Profit-function of two-unit dissimilar warm standby system with failure due to extremely high radiations and failure due to geo-thermal energy

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Abstract

Introduction: Geothermal energy is thermal energy generated and stored in the Earth. Thermal energy is the energy that determines the temperature of matter. The geothermal energy of the Earth's crust originates from the original formation of the planet (20%) and from radioactive decay of minerals (80%). The geothermal gradient, which is the difference in temperature between the core of the planet and its surface, drives a continuous conduction of thermal energy in the form of heat from the core to the surface. Earth's internal heat is thermal energy generated from radioactive decay and continual heat loss from Earth's formation. Temperatures at the core–mantle boundary may reach over 4000 °C (7,200 °F). The high temperature and pressure in Earth's interior cause some rock to melt and solid mantle to behave plastically, resulting in portions of mantle convecting upward since it is lighter than the surrounding rock. Rock and water is heated in the crust, sometimes up to 370 °C (700 °F). In Nuclear Reactor the leakage in form of radiations becomes highly dangerous to the lives of living beings. The radiations from the nuclear reactor are always under serious consideration due to fatal and miserable results to human race. Every precautions and extra care is taken to avoid any miss-happening due to radiations. But still due carelessness or due to failure of some equipment in Nuclear Reactors there occur leakage of radiations causing a major casualty. In the present paper we have taken two-dissimilar warm standby system with failure due to extremely high radiations. The unit fails due to extremely high radiations. When there are radiations of extremely high magnitude the working of unit stops automatically to avoid excessive damage of the units and when the unit comes in no normal position the repair of the units starts immediately. The failure time distribution is taken as exponential and repair time distribution as general. Using Markov regenerative point technique we have calculated different reliability characteristics such as MTSF, reliability of the system, availability analysis in steady state, busy period analysis of the system under repair, expected number of visits by the repairman in the long run and profit-function. Special case by taking failure and repair as exponential have been derived and graphs are drawn.

Keyword: warm standby, extremely high radiations, geo-thermal energy, MTSF, Availability, busy period, profit-function.

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INTRODUCTION

From hot springs, geothermal energy has been used for bathing since Paleolithic times and for space heating since ancient Roman times, but it is now better known for electricity generation. Worldwide, 11,400 megawatts (MW) of
geothermal power is online in 24 countries in 2012. An additional 28 gig watts of direct geothermal heating capacity is installed for district heating, space heating, spas, industrial processes, desalination and agricultural applications in 2010. Geothermal power is cost effective, reliable, sustainable, and environmentally friendly, but has historically been limited to areas near tectonic plate boundaries. Recent technological advances have dramatically expanded the range and size of viable resources, especially for applications such as home heating, opening a potential for widespread exploitation. Geothermal wells release greenhouse gases trapped deep within the earth, but these emissions are much lower per energy unit than those of fossil fuels. As a result, geothermal power has the potential to help mitigate global warming if widely deployed in place of fossil fuels. In Nuclear Reactor the leakage in form of radiations becomes highly dangerous to the lives of living beings. The radiations from the nuclear reactor are always under serious consideration due to fatal and miserable results to human race. In the present paper we have taken two-dissimilar warm standby system with failure due to extremely high radiations and geo-thermal power.

**Assumptions**
1. The failure time distribution is exponential whereas the repair time distribution is arbitrary of two non-identical units.
2. The repair starts immediately upon failure of units and the repair discipline is FCFS.
3. The repairs are perfect and start immediately as soon as the extremely high radiations of the system become normal. The radiations of both the units do not go extremely high.
4. The failure of a unit is detected immediately and perfectly.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.

**Symbols for states of the System**

**Superscripts:** O, WS, SO, FEHR, FGE  
Operative, Warm Standby, Stops the operation, Failure due to extremely high radiations, failure due to geo-thermal energy respectively  

**Subscripts:** nehr, ehr, ur, wr, uR  
No extremely high radiations. Extremely high radiations, under repair, waiting for repair, under repair continued respectively

**Up states:** 0, 1, 2, 9;  
**Down states:** 3, 4, 5, 6, 7, 8, 10, 11  
**Regeneration point:** 0, 1, 2, 4, 7, 10

**States of the System**
0(O nehr, WS nehr ) One unit is operative and the other unit is warm standby and there is no extremely high radiations in both the units.
1(SO nehr, O nehr )  
The operation of the first unit stops automatically due to extremely high radiations and warm standby units starts operating and there is no extremely high radiations.

2(FEHR ur, O nehr )  
The first unit fails and undergoes repair after failure due to extremely high radiations are over and the second unit continues to be operative with no extremely high radiations.

3(FEHR ur, SO ehr )  
The repair of the first unit is continued from state 2 and in the other unit extremely high radiations occur and stops automatically due to extremely high radiations.

4(FEHR ur, SO nehr )  
The one unit fails and undergoes repair after the extremely high radiations are over and the other unit also stops automatically due to extremely high radiations.

5(FEHR ur, FEHR ur )  
The repair of the first unit is continued from state 4 and the other unit is failed due to extremely high radiations in it and is waiting for repair.

6 (O nehr, FEHR ehr )  
The first unit is operative with no extremely high radiations and the second unit failed due to extremely high radiations is under repair.

7(SO nehr, FGE nehr, ur )  
The operation of the first unit stops automatically due to extremely high radiations and the second unit fails due to geo-thermal energy and undergoes repair.
8(FEHR wr, FGE nehr, ur)
The repair of failed unit due to geo-thermal energy is continued from state 7 and the first unit is failed after extremely high radiations and waiting for repair.

9(O nehr, SO uehr)
The first unit is operative and the warm standby dissimilar unit is under extremely high radiations

10(SO nehr, FGE ur)
The operation of the first unit stops automatically due to extremely high radiations and the second unit fails due to geothermal energy and undergoes repair after the extremely high radiations is over.

11(FEHR wr, FGE ur)
The repair of the second unit is continued from state 10 and the first unit is failed due to extremely high radiations is waiting for repair.

![Figure 1: The State Transition Diagram](image)

**TRANSITION PROBABILITIES**

Simple probabilistic considerations yield the following expressions:

\[ p_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}, \quad p_{07} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}, \quad p_{09} = \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \]

\[ P_{20} = G_1(\lambda_1), \quad P_{22} = G_1(\lambda_1), \quad \lambda_2 = G_2(\lambda_4), \quad P_{27} = G_2(\lambda_4) = P_{78} \]

We can easily verify that

\[ p_{01} + p_{07} + p_{09} = 1, \quad p_{12} + p_{14} = 1, \quad p_{20} + p_{23} = P_{22}^{(3)} = 1, \quad p_{46} = 1, \quad p_{60} = 1 \]

\[ p_{72} + p_{74} = 1, \quad p_{9,10} = 1, \quad p_{10,2} + p_{10,2}^{(11)} = 1 \]

And mean sojourn time are

\[ \mu_0 = E(T) = \int_0^\infty p[T > t]dt \]

**Mean Time To System Failure**

We can regard the failed state as absorbing

\[ \theta_0(t) = Q_{01}(t)[s] \theta_1(t) + Q_{09}(t)[s] \theta_3(t) + Q_{07}(t) \]

\[ \theta_1(t) = Q_{12}(t)[s] \theta_2(t) + Q_{14}(t)[s] \theta_3(t) \]

\[ \theta_2(t) = Q_{20}(t)[s] \theta_0(t) + Q_{22}^{(3)}(t) \]

\[ \theta_3(t) = Q_{9,10}(t) \]

Taking Laplace-Stiltjes transform of eq. (3-5) and solving for

\[ Q_0(s) = \frac{N_1(s)}{D_1(s)} \]

Where
\[ N_i(s) = Q_{0i}^*(s) \{ Q_{12}^{(s)}(s) + Q_{14}^{(s)}(s) \} + Q_{09}^*(s) Q_{2,10}^{(s)}(s) + Q_{07}^{(s)} \]  
\[ D_i(s) = 1 - Q_{0i}^*(s) \]  
\[ Q_{12}^{(s)}(s) = \frac{D_i(s)}{D_i(0)} \]  

Making use of relations (1) and (2) it can be shown that \( \theta(0) = 1 \), which implies that \( \theta(t) \) is a proper distribution.

\[ \text{MTSF} = E[T] = \frac{d}{ds} \theta_0(0) + \frac{(D_i(0) - N_i(0)) / D_i(0)}{s} \]  
where
\[ \mu_0 = \mu_{01} + \mu_{09}, \quad \mu_1 = \mu_{12} + \mu_{14}, \quad \mu_2 = \mu_{20} + \mu_{22}^{(3)}, \quad \mu_9 = \mu_{9,10} \]

### AVAILABILITY ANALYSIS

Let \( M_i(t) \) be the probability of the system having started from state \( i \) is up at time \( t \) without making any other regenerative state belonging to \( E \). By probabilistic arguments, we have

The value of \( M_0(t) \), \( M_1(t) \), \( M_2(t) \), \( M_9(t) \) can be found easily.

The point wise availability \( A_i(t) \) have the following recursive relations
\[ A_0(t) = M_0(t) + q_{01}(t) A_1(t) + q_{09}(t) A_9(t) \]
\[ A_1(t) = M_1(t) + q_{12}(t) A_2(t) + q_{14}(t) A_4(t), \quad A_2(t) = M_2(t) + q_{20}(t) A_0(t) + q_{22}^{(3)}(t) A_2(t) \]
\[ A_4(t) = q_{46}^{(3)}(t) A_6(t), \quad A_6(t) = q_{60}(t) A_0(t) + q_{62}(t) A_2(t) \]
\[ A_2(t) = q_{27}(t) + q_{22}^{(3)}(t) A_2(t) + q_{24}(t) A_4(t) \]
\[ A_9(t) = M_9(t) + q_{9,10}(t) A_{10}(t), \quad A_{10}(t) = q_{10,2}(t) A_2(t) + q_{10,2}^{(1)}(t) A_2(t) \]

Taking Laplace Transform of eq. (7-14) and solving for \( \hat{A}_0(s) \)

\[ \hat{A}_0(s) = N_2(s) / D_2(s) \]

Where
\[ N_2(s) = \left( 1 - \hat{q}_{22}^{(3)}(s) \right) \left\{ \hat{M}_0(s) + \hat{q}_{01}(s) \hat{M}_1(s) + \hat{q}_{09}(s) \hat{M}_9(s) \right\} + \hat{M}_2(s) \left\{ \hat{q}_{01}(s) \hat{q}_{42}(s) + \hat{q}_{07}(s) \left( \hat{q}_{72}(s) + \hat{q}_{73}^{(3)}(s) \right) + \hat{q}_{09}(s) \left( \hat{q}_{9,10}(s) + \hat{q}_{9,12}(s) \right) \right\} \]
\[ D_2(s) = \left( 1 - \hat{q}_{22}^{(3)}(s) \right) \left\{ 1 - \hat{q}_{46}^{(5)}(s) \hat{q}_{46}(s) \left( \hat{q}_{41}(s) \hat{q}_{44}(s) + \hat{q}_{47}(s) \hat{q}_{48}(s) \right) - \hat{q}_{20}(s) \hat{q}_{10}(s) \hat{q}_{12}(s) + \hat{q}_{47}(s) \hat{q}_{72}(s) + \hat{q}_{9,10}(s) \hat{q}_{9,12}(s) \right\} \]

The steady state availability
\[ A_0 = \lim_{s \to \infty} [A_0(t)] = \lim_{s \to \infty} [s \hat{A}_0(s)] = \lim_{s \to \infty} \frac{s N_2(s)}{D_2(s)} \]

Using L’ Hospital’s rule, we get
\[ A_0 = \lim_{s \to \infty} \frac{N_2(s) + s N_2(s)}{D_2(s)} = \frac{N_2(0)}{D_2(0)} \]

Where
\[ N_2(0) = p_{20}(\hat{M}_0(0) + p_{01} \hat{M}_1(0) + p_{09} \hat{M}_9(0)) \right\}, \quad \hat{M}_2(0) = p_{10}(p_{11} + p_{07} \hat{p}_{74}) \]

\[ \mu_4 = \mu_{46}, \quad \mu_7 = \mu_{72} \]

The expected up time of the system in \( (0, t] \) is
\[ \lambda_u(t) = \int_0^t A_0(z) dz \]  
\[ \lambda_u(t) = \frac{N_2(5)}{s D_2(s)} \]

The expected down time of the system in \( (0, t] \) is
\[ \lambda_d(t) = t - \lambda_u(t) \]  
\[ \lambda_d(t) = \frac{N_2(5)}{s D_2(s)} - \lambda_u(t) \]

### The expected busy period of the server for repairing the failed unit under extremely high radiations in \( (0, t] \)

\[ R_0(t) = S_0(t) + q_{01}(t) R_1(t) + q_{09}(t) R_9(t) + q_{07}(t) R_7(t) \]
\[ R_1(t) = S_1(t) + q_{12}(t) R_2(t) + q_{14}(t) R_4(t) \]
\[ R_2(t) = q_{20}(t) R_0(t) + q_{22}^{(3)}(t) R_2(t) \]
\[ R_4(t) = q_{46}^{(5)}(t) R_6(t), \quad R_6(t) = q_{60}(t) R_0(t) \]
\[ R_7(t) = (q_{72}(t) + q_{73}(t)) [R_2(t) + q_{74}(t) R_4(t) \]

\[ R_9(t) = q_{9,10}(t) R_{10}(t), \quad R_{10}(t) = q_{10,2}(t) R_2(t) + q_{10,2}^{(1)}(t) R_2(t) \]
Taking Laplace Transform of eq. (19-26) and solving for $R_0(s)$

\[
R_0(s) = N_0(s) / D_2(s)
\]

(27)

Where

\[
N_2(s) = (1 - \hat{q}_2(3)(s)) \{ \hat{q}_0(s) + \hat{q}_0(s) \hat{S}(s) + \hat{q}_0(s) \hat{S}_0(s) \}
\]

and $D_2(s)$ is already defined.

In the long run, $R_0 = \frac{N_0(s)}{D_2(0)}$ (28)

where $N_0(0) = p_{20}(\hat{S}_0(0) + p_{00} \hat{S}_0(0))$ and $D_2(0)$ is already defined.

The expected period of the system under extremely high radiations in $(0, t]$ is

\[
\lambda_{ru}(t) = \int_0^\infty R_0(z)dz
\]

So that $\lambda_{ru}(s) = \frac{R_0(s)}{s}$

(37)

The expected busy period of the server for repair of dissimilar units by the repairman in $(0, t]$

\[
B_0(t) = q_{01}(0)N_t(0) + q_{02}(0)N_t(0) + p_{00}(0)N_t(0)
\]

\[
B_1(t) = q_{12}(0)N_t(0) + q_{14}(0)N_t(0) + p_{10}(0)N_t(0)
\]

\[
B_2(t) = q_{22}(0)N_t(0) + q_{24}(0)N_t(0) + p_{20}(0)N_t(0)
\]

\[
B_3(t) = q_{32}(0)N_t(0) + q_{34}(0)N_t(0) + p_{30}(0)N_t(0)
\]

\[
B_4(t) = q_{42}(0)N_t(0) + q_{44}(0)N_t(0) + p_{40}(0)N_t(0)
\]

(29-36)

Taking Laplace Transform of eq. (29-36) and solving for $B_0(s)$

\[
B_0(s) = N_0(s) / D_2(s)
\]

(37)

The expected busy period of the server for repair when the failure of the unit is due to Geo-thermal energy in $(0, t]$

\[
P_0(t) = q_{01}(0)P_0(t) + q_{02}(0)P_0(t) + q_{00}(0)P_0(t)
\]

\[
P_1(t) = q_{12}(0)P_1(t) + q_{14}(0)P_1(t) + q_{10}(0)P_1(t)
\]

\[
P_2(t) = q_{22}(0)P_2(t) + q_{24}(0)P_2(t) + q_{20}(0)P_2(t)
\]

\[
P_3(t) = q_{32}(0)P_3(t) + q_{34}(0)P_3(t) + q_{30}(0)P_3(t)
\]

\[
P_4(t) = q_{42}(0)P_4(t) + q_{44}(0)P_4(t) + q_{40}(0)P_4(t)
\]

(40-47)

Taking Laplace Transform of eq. (40-47) and solving for $B_0(s)$

\[
B_0(s) = N_0(s) / D_2(s)
\]

(38)

where $N_0(0) = p_{20}(0) \{P_0(1) + p_{02}(0) \{ \hat{T}_0(0) + \hat{T}_0(0) \} + p_{02}(0) \hat{T}_0(0) \}$ and $D_2(0)$ is already defined.

The expected busy period of the server for repair in $(0, t]$ is

\[
\lambda_{ru}(t) = \int_0^\infty R_0(z)dz
\]

So that $\lambda_{ru}(s) = \frac{R_0(s)}{s}$

(39)

The expected Busy period of the server for repair when the failure of the unit is due to Geo-thermal energy in $(0, t]$

\[
H_0(t) = Q_{02}(0)H_0(t) + Q_{04}(0)H_0(t) + Q_{00}(0)H_0(t)
\]

\[
H_1(t) = Q_{12}(0)H_1(t) + Q_{14}(0)H_1(t) + Q_{11}(0)H_1(t) + Q_{10}(0)H_1(t) + Q_{12}(0)H_2(t)
\]

\[
H_2(t) = Q_{22}(0)H_2(t) + Q_{24}(0)H_2(t)
\]

\[
H_3(t) = Q_{32}(0)H_3(t) + Q_{34}(0)H_3(t) + Q_{32}(0)H_4(t)
\]

\[
H_4(t) = Q_{42}(0)H_4(t) + Q_{44}(0)H_4(t) + Q_{42}(0)H_5(t)
\]

(51-58)

Taking Laplace Transform of eq. (51-58) and solving for $H_0(s)$

\[
H_0(s) = N_0(s) / D_2(s)
\]

(59)

Where

\[
N_0(s) = (1 - q_2(3)(s)) \{ q_0(s) \hat{T}(s) + q_1(s) \hat{T}_0(s) + q_2(s) \hat{T}_1(s) + q_3(s) \hat{T}_2(s) + q_4(s) \hat{T}_3(s) \}
\]

\[
D_2(s) = (1 - q_2(3)(s)) \{ 1 - (q_0(s)q_1(s) + q_2(s)q_3(s) + q_4(s)q_5(s)) \}
\]
\[ C = \text{The expected total cost per unit time in steady state is} \]
\[ V = K V \]
\[ \text{Where} \]
\[ \text{Taking Laplace-Stieltjes transform of eq. (61-68) and solving for} \]
\[ N = K N \]
\[ \text{where} N(0) = \text{p}_{20} (\text{p}_{20} + \text{p}_{20}) \text{and D'}(0) \text{is already defined.} \]

**COST BENEFIT ANALYSIS**

The cost-benefit function of the system considering mean up-time, expected busy period of the server for repair of unit and failure due to geo-thermal energy, expected number of visits by the repairman for unit failure, expected number of visits by the repairman for failure due to geo-thermal energy.

The expected total cost-benefit incurred in (0, t] is

\[ C(t) = \text{Expected total revenue in (0, t] - expected total repair cost for failure due to geo-thermal energy in (0, t]} \]

- expected total repair cost for repairing the units in (0, t]
- expected busy period of the system under extremely high radiations when the units automatically stop in (0, t]
- expected number of visits by the repairman for repairing the failure due to geo-thermal power in (0, t]
- expected number of visits by the repairman for repairing of the units in (0, t]

The expected total cost per unit time in steady state is

\[ C = \lim_{t \rightarrow \infty} \frac{C(t)}{t} = \lim_{k \rightarrow 0} (s^2 C(s)) \]

\[ = K_1 A_0 - K_2 P_0 - K_3 B_0 - K_4 R_0 - K_5 V_0 - K_6 H_0 \]

Where

- \( K_1 \): revenue per unit up-time.
- \( K_2 \): cost per unit time for which the system is under repair when failure of the unit due to geo-thermal energy.
- \( K_3 \): cost per unit time for which the system is under repair.
- \( K_4 \): cost per unit time for which the system stops automatically.
- \( K_5 \): cost per unit time for which the system is under extremely high radiations.
- \( K_6 \): cost per visit by the repairman for repair the unit failure due to geo-thermal energy.

**CONCLUSION**

After studying the system, we have analyzed graphically that when the failure rate due to geo-thermal energy, extremely high radiations rate increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

**REFERENCES**


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