

# Gain-function of two-unit non-identical warm standby nuclear power system with failure due to extremely high radiations and failure due to non-availability of heavy water $D_2O$ in nuclear power reactors

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## Abstract

**Introduction:** A **pressurized heavy-water reactor** (PHWR) is a nuclear power reactor, commonly using enriched natural uranium as its fuel, that uses heavy water (deuterium oxide  $D_2O$ ) as its coolant and moderator. The heavy-water coolant is kept under pressure, allowing it to be heated to higher temperatures without boiling, much as in a PWR. While heavy water is significantly more expensive than ordinary light water, it yields greatly enhanced neutron economy, allowing the reactor to operate without fuel-enrichment facilities (mitigating the additional capital cost of the heavy water) and generally enhancing the ability of the reactor to efficiently make use of alternate fuel cycles. The 1979 Three Mile Island accident and 1986 Chernobyl disaster, along with high construction costs, ended the rapid growth of global nuclear power capacity. A further disastrous release of radioactive materials followed the 2011 Japanese tsunami which damaged the Fukushima I Nuclear Power Plant, resulting in hydrogen gas explosions and partial meltdowns classified as a Level 7 event. The large-scale release of radioactivity resulted in people being evacuated from a 20 km exclusion zone set up around the power plant, similar to the 30 km radius Chernobyl Exclusion Zone still in effect. In **Nuclear Reactor** the leakage in form of radiations becomes highly dangerous to the lives of living beings. The radiations from the nuclear reactor are always under serious consideration due to fatal and miserable results to human race. Every precautions and extra care is taken to avoid any miss-happening due to radiations. But still due carelessness or due to failure of some equipment in Nuclear Reactors there occur leakage of radiations causing a major casualty. In the present paper we have taken two-dissimilar warm standby nuclear power system with failure due to extremely high radiations which we abbreviated as **FEHR** and failure due to Non-availability of heavy water  $D_2O$  in nuclear power reactor abbreviated as **FNAHW**. When there are radiations of extremely high magnitude the working of unit stops automatically to avoid excessive damage of the units and when the unit comes in no normal position the repair of the units' starts immediately. The failure time distribution is taken as exponential and repair time distribution as general. Using Markov regenerative point technique we have calculated different reliability characteristics such as MTSF, reliability of the system, availability analysis in steady state, busy period analysis of the system under repair, expected number of visits by the repairman in the long run and Gain-function. Special case by taking failure and repair as exponential have been derived and graphs are drawn.

**Keywords:** warm standby, extremely high radiations, non-availability of heavy water in nuclear power reactor, MTSF, Availability, busy period, Gain - function.

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## INTRODUCTION

Water makes an excellent moderator; the hydrogen atoms in the water molecules are very close in mass to a single neutron, and the collisions thus have a very efficient momentum transfer, similar conceptually to the collision of two billiard balls. However, in addition to being a good moderator, water is relatively effective at absorbing neutrons. Using water as a moderator will absorb enough neutrons that there will be too few left over to react with the small amount of  $^{235}\text{U}$  in the fuel, again precluding criticality in natural uranium. Instead, in order to fuel a light-water reactor, first the amount of  $^{235}\text{U}$  in the uranium must be increased, producing enriched uranium, which generally contains between 3% and 5%  $^{235}\text{U}$  by weight (the waste from this process is known as depleted uranium, consisting primarily of  $^{238}\text{U}$ ). In this enriched form there is enough  $^{235}\text{U}$  to react with the water-moderated neutrons to maintain criticality. An alternative solution to the problem is to use a moderator that does not absorb neutrons as readily as water. In this case potentially all of the neutrons being released can be moderated and used in reactions with the  $^{235}\text{U}$ , in which case there is enough  $^{235}\text{U}$  in natural uranium to sustain criticality. One such moderator is heavy water, or deuterium-oxide. Although it reacts dynamically with the neutrons in a similar fashion to light water (albeit with less energy transfer on average, given that heavy hydrogen, or deuterium, is about twice the mass of hydrogen), it already has the extra neutron that light water would normally tend to absorb. **Nuclear power** is the fourth-largest source of electricity in India after thermal, hydroelectric and renewable sources of electricity. As of 2013, India has 21 nuclear reactors in operation in 7 nuclear power plants, having an installed capacity of 5308 MW and producing a total of 30,292.91 GW h of electricity while seven other reactors are under construction and are expected to generate an additional 6,100 MW. Despite the opposition, the capacity factor of Indian reactors was at 79% in the year 2011-12 compared to 71% in 2010-11. Nine out of twenty Indian reactors recorded an unprecedented 97% Capacity factor during 2011-12. With the imported uranium from France, the 220 MW Kakrapar 2 PHWR reactors recorded 99% capacity factor during 2011-12. The Availability factor for the year 2011-12 was at 89%. In **Nuclear Reactor** the leakage in form of radiations becomes highly dangerous to the lives of living beings. The radiations from the nuclear reactor are always under serious consideration due to fatal and miserable results to human race. In the present paper we have taken two-dissimilar warm standby system with failure due to extremely high radiations- **FHER** and failure due to non-availability of heavy water in nuclear power plant -**FNAHW**

### Assumptions

1. The failure time distribution is exponential whereas the repair time distribution is arbitrary of two non-identical units.
2. The repair starts immediately upon failure of units and the repair discipline is FCFS.
3. The repairs are perfect and start immediately as soon as the extremely high radiations of the system become normal. The radiations of both the units do not go extremely high.
4. The failure of a unit is detected immediately and perfectly.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.

## SYMBOLS FOR STATES OF THE SYSTEM

**Superscripts:** O, WS, SO, FEHR, FNAHW

Operative, Warm Standby, Stops the operation, Failure due to extremely high radiations, failure due to non-availability of heavy water in nuclear power plant respectively

**Subscripts:** nehr, ehr, nahw, ur, wr, uR

No extremely high radiations. Extremely high radiations, non-availability due to heavy water, under repair, waiting for repair, under repair continued respectively

**Up states:** 0, 1, 2, 9;

**Down states:** 3, 4, 5, 6, 7, 8, 10, 11

**Regeneration point:** 0, 1, 2, 4, 7, 10

### States of the System

**0( $O_{\text{nehr}}$ ,  $WS_{\text{nehr}}$ )** One unit is operative and the other unit is warm standby and there is no extremely high radiations in both the units.

**1( $SO_{\text{nehr}}$ ,  $O_{\text{nehr}}$ )**

The operation of the first unit stops automatically due to extremely high radiations and warm standby units starts operating and there is no extremely high radiations.

**2( $FEHR_{\text{ehr,ur}}$ ,  $O_{\text{nehr}}$ )**

The first unit fails and undergoes repair after failure due to extremely high radiations are over and the second unit continues to be operative with no extremely high radiations.

**3(FEHR<sub>chr,uR</sub>, SO<sub>chr</sub>)**

The repair of the first unit is continued from state 2 and in the other unit extremely high radiations occur and stops automatically due to extremely high radiations.

**4(FEHR<sub>chr,ur</sub>, SO<sub>uehr</sub>)**

The one unit fails and undergoes repair after the extremely high radiations are over and the other unit also stops automatically due to extremely high radiations.

**5(FEHR<sub>chr,uR</sub>, FEHR<sub>chr,wr</sub>)** The repair of the first unit is continued from state 4 and the other unit is failed due to extremely high radiations in it and is waiting for repair.

**6(O<sub>nehr</sub>, FEHR<sub>chr,ur</sub>)**

The first unit is operative with no extremely high radiations and the second unit failed due to extremely high radiations is under repair.

**7(SO<sub>nehr</sub>, FNAF<sub>nahw,ur</sub>)**

The operation of the first unit stops automatically due to extremely high radiations and the second unit fails due non-availability of heavy water in nuclear power reactor and undergoes repair.

**8(FEHR<sub>chr,wr</sub>, FNAHW<sub>nahw,uR</sub>)**

The repair of failed switch is continued from state 7 and the first unit is failed after extremely high radiations and waiting for repair.

**9(O<sub>nehr</sub>, SO<sub>uehr</sub>)**

The first unit is operative and the warm standby dissimilar unit is under extremely high radiations

**10(SO<sub>nehr</sub>, FNAHW<sub>nahw,ur</sub>)**

The operation of the first unit stops automatically due to extremely high radiations and the second unit fails due to non-availability of heavy water in nuclear power reactor and undergoes repair after the extremely high radiations is over.

**11(FEHR<sub>chr,wr</sub>, FNAHW<sub>nahw,uR</sub>)**

The repair of the second unit is continued from state 10 and the first unit is failed due to extremely high radiations is waiting for repair.

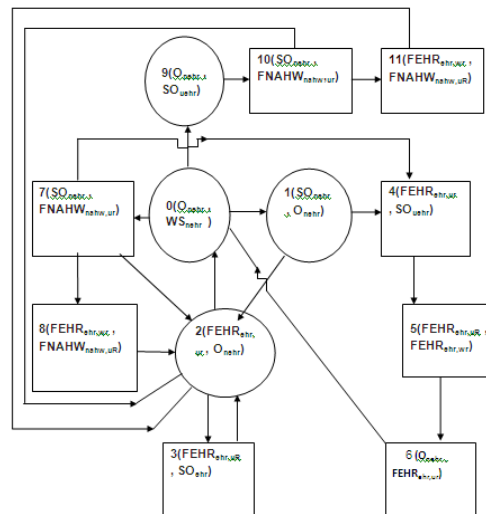


Figure 1: The State Transition Diagram

○ Up state    □ Down state

## TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$$P_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}, P_{07} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$P_{09} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}, P_{12} = \frac{\lambda_1}{\lambda_1 + \lambda_3}, P_{14} = \frac{\lambda_3}{\lambda_1 + \lambda_3}$$

$$P_{20} = G_1^*(\lambda_1), P_{22}^{(3)} = G_1^*(\lambda_1) = p_{23}, \overline{P}_{72} = G_2^*(\lambda_4), \\ P_{72}^{(8)} = G_2^*(\lambda_4) = P_{78}$$

We can easily verify that

$$p_{01} + p_{07} + p_{09} = 1, p_{12} + p_{14} = 1, p_{20} + p_{23} = p_{22}^{(3)} = 1, p_{46}^{(6)} = 1, p_{60} = 1, \\ p_{72} + P_{72}^{(5)} + p_{74} = 1, p_{9,10} = 1, p_{10,2} + p_{10,2}^{(11)} = 1 \quad (1)$$

And mean sojourn time is

$$\mu_0 = E(T) = \int_0^\infty P[T > t] dt \quad (2)$$

### Mean Time to System Failure

We can regard the failed state as absorbing

$$\theta_0(t) = Q_{01}(t)[s]\theta_1(t) + Q_{09}(t)[s]\theta_9(t) + Q_{07}(t) \\ \theta_1(t) = Q_{12}(t)[s]\theta_2(t) + Q_{14}(t), \theta_2(t) = Q_{20}(t)[s]\theta_0(t) + Q_{22}^{(3)}(t) \\ \theta_4(t) = Q_{9,10}(t) \quad (3-5)$$

Taking Laplace-Stiltjes transform of eq. (3-5) and solving for

$$Q_0^*(s) = N_1(s) / D_1(s) \quad (6)$$

Where

$$N_1(s) = Q_{01}^*(s) \{ Q_{12}^*(s) Q_{22}^{(3)*}(s) + Q_{14}^*(s) \} + Q_{09}^*(s) Q_{9,10}^*(s) + Q_{07}^*(s) \\ D_1(s) = 1 - Q_{01}^*(s) Q_{12}^*(s) Q_{20}^*(s)$$

Making use of relations (1) and (2) it can be shown that  $\theta_0(0) = 1$ , which implies that  $\theta_0(t)$  is a proper distribution.

$$MTSF = E[T] = d/ds \theta_0^*(0) \Big|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$$= (\mu_0 + p_{01} \mu_1 + p_{01} p_{12} \mu_2 + p_{09} \mu_9) / (1 - p_{01} p_{12} p_{20})$$

Where

$$\mu_0 = \mu_{01} + \mu_{07} + \mu_{09}, \mu_1 = \mu_{12} + \mu_{14}, \mu_2 = \mu_{20} + \mu_{22}^{(3)}, \mu_9 = \mu_{9,10}$$

### AVAILABILITY ANALYSIS

Let  $M_i(t)$  be the probability of the system having started from state I is up at time t without making any other regenerative state. By probabilistic arguments, we have

The value of  $M_0(t)$ ,  $M_1(t)$ ,  $M_2(t)$ ,  $M_4(t)$  can be found easily.

The point wise availability  $A_i(t)$  have the following recursive relations

$$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{07}(t)[c]A_7(t) + q_{09}(t)[c]A_9(t) \\ A_1(t) = M_1(t) + q_{12}(t)[c]A_2(t) + q_{14}(t)[c]A_4(t), A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + q_{22}^{(3)}(t)[c]A_2(t) \\ A_4(t) = q_{46}^{(3)}(t)[c]A_6(t), A_6(t) = q_{60}(t)[c]A_0(t) \\ A_7(t) = (q_{72}(t) + q_{72}^{(8)}(t)) [c]A_2(t) + q_{74}(t)[c]A_4(t) \\ A_9(t) = M_9(t) + q_{9,10}(t)[c]A_{10}(t), A_{10}(t) = q_{10,2}(t)[c]A_2(t) + q_{10,2}^{(11)}(t)[c]A_2(t) \quad (7-14)$$

Taking Laplace Transform of eq. (7-14) and solving for  $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \quad (15)$$

Where

$$N_2(s) = (1 - \hat{q}_{22}^{(3)}(s)) \{ \hat{M}_0(s) + \hat{q}_{01}(s) \hat{M}_1(s) + \hat{q}_{09}(s) \hat{M}_9(s) \} + \hat{M}_2(s) \{ \hat{q}_{01}(s) \hat{q}_{42}(s) + \hat{q}_{07}(s) (\hat{q}_{72}(s) + \hat{q}_{72}^{(8)}(s)) + \hat{q}_{09} \\ (s) \hat{q}_{9,10}(s) (\hat{q}_{10,2}(s) + \hat{q}_{10,2}^{(11)}(s)) \} \\ D_2(s) = (1 - \hat{q}_{22}^{(3)}(s)) \{ 1 - \hat{q}_{46}^{(5)}(s) \hat{q}_{60}(s) (\hat{q}_{01}(s) \hat{q}_{44}(s) + \hat{q}_{07}(s) \hat{q}_{74}(s)) \\ - \hat{q}_{20}(s) \{ \hat{q}_{01}(s) \hat{q}_{12}(s) + \hat{q}_{07}(s) (\hat{q}_{72}(s) + \hat{q}_{72}^{(8)}(s) + \hat{q}_{09}(s) \hat{q}_{9,10}(s) \\ (\hat{q}_{10,2}(s) + \hat{q}_{10,2}^{(11)}(s)) \} \}$$

The steady state availability

$$A_0 = \lim_{t \rightarrow \infty} [A_0(t)] = \lim_{s \rightarrow 0} [s \hat{A}_0(s)] = \lim_{s \rightarrow 0} \frac{s N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s \rightarrow 0} \frac{N_2(s) + s N_2'(s)}{D_2(s)} = \frac{N_2(0)}{D_2'(0)} \quad (16)$$

Where

$$N_2(0) = p_{20}(\hat{M}_0(0) + p_{01}\hat{M}_1(0) + p_{09}\hat{M}_9(0)) + \hat{M}_2(0) (p_{01}p_{12} + p_{07}(p_{72} \\ + p_{72}^{(8)} + p_{09}))$$

$$D_2'(0) = p_{20} \{ \mu_0 + p_{01} \mu_1 + (p_{01} p_{14} + p_{07} p_{74}) \mu_4 + p_{07} \mu_7 + p_{07} \mu_7 + p_{09}(\mu_9 + \mu_{10}) + \mu_2 \{ 1 - ((p_{01} p_{14} + p_{07} p_{74})) \} \}$$

$$\mu_4 = \mu_{46}^{(5)}, \mu_7 = \mu_{72} + \mu_{72}^{(8)} + \mu_{74}, \mu_{10} = \mu_{10,2} + \mu_{10,2}^{(11)}$$

The expected up time of the system in (0, t] is

$$\lambda_u(t) = \int_0^\infty A_0(z) dz \text{ So that } \widehat{\lambda}_u(s) = \frac{\widehat{A}_0(s)}{s} = \frac{N_2(s)}{SD_2(s)} \quad (17)$$

The expected down time of the system in (0, t] is

$$\lambda_d(t) = t - \lambda_u(t) \text{ So that } \widehat{\lambda}_d(s) = \frac{1}{s^2} - \widehat{\lambda}_u(s) \quad (18)$$

**The expected busy period of the server for repairing the failed unit under extremely high radiations in (0, t]**

$$R_0(t) = S_0(t) + q_{01}(t)[c]R_1(t) + q_{07}(t)[c]R_7(t) + q_{09}(t)[c]R_9(t)$$

$$R_1(t) = S_1(t) + q_{12}(t)[c]R_2(t) + q_{14}(t)[c]R_4(t),$$

$$R_2(t) = q_{20}(t)[c]R_0(t) + q_{22}^{(3)}(t)[c]R_2(t)$$

$$R_4(t) = q_{46}^{(3)}(t)[c]R_6(t), R_6(t) = q_{60}(t)[c]R_0(t)$$

$$R_7(t) = (q_{72}(t) + q_{72}^{(8)}(t)) [c]R_2(t) + q_{74}(t)[c]R_4(t)$$

$$R_9(t) = S_9(t) + q_{9,10}(t)[c]R_{10}(t), R_{10}(t) = q_{10,2}(t) + q_{10,2}^{(11)}(t)[c]R_2(t) \quad (19-26)$$

Taking Laplace Transform of eq. (19-26) and solving for  $\widehat{R}_0(s)$

$$\widehat{R}_0(s) = N_3(s) / D_2(s) \quad (27)$$

Where

$$N_2(s) = (1 - \widehat{q}_{22}^{(3)}(s)) \{ \widehat{S}_0(s) + \widehat{q}_{01}(s) \widehat{S}_1(s) + \widehat{q}_{09}(s) \widehat{S}_9(s) \} \text{ and } D_2(s) \text{ is already defined.}$$

$$\text{In the long run, } R_0 = \frac{N_3(0)}{D_2'(0)} \quad (28)$$

where  $N_3(0) = p_{20}(\widehat{S}_0(0) + p_{01}\widehat{S}_1(0) + p_{09}\widehat{S}_9(0))$  and  $D_2'(0)$  is already defined.

The expected period of the system under extremely high radiations in (0, t] is

$$\lambda_{rv}(t) = \int_0^\infty R_0(z) dz \text{ So that } \widehat{\lambda}_{rv}(s) = \frac{\widehat{R}_0(s)}{s}$$

**The expected Busy period of the server for repair of dissimilar units by the repairman in (0, t]**

$$B_0(t) = q_{01}(t)[c]B_1(t) + q_{07}(t)[c]B_7(t) + q_{09}(t)[c]B_9(t)$$

$$B_1(t) = q_{12}(t)[c]B_2(t) + q_{14}(t)[c]B_4(t), B_2(t) = q_{20}(t)[c]B_0(t) + q_{22}^{(3)}(t)[c]B_2(t)$$

$$B_4(t) = T_4(t) + q_{46}^{(3)}(t)[c]B_6(t), B_6(t) = T_6(t) + q_{60}(t)[c]B_0(t)$$

$$B_7(t) = (q_{72}(t) + q_{72}^{(8)}(t)) [c]B_2(t) + q_{74}(t)[c]B_4(t)$$

$$B_9(t) = q_{9,10}(t)[c]B_{10}(t), B_{10}(t) = T_{10}(t) + (q_{10,2}(t) + q_{10,2}^{(11)}(t)[c]B_2(t) \quad (29-36)$$

Taking Laplace Transform of eq. (29-36) and solving for  $\widehat{B}_0(s)$

$$\widehat{B}_0(s) = N_4(s) / D_2(s) \quad (37)$$

Where

$$N_4(s) = (1 - \widehat{q}_{22}^{(3)}(s)) \{ \widehat{q}_{01}(s) \widehat{q}_{14}(s) (\widehat{T}_4(s) + \widehat{q}_{46}^{(5)}(s) \widehat{T}_6(s)) + \widehat{q}_{07}^{(3)}(s) \widehat{q}_{74}(s) (\widehat{T}_4(s) + \widehat{q}_{46}^{(5)}(s) \widehat{T}_6(s)) + \widehat{q}_{09}(s) \widehat{q}_{9,10}(s) \widehat{T}_{10}(s) \}$$

And  $D_2(s)$  is already defined.

$$\text{In steady state, } B_0 = \frac{N_4(0)}{D_2'(0)} \quad (38)$$

where  $N_4(0) = p_{20} \{ (p_{01} p_{14} + p_{07} p_{74}) (\widehat{T}_4(0) + \widehat{T}_6(0)) + p_{09} \widehat{T}_{10}(0) \}$  and  $D_2'(0)$  is already defined.

The expected busy period of the server for repair in (0, t] is

$$\lambda_{ru}(t) = \int_0^\infty B_0(z) dz \text{ So that } \widehat{\lambda}_{ru}(s) = \frac{\widehat{B}_0(s)}{s} \quad (39)$$

**The expected Busy period of the server for repair of unit for failure due non-availability of heavy water in nuclear power reactors in (0, t]**

$$P_0(t) = q_{01}(t)[c]P_1(t) + q_{07}(t)[c]P_7(t) + q_{09}(t)[c]P_9(t)$$

$$P_1(t) = q_{12}(t)[c]P_2(t) + q_{14}(t)[c]P_4(t), P_2(t) = q_{20}(t)[c]P_0(t) + q_{22}^{(3)}(t)[c]P_2(t)$$

$$P_4(t) = q_{46}^{(3)}(t)[c]P_6(t), P_6(t) = q_{60}(t)[c]P_0(t)$$

$$P_7(t) = L_7(t) + (q_{72}(t) + q_{72}^{(8)}(t)) [c]P_2(t) + q_{74}(t)[c]P_4(t)$$

$$P_9(t) = q_{9,10}(t)[c]P_{10}(t), P_{10}(t) = (q_{10,2}(t) + q_{10,2}^{(11)}(t)[c]P_2(t) \quad (40-47)$$

Taking Laplace Transform of eq. (40-47) and solving for

$$\widehat{P}_0(s) = N_5(s) / D_2(s) \quad (48)$$

where  $N_2(s) = \hat{Q}_{07}(s) \hat{L}_7(s) (1 - \hat{q}_{22}^{(3)}(s))$  and  $D_2(s)$  is defined earlier.

$$\text{In the long run, } P_0 = \frac{N_5(0)}{D_2'(0)} \quad (49)$$

where  $N_5(0) = p_{20} p_{07} \hat{L}_4(0)$  and  $D_2'(0)$  is already defined.

The expected busy period of the server for repair of the in  $(0, t]$  is

$$\lambda_{rs}(t) = \int_0^\infty P_0(z) dz \text{ So that } \widehat{\lambda_{rs}}(s) = \frac{\hat{P}_0(s)}{s} \quad (50)$$

**The expected number of visits by the repairman for repairing the different units in  $(0, t]$**

$$\begin{aligned} H_0(t) &= Q_{01}(t)[c]H_1(t) + Q_{07}(t)[c]H_7(t) + Q_{09}(t)[c]H_9(t) \\ H_1(t) &= Q_{12}(t)[c][1+H_2(t)] + Q_{14}(t)[c][1+H_4(t)], H_2(t) = Q_{20}(t)[c]H_0(t) + Q_{22}^{(3)}(t)[c]H_2(t) \\ H_4(t) &= Q_{46}^{(3)}(t)[c]H_6(t), H_6(t) = Q_{60}(t)[c]H_0(t) \\ H_7(t) &= (Q_{72}(t) + Q_{72}^{(8)}(t)) [c]H_2(t) + Q_{74}(t)[c]H_4(t) \\ H_9(t) &= Q_{9,10}(t)[c][1+H_{10}(t)], H_{10}(t) = (Q_{10,2}(t)[c] + Q_{10,2}^{(11)}(t))[c]H_2(t) \end{aligned} \quad (51-58)$$

Taking Laplace Transform of eq. (51-58) and solving for  $H_0^*(s)$

$$H_0^*(s) = N_6(s) / D_3(s) \quad (59)$$

Where

$$\begin{aligned} N_6(s) &= (1 - Q_{22}^{(3)*}(s)) \{ Q_{01}^*(s)(Q_{12}^*(s) + Q_{14}^*(s)) + Q_{09}^*(s) Q_{9,10}^*(s) \} \\ D_3(s) &= (1 - Q_{22}^{(3)*}(s)) \{ 1 - (Q_{01}^*(s) Q_{14}^*(s) + Q_{07}^*(s) Q_{74}^*(s)) Q_{46}^{(5)*}(s) Q_{60}^*(s) \} - Q_{20}^*(s) \{ Q_{01}^*(s) Q_{12}^*(s) + Q_{07}^*(s) \\ &\quad (Q_{72}^*(s) + Q_{72}^{(8)*}(s)) + \\ &\quad Q_{09}^*(s) Q_{9,10}^*(s) (Q_{10,2}^*(s) + Q_{10,2}^{(11)*}(s)) \} \end{aligned}$$

$$\text{In the long run, } H_0 = \frac{N_6(0)}{D_3'(0)} \quad (60)$$

where  $N_6(0) = p_{20} (p_{01} + p_{09})$  and  $D_3'(0)$  is already defined.

**The expected number of visits by the repairman for repairing the unit for failure due non-availability of heavy water in nuclear power reactors in  $(0, t]$**

$$\begin{aligned} V_0(t) &= Q_{01}(t)[c]V_1(t) + Q_{07}(t)[c]V_7(t) + Q_{09}(t)[c]V_9(t) \\ V_1(t) &= Q_{12}(t)[c]V_2(t) + Q_{14}(t)[c]V_4(t), V_2(t) = Q_{20}(t)[c]V_0(t) + Q_{22}^{(3)}(t)[c]V_2(t) \\ V_4(t) &= Q_{46}^{(3)}(t)[c]V_6(t), V_6(t) = Q_{60}(t)[c]V_0(t) \\ V_7(t) &= (Q_{72}(t)[1+V_2(t)] + Q_{72}^{(8)}(t)) [c]V_2(t) + Q_{74}(t)[c]V_4(t) \\ V_9(t) &= Q_{9,10}(t)[c]V_{10}(t), V_{10}(t) = (Q_{10,2}(t) + Q_{10,2}^{(11)}(t))[c]V_2(t) \end{aligned} \quad (61-68)$$

Taking Laplace-Stieltjes transform of eq. (61-68) and solving for  $V_0^*(s)$

$$V_0^*(s) = N_7(s) / D_4(s) \quad (69)$$

where  $N_7(s) = Q_{07}^*(s) Q_{72}^*(s) (1 - Q_{22}^{(3)*}(s))$  and  $D_4(s)$  is the same as  $D_3(s)$

$$\text{In the long run, } V_0 = \frac{N_7(0)}{D_4'(0)} \quad (70)$$

where  $N_7(0) = p_{20} p_{07} p_{72}$  and  $D_4'(0)$  is already defined.

## COST BENEFIT ANALYSIS

The cost-benefit function of the system considering mean up-time, expected busy period of the system under extremely high radiations when the units stops automatically, expected busy period of the server for repair of unit for failure due non-availability of heavy water in nuclear power reactors, expected number of visits by the repairman for unit failure, expected number of visits by the repairman for failure due non-availability of heavy water in nuclear power reactors. The expected total cost-benefit incurred in  $(0, t]$  is

$C(t) =$  Expected total revenue in  $(0, t]$

- expected total repair cost for unit failure due non-availability of heavy water in nuclear power reactors in  $(0, t]$  - expected total repair cost for repairing the units in  $(0, t]$
- expected busy period of the system under extremely high radiations when the units automatically stop in  $(0, t]$
- expected number of visits by the repairman for repairing the unit failure due non-availability of heavy water in nuclear power reactors in  $(0, t]$
- expected number of visits by the repairman for repairing of the units in  $(0, t]$

The expected total cost per unit time in steady state is

$$\begin{aligned} C &= \lim_{t \rightarrow \infty} (C(t)/t) = \lim_{s \rightarrow 0} (s^2 C(s)) \\ &= K_1 A_0 - K_2 P_0 - K_3 B_0 - K_4 R_0 - K_5 V_0 - K_6 H_0 \end{aligned}$$

Where

$K_1$ : revenue per unit up-time,

$K_2$ : cost per unit time for which the system is under repair failure due non-availability of heavy water in nuclear power

$K_3$ : cost per unit time for which the system is under unit repair

$K_4$ : when units automatically stop cost per unit time for which the system is under extremely high radiations

$K_5$ : cost per visit by the repairman for which unit under repair for failure due non-availability of heavy water in nuclear power reactors

$K_6$ : cost per visit by the repairman for units repair.

## CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to non-availability of heavy water in nuclear power reactors, failure rate due to extremely high radiations increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

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