

Gain-function of two non-identical warm standby systems with failure due to non-availability of sunlight and failure due to ultra-violet radiations

Ashok Kumar Saini

Associate Professor, Department of Mathematics, B. L. J. S. College, Tosham, Bhiwani, Haryana, INDIA.

Email: drashokksaini2009@gmail.com

Abstract

Introduction: Ultraviolet of wavelengths from 10 nm to 125 nm ionizes air molecules, and this interaction causes it to be strongly absorbed by air, ozone (O_3) in particular. Ionizing UV therefore does not penetrate Earth's atmosphere to a significant degree, and is sometimes referred to as vacuum ultraviolet. There is a zone of the atmosphere in which ozone absorbs some 98% of UV, starting about 20 miles (32 km) high and extending upward. Although present in space, this part of the UV spectrum is not of biological importance, because it does not reach living organisms on Earth, thanks to this ozone layer. In **Solar System** the sunlight plays a important and vital role. The non-conventional renewable **solar energy** which is cheap and readily available for use in institutions, hospitals, industries and all sort of equipments and places where energy is required. But for solar energy **Sun** is the prime source from where solar energy can be generated. During rainy and winter seasons the sun is under the cover of clouds regularly resulting solar penal cells are unable to receive sunlight which causing failure of the system. In the present paper we have taken two non-identical warm standby system with failure due to non- availability of sunlight. When there is non-availability of sunlight the working of unit stops automatically. The failure time distribution is taken as exponential and repair time distribution as general. Using Semi Markov regenerative point technique we have calculated different reliability characteristics such as MTSF, reliability of the system, availability analysis in steady state, busy period analysis of the system under repair, expected number of visits by the repairman in the long run and profit-function. Special case by taking repair as exponential has been derived and graphs are drawn.

Keyword: warm standby, non-availability of sunlight, ultra-violet radiations, MTSF, Availability, busy period, Gain-function.

*Address for Correspondence:

Dr. Ashok Kumar Saini, Associate Professor, Department of Mathematics, BLJS College, Tosham, Bhiwani, Haryana, INDIA.

Email: drashokksaini2009@gmail.com

Received Date: 25/09/2014 Accepted Date: 07/10/2014

Access this article online

Quick Response Code:



Website:

www.statperson.com

DOI: 10 October 2014

INTRODUCTION

Some of the ultraviolet spectrum that does reach the ground (the part that begins above energies of 3.1 eV, or wavelength less than 400 nm) is non-ionizing, but is still biologically hazardous due to the ability of single photons of this energy to cause electronic excitation in biological molecules, and thus damage them by means of unwanted reactions. An example is the formation of **pyrimidine dimers** in DNA, which begins at wavelengths below 365 nm (3.4 eV), which is well below ionization energy. This property gives the ultraviolet spectrum some of the dangers of ionizing radiation in biological systems without actual ionization occurring. In contrast, visible light and longer-wavelength electromagnetic radiation,

such as infrared, microwaves, and radio waves, consists of photons with too little energy to cause damaging molecular excitation, and thus this radiation is far less hazardous per unit of energy. In **Solar System** the sunlight plays the pivotal and vital role. The non-conventional renewable **solar energy** which is cheap and easily available for use in institutions, hospitals, industries and all sort of places and equipments where energy is required. But for solar energy **Sun** is the primary source from where solar energy can be generated. The sun is under the cover of clouds almost every day during rainy and winter seasons causing solar penal cells not receiving sunlight and becomes inactive unable to generate solar energy resulting failure of the system.

Assumptions

1. The failure time distribution is exponential whereas the repair time distribution is arbitrary of two non-identical units.
2. The repairs are perfect and starts immediately upon failure of units with repair discipline are FCFS.
3. The operation of the unit stops as soon as there is non-availability of sunlight.
4. The failure of a unit is detected immediately and perfectly.
5. The switches are instantaneous and perfect.
6. All random variables are mutually independent.

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts: O, WS, SO, FNASL, FEHR

Operative, Warm Standby, Stops the operation, Failure due to non-availability of sunlight, ultra-violet radiations respectively

Subscripts: $nasl$, asl , uvr , ur , wr , uR

Non-availability of sunlight, availability of sunlight, ultra-violet radiations, under repair, waiting for repair, under repair continued respectively.

Up States: 0, 1, 2, 9;

Down states: 3, 4, 5, 6, 7, 8, 10, 11

Regeneration point: 0, 1, 2, 4, 7, 10

States of the System

0(O_{asl} , WS_{nasl})

The first unit is operative due to availability of sunlight and the second unit is warm standby with non-availability of sunlight.

1(SO_{nasl} , O_{asl})

The operation of the first unit stops automatically due to non-availability of sunlight and warm standby unit starts operating due to availability of sunlight.

2($FNASL_{ur}$, O_{asl})

The first unit fails due to non-availability of sunlight undergoes repair and the second unit continues to be operative due to availability of sunlight.

3($FNASL_{uR}$, SO_{nasl})

The repair of the first unit is continued from state 2 and in the other unit the operation of the unit stops automatically due to non-availability of sunlight.

4($FNASL_{ur}$, SO_{nasl})

The one unit fails due to non-availability of sunlight and undergoes repair and the other unit also stops automatically due to non-availability of sunlight.

5($FNASL_{uR}$, $FNASL_{wr}$)

The repair of the first unit is continued from state 4 and the other unit is failed due to non-availability of sunlight in it and is waiting for repair.

6 (O_{asl} , $FNASL_{ur}$)

The first unit is operative due to availability of sunlight and the second unit failed due to non-availability of sunlight is under repair.

7(SO_{nasl} , $FUVR_{uvr, ur}$)

The operation of the first unit stops automatically due to non-availability of sunlight and the second unit fails due to ultra-violet radiations and undergoes repair.

8($FNASL_{wr}$, $FUVR_{uvr, uR}$)

Making use of relations (1) and (2) it can be shown that $\theta_0(0) = 1$, which implies that $\theta_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = d/ds \theta_0^*(0) \Big|_{s=0} = (D_1(0) - N_1(0)) / D_1(0)$$

$$= (\mu_0 + p_{01} \mu_1 + p_{01} p_{12} \mu_2 + p_{09} \mu_9) / (1 - p_{01} p_{12} p_{20})$$

where

$$\mu_0 = \mu_{01} + \mu_{07} + \mu_{09}, \mu_1 = \mu_{12} + \mu_{14}, \mu_2 = \mu_{20} + \mu_{22}^{(3)}, \mu_9 = \mu_{9,10}$$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have The value of $M_0(t)$, $M_1(t)$, $M_2(t)$, $M_4(t)$ can be found easily.

The point wise availability $A_i(t)$ have the following recursive relations

$$\begin{aligned} A_0(t) &= M_0(t) + q_{01}(t)[c]A_1(t) + q_{07}(t)[c]A_7(t) + q_{09}(t)[c]A_9(t) \\ A_1(t) &= M_1(t) + q_{12}(t)[c]A_2(t) + q_{14}(t)[c]A_4(t), A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + q_{22}^{(3)}(t)[c]A_2(t) \\ A_4(t) &= q_{46}^{(3)}(t)[c]A_6(t), A_6(t) = q_{60}(t)[c]A_0(t) \\ A_7(t) &= (q_{72}(t) + q_{72}^{(8)}(t)) [c]A_2(t) + q_{74}(t)[c]A_4(t) \\ A_9(t) &= M_9(t) + q_{9,10}(t)[c]A_{10}(t), A_{10}(t) = q_{10,2}(t)[c]A_2(t) + q_{10,2}^{(11)}(t)[c]A_2(t) \end{aligned} \quad (7-14)$$

Taking Laplace Transform of eq. (7-14) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \quad (15)$$

Where

$$\begin{aligned} N_2(s) &= (1 - \hat{q}_{22}^{(3)}(s)) \{ \hat{M}_0(s) + \hat{q}_{01}(s) \hat{M}_1(s) + \hat{q}_{09}(s) \hat{M}_9(s) \} + \hat{M}_2(s) \{ \hat{q}_{01}(s) \hat{q}_{42}(s) + \hat{q}_{07}(s) (\hat{q}_{72}(s) + \hat{q}_{73}^{(8)}(s)) + \hat{q}_{09} \\ &\quad (s) \hat{q}_{9,10}(s) (\hat{q}_{10,2}(s) + \hat{q}_{10,2}^{(11)}(s)) \} \\ D_2(s) &= (1 - \hat{q}_{22}^{(3)}(s)) \{ 1 - \hat{q}_{46}^{(5)}(s) \hat{q}_{60}(s) (\hat{q}_{01}(s) \hat{q}_{44}(s) + \hat{q}_{07}(s) \hat{q}_{74}(s)) \\ &\quad - \hat{q}_{20}(s) \{ \hat{q}_{01}(s) \hat{q}_{12}(s) + \hat{q}_{07}(s) (\hat{q}_{72}(s)) + \hat{q}_{72}^{(8)}(s) + \hat{q}_{09}(s) \hat{q}_{9,10}(s) \\ &\quad (\hat{q}_{10,2}(s) + \hat{q}_{10,2}^{(11)}(s)) \} \} \end{aligned}$$

The steady state availability

$$A_0 = \lim_{t \rightarrow \infty} [A_0(t)] = \lim_{s \rightarrow 0} [s \hat{A}_0(s)] = \lim_{s \rightarrow 0} \frac{s N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s \rightarrow 0} \frac{N_2(s) + s N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \quad (16)$$

Where

$$N_2(0) = p_{20}(\hat{M}_0(0) + p_{01}\hat{M}_1(0) + p_{09}\hat{M}_9(0)) + \hat{M}_2(0) (p_{01}p_{12} + p_{07}(p_{72} + p_{72}^{(8)} + p_{09}))$$

$$D_2(0) = p_{20} \{ \mu_0 + p_{01} \mu_1 + (p_{01} p_{14} + p_{07} p_{74}) \mu_4 + p_{07} \mu_7 + p_{07} \mu_7 + p_{09}(\mu_9 + \mu_{10}) + \mu_2 \{ 1 - ((p_{01} p_{14} + p_{07} p_{74})) \}$$

$$\mu_4 = \mu_{46}^{(5)}, \mu_7 = \mu_{72} + \mu_{72}^{(8)} + \mu_{74}, \mu_{10} = \mu_{10,2} + \mu_{10,2}^{(11)}$$

The expected up time of the system in $(0, t]$ is

$$\lambda_u(t) = \int_0^\infty A_0(z) dz \text{ So that } \widehat{\lambda}_u(s) = \frac{\widehat{A}_0(s)}{s} = \frac{N_2(s)}{s D_2(s)} \quad (17)$$

The expected down time of the system in $(0, t]$ is

$$\lambda_d(t) = t - \lambda_u(t) \text{ So that } \widehat{\lambda}_d(s) = \frac{1}{s^2} - \widehat{\lambda}_u(s) \quad (18)$$

The expected busy period of the server for repairing the failed unit under non-availability of sunlight in $(0, t]$

$$R_0(t) = S_0(t) + q_{01}(t)[c]R_1(t) + q_{07}(t)[c]R_7(t) + q_{09}(t)[c]R_9(t)$$

$$R_1(t) = S_1(t) + q_{12}(t)[c]R_2(t) + q_{14}(t)[c]R_4(t),$$

$$R_2(t) = q_{20}(t)[c]R_0(t) + q_{22}^{(3)}(t)[c]R_2(t)$$

$$R_4(t) = q_{46}^{(3)}(t)[c]R_6(t), R_6(t) = q_{60}(t)[c]R_0(t)$$

$$R_7(t) = (q_{72}(t) + q_{72}^{(8)}(t)) [c]R_2(t) + q_{74}(t)[c]R_4(t)$$

$$R_9(t) = S_9(t) + q_{9,10}(t)[c]R_{10}(t), R_{10}(t) = q_{10,2}(t) + q_{10,2}^{(11)}(t)[c]R_2(t) \quad (19-26)$$

Taking Laplace Transform of eq. (19-26) and solving for $\widehat{R}_0(s)$

$$\widehat{R}_0(s) = N_3(s) / D_2(s) \quad (27)$$

Where

$N_2(s) = (1 - \hat{q}_{22}^{(3)}(s)) \{ \hat{S}_0(s) + \hat{q}_{01}(s) \hat{S}_1(s) + \hat{q}_{09}(s) \hat{S}_9(s) \}$ and $D_2(s)$ is already defined.

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (28)

where $N_3(0) = p_{20}(\hat{S}_0(0) + p_{01}\hat{S}_1(0) + p_{09}\hat{S}_9(0))$ and $D_2'(0)$ is already defined.

The expected period of the system under non-availability of sunlight in $(0, t]$ is

$$\lambda_{rv}(t) = \int_0^\infty R_0(z) dz \text{ So that } \widehat{\lambda_{rv}}(s) = \frac{\widehat{R}_0(s)}{s}$$

The expected Busy period of the server for repair of dissimilar units by the repairman in $(0, t]$

$$B_0(t) = q_{01}(t)[c]B_1(t) + q_{07}(t)[c]B_7(t) + q_{09}(t)[c]B_9(t)$$

$$B_1(t) = q_{12}(t)[c]B_2(t) + q_{14}(t)[c]B_4(t), B_2(t) = q_{20}(t)[c]B_0(t) + q_{22}^{(3)}(t)[c]B_2(t)$$

$$B_4(t) = T_4(t) + q_{46}^{(3)}(t)[c]B_6(t), B_6(t) = T_6(t) + q_{60}(t)[c]B_0(t)$$

$$B_7(t) = (q_{72}(t) + q_{72}^{(8)}(t)) [c]B_2(t) + q_{74}(t)[c]B_4(t)$$

$$B_9(t) = q_{9,10}(t)[c]B_{10}(t), B_{10}(t) = T_{10}(t) + (q_{10,2}(t) + q_{10,2}^{(11)}(t))[c]B_2(t)$$

Taking Laplace Transform of eq. (29-36) and solving for $\widehat{B}_0(s)$

$$\widehat{B}_0(s) = N_4(s) / D_2(s) \quad (37)$$

Where

$$N_4(s) = (1 - \hat{q}_{22}^{(3)}(s)) \{ \hat{q}_{01}(s) \hat{q}_{14}(s) (\hat{T}_4(s) + \hat{q}_{46}^{(5)}(s) \hat{T}_6(s)) + \hat{q}_{07}^{(3)}(s) \hat{q}_{74}(s) (\hat{T}_4(s) + \hat{q}_{46}^{(5)}(s) \hat{T}_6(s)) + \hat{q}_{09}(s) \hat{q}_{09,10}(s) \hat{T}_{10}(s) \}$$

And $D_2(s)$ is already defined.

$$\text{In steady state, } B_0 = \frac{N_4(0)}{D_2'(0)} \quad (38)$$

where $N_4(0) = p_{20} \{ (p_{01} p_{14} + p_{07} p_{74}) (\hat{T}_4(0) + \hat{T}_6(0)) + p_{09} \hat{T}_{10}(0) \}$ and $D_2'(0)$ is already defined.

The expected busy period of the server for repair in $(0, t]$ is

$$\lambda_{ru}(t) = \int_0^\infty B_0(z) dz \text{ So that } \widehat{\lambda_{ru}}(s) = \frac{\widehat{B}_0(s)}{s} \quad (39)$$

The expected Busy period of the server for repair when unit failed due to ultra-violet radiations in $(0, t]$

$$P_0(t) = q_{01}(t)[c]P_1(t) + q_{07}(t)[c]P_7(t) + q_{09}(t)[c]P_9(t)$$

$$P_1(t) = q_{12}(t)[c]P_2(t) + q_{14}(t)[c]P_4(t), P_2(t) = q_{20}(t)[c]P_0(t) + q_{22}^{(3)}(t)[c]P_2(t)$$

$$P_4(t) = q_{46}^{(3)}(t)[c]P_6(t), P_6(t) = q_{60}(t)[c]P_0(t)$$

$$P_7(t) = L_7(t) + (q_{72}(t) + q_{72}^{(8)}(t)) [c]P_2(t) + q_{74}(t)[c]P_4(t)$$

$$P_9(t) = q_{9,10}(t)[c]P_{10}(t), P_{10}(t) = (q_{10,2}(t) + q_{10,2}^{(11)}(t))[c]P_2(t) \quad (40-47)$$

Taking Laplace Transform of eq. (40-47) and solving for

$$\widehat{P}_0(s) = N_5(s) / D_2(s) \quad (48)$$

where $N_2(s) = \hat{q}_{07}(s) \hat{L}_7(s) (1 - \hat{q}_{22}^{(3)}(s))$ and $D_2(s)$ is defined earlier.

$$\text{In the long run, } P_0 = \frac{N_5(0)}{D_2'(0)} \quad (49)$$

where

$$N_5(0) = p_{20} p_{07} \hat{L}_4(0)$$

and $D_2(0)$ is already defined.

The expected busy period of the server for repair of the failed unit due to ultra-violet radiations in $(0, t]$ is

$$\lambda_{rs}(t) = \int_0^\infty P_0(z) dz \text{ So that } \widehat{\lambda_{rs}}(s) = \frac{\widehat{P}_0(s)}{s} \quad (50)$$

The expected number of visits by the repairman for repairing the non-identical units in $(0, t]$

$$H_0(t) = Q_{01}(t)[c]H_1(t) + Q_{07}(t)[c]H_7(t) + Q_{09}(t)[c]H_9(t)$$

$$H_1(t) = Q_{12}(t)[c][1+H_2(t)] + Q_{14}(t)[c][1+H_4(t)], H_2(t) = Q_{20}(t)[c]H_0(t) + Q_{22}^{(3)}(t)[c]H_2(t)$$

$$H_4(t) = Q_{46}^{(3)}(t)[c]H_6(t), H_6(t) = Q_{60}(t)[c]H_0(t)$$

$$H_7(t) = (Q_{72}(t) + Q_{72}^{(8)}(t)) [c]H_2(t) + Q_{74}(t)[c]H_4(t)$$

$$H_9(t) = Q_{9,10}(t)[c][1+H_{10}(t)], H_{10}(t) = (Q_{10,2}(t)[c] + Q_{10,2}^{(11)}(t))[c]H_2(t) \quad (51-58)$$

Taking Laplace Transform of eq. (51-58) and solving for $H_0^*(s)$

$$H_0^*(s) = N_6(s) / D_3(s) \quad (59)$$

Where

$$N_6(s) = (1 - Q_{22}^{(3)*}(s)) \{ Q_{01}^*(s)(Q_{12}^*(s) + Q_{14}^*(s)) + Q_{09}^*(s) Q_{9,10}^*(s) \}$$

$$D_3(s) = (1 - Q_{22}^{(3)*}(s)) \{ 1 - (Q_{01}^*(s) Q_{14}^*(s) + Q_{07}^*(s) Q_{74}^*(s)) Q_{46}^{(5)*}(s) Q_{60}^*(s) \}$$

$$- Q_{20}^*(s) \{ Q_{01}^*(s) Q_{12}^*(s) + Q_{07}^*(s) (Q_{72}^*(s)) + Q_{72}^{(8)}(s) + Q_{09}^*(s) Q_{9,10}^*(s) (Q_{10,2}^*(s) + Q_{10,2}^{(11)}(s)) \}$$

$$\text{In the long run, } H_0 = \frac{N_6(0)}{D_3'(0)} \quad (60)$$

where $N_6(0) = p_{20} (p_{01} + p_{09})$ and $D_3'(0)$ is already defined.

The expected number of visits by the repairman for repairing the failed unit due to ultra-violet radiations in $(0, t]$

$$V_0(t) = Q_{01}(t)[c]V_1(t) + Q_{07}(t)[c]V_7(t) + Q_{09}(t)[c]V_9(t)$$

$$V_1(t) = Q_{12}(t)[c]V_2(t) + Q_{14}(t)[c]V_4(t), V_2(t) = Q_{20}(t)[c]V_0(t) + Q_{22}^{(3)}(t)[c]V_2(t)$$

$$V_4(t) = Q_{46}^{(3)}(t)[c]V_6(t), V_6(t) = Q_{60}(t)[c]V_0(t)$$

$$V_7(t) = (Q_{72}(t)[1+V_2(t)] + Q_{72}^{(8)}(t)) [c]V_2(t) + Q_{74}(t)[c]V_4(t)$$

$$V_9(t) = Q_{9,10}(t)[c]V_{10}(t), V_{10}(t) = (Q_{10,2}(t) + Q_{10,2}^{(11)}(t))[c]V_2(t) \quad (61-68)$$

Taking Laplace-Stieltjes transform of eq. (61-68) and solving for $V_0^*(s)$

$$V_0^*(s) = N_7(s) / D_4(s) \quad (69)$$

where $N_7(s) = Q_{07}^*(s) Q_{72}^*(s) (1 - Q_{22}^{(3)*}(s))$ and $D_4(s)$ is the same as $D_3(s)$

$$\text{In the long run, } V_0 = \frac{N_7(0)}{D_4'(0)} \quad (70)$$

where $N_7(0) = p_{20} p_{07} p_{72}$ and $D_4'(0)$ is already defined.

GAIN-FUNCTION ANALYSIS

The Gain- function of the system considering mean up-time, expected busy period of the system under non-availability of sunlight when the units stops automatically, expected busy period of the server for repair of unit when failure is due to ultra-violet radiations and, expected number of visits by the repairman for non-identical units failure, expected number of visits by the repairman for failure due to ultra-violet radiations.

The expected total Gain-function incurred in $(0, t]$ is

$C(t)$ = Expected total revenue in $(0, t]$

- expected total repair cost for failure of the unit due to ultra-violet radiations in $(0, t]$
- expected total repair cost for repairing the units in $(0, t]$
- expected busy period of the system under non-availability of sunlight when the units automatically stop in $(0, t]$
- expected number of visits by the repairman for repairing the failed unit due to ultra-violet radiations in $(0, t]$
- expected number of visits by the repairman for repairing of the non-identical units in $(0, t]$

The expected total cost per unit time in steady state is

$$C = \lim_{t \rightarrow \infty} (C(t)/t) = \lim_{s \rightarrow 0} (s^2 C(s)) \\ = K_1 A_0 - K_2 P_0 - K_3 B_0 - K_4 R_0 - K_5 V_0 - K_6 H_0$$

Where

K₁: revenue per unit up-time,

K₂: cost per unit time for which the system fails due to ultra-violet radiations is under repair

K₃: cost per unit time for which the system is under unit repair

K₄: when units automatically stop cost per unit time for which the system is under non-availability of sunlight

K₅: cost per visit by the repairman for which the unit fails due to ultra-violet radiations is under repair,

K₆: cost per visit by the repairman for non-identical units repair.

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to non-availability of sunlight and failure rate due to ultra-violet radiations increases, the MTSF and steady state availability decreases and the Gain-Function also decreased as the failure increases.

REFERENCES

1. Barlow, R.E. and Proschan, F., Mathematical theory of Reliability, 1965; John Wiley, New York.
2. Dhillon, B.S. and Natesen, J, Stochastic Anaylsis of outdoor Power Systems in fluctuating environment, Microelectron. Reliab.. 1983; 23, 867-881.
3. Gnedenko, B.V., Belyayev, Yu.K. and Soloyer, A.D., Mathematical Methods of Reliability Theory, 1969 ; Academic Press, New York.
4. Goel, L.R., Sharma,G.C. and Gupta, Rakesh Cost Analysis of a Two-Unit standby system with different weather conditions, Microelectron. Reliab., 1985; 25, 665-659.
5. Gupta, Rakesh and Goel, Rakesh, Profit Analysis of a Two-Unit ColdStandby system under Different Weather Conditions, Microelectron. Reliab., 1991;31, pp.18-27.

Source of Support: None Declared
Conflict of Interest: None Declared