}_0(s)}{s}$

The expected busy period of the server when there is Thermal radiation in (0, t]

$B_0(t) = q_{01}(t)[c]B_1(t) + q_{02}(t)[c]B_2(t)$

$B_1(t) = q_{01}(t)[c]B_1(t) + [q_{11}^{(3)}(t) + q_{11}^{(4)}(t)]\,[c]B_1(t),$

$B_2(t) = T_2(t) + q_{02}(t)[c]\,B_2(t) + [q_{22}^{(5)}(t) + q_{22}^{(6)}(t)]\,[c]B_2(t)$

$T_2(t) = e^{-\lambda_1 t}G_2(t)$ (19- 21)

Taking Laplace Transform of eq. (19-21) and solving for $\bar{B}_0^{*}(s)$

$$\bar{B}_0^{*}(s) = N_4(s) / D_2(s) \quad (22)$$

Where

$N_4(s) = \hat{q}_{02}(s)\,\hat{T}_2(s))$

and $D_2(s)$ is already defined.

In steady state, $B_0 = \frac{N_4(0)}{D_2'(0)}$ (23)

The expected busy period of the server for repair in (0, t] is

$\lambda_{ru}(t) = \int_0^{\infty} B_0(z)dz$ So that $\bar{\lambda}_{ru}(s) = \frac{\bar{B}_0(s)}{s}$ (24)

The expected number of visits by the repairman for repairing the identical units in (0, t]

$H_0(t) = Q_{01}(t)[s][1+ H_1(t)] + Q_{02}(t)[s][1+ H_2(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + [Q_{11}^{(3)}(t)+ Q_{11}^{(4)}(t)] [s]H_1(t),$

$H_2(t) = Q_{20}(t)[s]H_0(t) + [Q_{22}^{(5)}(t) +Q_{22}^{(6)}(t)] [c]H_2(t)$ (25-27)

Taking Laplace Transform of eq. (25-27) and solving for $H_0^{*}(s)$

$$H_0^{*}(s) = N_6(s) / D_3(s) \quad (28)$$

In the long run, $H_0 = \frac{N_6(0)}{D_3'(0)}$ (29)

BENEFIT- FUNCTION ANALYSIS

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under Thermal radiation when the units stops automatically, expected busy period of the server for repair of unit under Green House effect, expected number of visits by the repairman for unit failure.

The expected total Benefit-Function incurred in (0, t] is

C (t) = Expected total revenue in (0, t]

- expected total repair cost in (0,t] due to Thermal radiation failure
- expected total repair cost due to FGHE: failure resulting from Green house effect for repairing the units in (0,t]
- expected busy period of the system under Thermal radiation when the units automatically stop in (0, t]
- expected number of visits by the repairman for repairing of identical the units in (0, t]

The expected total cost per unit time in steady state is

$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$

$= K_1A_0 - K_2R_0 - K_3B_0 - K_4H_0$

Where

K_1: revenue per unit up-time,

K_2: cost per unit time for which the system is under repair of type- I

K_3: cost per unit time for which the system is under repair of type-II

K_4: cost per visit by the repairman for units repair.

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to green house effect and failure rate due to Thermal radiation increases, the MTSF and steady state availability decreases and the Benefit-function decreased as the failure increases.

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