Research Article

Cost-Benefit analysis of two identical cold standby system subject to non-availability of water resulting failure to produce hydroelectric power and failure due to rainfall

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Abstract

Introduction: Hydroelectricity is the term referring to electricity generated by hydropower the production of electrical power through the use of the gravitational force of falling or flowing water. It is the most widely used form of renewable energy, accounting for 16 percent of global electricity generation – 3,427 terawatt-hours of electricity production in 2010 and is expected to increase about 3.1% each year for the next 25 years. Hydropower is produced in 150 countries, with the Asia-Pacific region generating 32 percent of global hydropower in 2010. China is the largest hydroelectricity producer, with 721 terawatt-hours of production in 2010, representing around 17 percent of domestic electricity use. The cost of hydroelectricity is relatively low, making it a competitive source of renewable electricity. The average cost of electricity from a hydro plant larger than 10 megawatts is 3 to 5 U.S. cents per kilowatt-hour. It is also a flexible source of electricity since the amount produced by the plant can be changed up or down very quickly to adapt to changing energy demands. Water plays an important and pivotal role in producing hydroelectric power. Non-availability of water results failure to produce hydroelectric power. Reliability is a measure of how well a system performs or meets its design requirements. It is hence the prime concern of all scientists and engineers engaged in developing such a system.. In this paper we have taken two types of failures (1) **FNAW- non-availability of water resulting failure to produce Hydroelectric Power** (2) **FRF-failure due to Rainfall.** Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Cold Standby, FNAW- non-availability of water resulting failure to produce Hydroelectric Power, FRF-failure due to Rainfall, first come first serve, Availability, Busy period, Cost-Benefit analysis

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INTRODUCTION

Stochastic behavior of systems operating under changing environments has widely been studied.. Dhillon, B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment. Kan Cheng (1985) has studied

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reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken two types of failures (1) **FNAW- failure due to non-availability of water resulting failure to produce Hydroelectric Power** (2) **FRF-failure due to Rainfall**. When the main operative unit **fails due to Rainfall-FRF** then cold standby system becomes operative. After failure the unit undergoes repair facility of very high cost in case of **FRF-failure due to Rainfall** immediately. Failure due to non-availability of water resulting failure to produce hydroelectric power may disrupt the whole life style. The repair is done on the basis of first fail first repaired.

Assumptions

1. $F_1(t)$ and $F_2(t)$ are general failure time distributions due to **non-availability of water resulting failure to produce Hydroelectric power** and **Rainfall.** The repair is of two types -Type -I, Type-II with repair time distributions as $G_1(t)$ and $G_2(t)$ respectively.
2. The **Rainfall** is non-instantaneous and it cannot come simultaneously in both the units.
3. Whenever the Rainfall occur within specified limit of the unit, it works as normal as before. But as soon as there occur Rainfall of higher amount the operation of the unit stops automatically.
4. The repair starts immediately after detecting the Rainfall and works on the principle first fail first repaired basis.
5. The repair facility does no damage to the units and after repair units are as good as new.
6. The switches are perfect and instantaneous.
7. All random variables are mutually independent.
8. When both the units fail, we give priority to operative unit for repair.
9. Repairs are perfect and failure of a unit is detected immediately and perfectly.
10. The system is down when both the units are non-operative.

Symbols for states of the System

$F_1(t)$ and $F_2(t)$ are the **failure time distribution** due to **non-availability of water resulting failure to produce Hydroelectric power** and failure due to **Rainfall** respectively

$G_1(t)$, $G_2(t)$ – repair time distribution Type -I, Type-II due to **non-availability of water resulting failure to produce hydroelectric power** and failure due to **Rainfall** respectively

Superscripts: O, CS, FNAW, FRF

Operative, Cold Standby, Failure due to non**-availability of water resulting failure to produce hydroelectric power**, failure due to **Rainfall** respectively

Subscripts: nawf, nrff, rff ur, wr, uR

Non-availability of water resulting failure to produce hydroelectric power, No Rainfall failure, Rainfall failure, under repair, waiting for repair, under repair continued from previous state respectively

Up states: 0, 1, 2;

Down states: 3, 4

regeneration point – 0,1,2

Notations

$M_i(t)$ System having started from state I is up at time t without visiting any other regenerative state

$A_i(t)$ state is up state as instant t

$R_i(t)$ System having started from state I is busy for repair at time t without visiting any other regenerative state.

$B_i(t)$ the server is busy for repair at time t.

$H_i(t)$ Expected number of visits by the server for repairing given that the system initially starts from regenerative state i

States of the System

0(O_{nrff}, CS_{nrff})

One unit is operative and the other unit is cold standby and there are no Rainfall failures in both the units.

1($SOFRF_{rff, ur}$, O_{nrff})

The operating unit fails due to Rainfall and is under repair immediately of very costly Type- I and standby unit starts operating with no Rainfall.

2($FNAW_{nrff, nawf, ur}$, O_{nrff})

The operative unit fails to produce hydroelectric power due to FNAW resulting from non-availability of water and undergoes repair of type II and the standby unit becomes operative with no Rainfall.

3($FRF_{rff, uR}$, $FNAW_{nrff, nawf, wr}$)

The first unit fails due to Rainfall and under very costly Type-! repair is continued from state 1 and the other unit fails to produce hydroelectric power due to FNAW resulting from non-availability of water and is waiting for repair of Type -II.

4($FRF_{rff,\ uR}$, $FRF_{rff,\ wr}$)

The one unit fails due to Rainfall is continues under repair of very costly Type - I from state 1 and the other unit also fails due to Rainfall. is waiting for repair of very costly Type- I.

5($FNAW_{nrff,\ nawf,\ uR}$, $FRF_{rff,\ wr}$)

The operating unit fails to produce hydroelectric power due to non-availability of water (FNAW mode) and under repair of Type - II continues from the state 2 and the other unit fails due to Rainfall is waiting for repair of very costly Type- I.

6($FNAW_{nrff,nawf,uR}$, $FNAW_{nrff,nawf,wr}$)

The operative unit fails to produce hydroelectric power due to FNAW resulting from non- availability of water and under repair continues from state 2 of Type –II and the other unit is also failed to produce hydroelectric power due to FNAW resulting from non-availability of water and is waiting for repair of Type-II and there is no Rainfall.

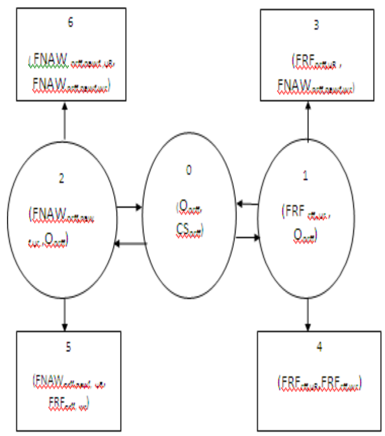

Figure 1: The State Transition Diagram

● Regeneration point - states 0, 1, 2 ○ Up State □ Down State

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$p_{01} = \int_0^\infty \overline{F}_2(t)dF_1(t)$, $p_{02} = \int_0^\infty \overline{F}_1(t)dF_2(t)$

$p_{10} = \int_0^\infty \overline{F}_2(t)dG_1(t)$, $p_{13} = p_{11}^{(3)} = p_{11}^{(4)} = \int_0^\infty \overline{G}_1(t)dF_2(t)$

$p_{25} = p_{22}^{(5)} = p_{22}^{(6)} = \int_0^\infty \overline{G}_2(t)dF_1(t)$

clearly

$p_{01} + p_{02} = 1,$

$p_{10} + p_{13} = (p_{11}^{(3)}) + p_{14} = (p_{11}^{(4)}) = 1,$

$p_{20} + p_{25} = (p_{22}^{(5)}) + p_{26} = (p_{22}^{(6)}) = 1$ (1)

And mean sojourn time are

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt$ (2)

Mean Time To System Failure

$Ø_0(t) = Q_{01}(t)[s]\ Ø_1(t) + Q_{02}(t)[s]\ Ø_2(t)$

$Ø_1(t) = Q_{10}(t)[s]\ Ø_0(t) + Q_{13}(t) + Q_{14}(t)$

$Ø_2(t) = Q_{20}(t)[s]\ Ø_0(t) + Q_{25}(t) + Q_{26}(t)$ (3-5)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-5) and solving for

$ø_0^*(s) = N_1(s) / D_1(s)$ (6)

where
$N_1(s) = Q_{01}^*[Q_{13}^*(s) + Q_{14}^*(s)] + Q_{02}^*[Q_{25}^*(s) + Q_{26}^*(s)]$
$D_1(s) = 1 - Q_{01}^* Q_{10}^* - Q_{02}^* Q_{20}^*$
Making use of relations (1) and (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0^*(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \left.\frac{d}{ds} ø_0^*(s)\right|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$$= (\mu_0 + p_{01}\mu_1 + p_{02}\mu_2) / (1 - p_{01} p_{10} - p_{02} p_{20})$$

where

$$\mu_0 = \mu_{01} + \mu_{02},\ \mu_1 = \mu_{10} + \mu_{13} + \mu_{14}$$
$$\mu_2 = \mu_{20} + \mu_{25} + \mu_{26}$$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state I is up at time t without making any other regenerative state. By probabilistic arguments, we have
The value of $M_0(t)$, $M_1(t)$, $M_2(t)$ can be found easily.
The point wise availability $A_i(t)$ have the following recursive relations

$$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$$
$$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{11}^{(3)}(t)[c]A_1(t)+ q_{11}^{(4)}(t)[c]A_1(t),$$
$$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + [q_{22}^{(5)}(t)[c]+ q_{22}^{(6)}(t)] [c]A_2(t) \quad (7\text{-}9)$$

Taking Laplace Transform of eq. (7-9) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \quad (10)$$

where

$$N_2(s) = \widehat{M}_0(s)(1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s)) (1- \hat{q}_{22}^{(5)}(s)- \hat{q}_{22}^{(6)}(s)) + \overline{q}_{01}(s)\widehat{M}_1(s)$$
$$[1- \hat{q}_{22}^{(5)}(s)- \hat{q}_{22}^{(6)}(s)] + \hat{q}_{02}(s)\widehat{M}_2(s)(1- \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))$$
$$D_2(s) = (1 - \hat{q}_{11}^{(3)}(s)- \hat{q}_{11}^{(4)}(s)) \{ 1 - \hat{q}_{22}^{(5)}(s) - \hat{q}_{22}^{(6)}(s))[1-(\hat{q}_{01}(s)\hat{q}_{10}(s))(1-$$
$$\hat{q}_{11}^{(3)}(s)- \hat{q}_{11}^{(4)}(s))]$$

The steady state availability

$$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\hat{A}_0(s)] = \lim_{s\to 0}\frac{s N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0}\frac{N_2(s)+s\ N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \quad (11)$$

The expected up time of the system in (0, t] is

$$\lambda_u(t) = \int_0^\infty A_0(z)dz \text{ So that } \overline{\lambda_u}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)} \quad (12)$$

The expected down time of the system in (0, t] is

$$\lambda_d(t) = t - \lambda_u(t) \text{ So that } \overline{\lambda_d}(s) = \frac{1}{s^2} - \overline{\lambda_u}(s) \quad (13)$$

The expected busy period of the server when there is FNAW-failure due to non-availability of water resulting not to produce hydroelectric power in (0, t]

$$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$$
$$R_1(t) = S_1(t) + q_{01}(t)[c]R_1(t) + [q_{11}^{(3)}(t) + q_{11}^{(4)}(t)[c]R_1(t),$$
$$R_2(t) = q_{20}(t)[c]R_0(t) + [q_{22}^{(6)}(t)+q_{22}^{(5)}(t)][c]R_2(t) \quad (14\text{-}16)$$

Taking Laplace Transform of eq. (14-16) and solving for $\overline{R_0}(s)$

$$\overline{R_0}(s) = N_3(s) / D_3(s) \quad (17)$$

Where

$N_3(s) = \hat{q}_{01}(s)\ \hat{S}_1(s)$ and

$D_3(s) = (1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s)) - \hat{q}_{01}(s)$ is already defined.

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (18)

The expected period of the system under FNAW-failure resulting from non availability of water not to produce hydropower in (0,t] is

$\lambda_{rv}(t) = \int_0^{\infty} R_0(z)dz$ So that $\bar{\lambda}_{rv}(s) = \frac{\hat{R}_0(s)}{s}$

The expected Busy period of the server when there is failure due to Rainfall when the units stops automatically in (0, t]

$B_0(t) = q_{01}(t)[c]B_1(t) + q_{02}(t)[c]B_2(t)$

$B_1(t) = q_{01}(t)[c]B_1(t) + [q_{11}^{(3)}(t) + q_{11}^{(4)}(t)]\ [c]B_1(t),$

$B_2(t) = T_2(t) + q_{02}(t)[c]\ B_2(t) + [q_{22}^{(5)}(t) + q_{22}^{(6)}(t)]\ [c]B_2(t)$

$T_2(t) = e^{-\lambda_1 t}G_2(t)$ (19- 21)

Taking Laplace Transform of eq. (19-21) and solving for $\bar{B}_0(s)$

$\bar{B}_0(s) = N_4(s) / D_2(s)$ (22)

Where

$N_4(s) = \hat{q}_{02}(s)\ \hat{T}_2(s))$

And $D_2(s)$ is already defined.

In steady state, $B_0 = \frac{N_4(0)}{D_2'(0)}$ (23)

The expected busy period of the server for repair in (0, t] is

$\lambda_{ru}(t) = \int_0^{\infty} B_0(z)dz$ So that $\bar{\lambda}_{ru}(s) = \frac{\hat{B}_0(s)}{s}$ (24)

The expected number of visits by the repairman for repairing the identical units in (0, t]

$H_0(t) = Q_{01}(t)[s][1+ H_1(t)] + Q_{02}(t)[s][1+ H_2(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + [Q_{11}^{(3)}(t) + Q_{11}^{(4)}(t)]\ [s]H_1(t),$

$H_2(t) = Q_{20}(t)[s]H_0(t) + [Q_{22}^{(5)}(t) + Q_{22}^{(6)}(t)]\ [c]H_2(t)$ (25-27)

Taking Laplace Transform of eq. (25-27) and solving for $H_0^*(s)$

$H_0^*(s) = N_6(s) / D_3(s)$ (28)

In the long run, $H_0 = \frac{N_6(0)}{D_3'(0)}$ (29)

COST-BENEFIT ANALYSIS

The Cost-Benefit analysis of the system considering mean up-time, expected busy period of the system under Rainfall when the units stops automatically, expected busy period of the server for repair of unit under non-availability of water resulting not to produce Hydroelectric power, expected number of visits by the repairman for unit failure.

The expected total Benefit-Function incurred in (0, t] is

C (t) = Expected total revenue in (0, t]

- expected total repair cost repairing the units in (0,t] due to FNAW- failure due to non-availability of water resulting not to produce hydroelectric power
- expected busy period of the system under Rainfall when the units automatically stop in (0,t]
- expected number of visits by the repairman for repairing of identical the units in (0,t]

The expected total cost per unit time in steady state is

$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$

$= K_1A_0 - K_2R_0 - K_3B_0 - K_4H_0$

Where

K_1: revenue per unit up-time,

K_2: cost per unit time for which the system is under repair of type- I
K_3: cost per unit time for which the system is under repair of type-II
K_4: cost per visit by the repairman for units repair.

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to non-availability of water resulting not to produce hydroelectric power and failure rate due to Rainfall increases, the MTSF and steady state availability decreases and the Cost-Benefit function decreased as the failure increases.

REFERENCES

1. Dhillon, B.S. and Natesen, J, Stochastic Anaysis of outdoor Power Systems in fluctuating environment, Microelectron. Reliab.. 1983; 23, 867-881.
2. Kan, Cheng, Reliability analysis of a system in a randomly changing environment, Acta Math. Appl. Sin. 1985, 2, pp.219-228.
3. Cao, Jinhua, Stochatic Behaviour of a Man Machine System operating under changing environment subject to a Markov Process with two states, Microelectron. Reliab., 1989; 28, pp. 373-378.
4. Barlow, R.E. and Proschan, F., Mathematical theory of Reliability, 1965; John Wiley, New York.
5. Gnedanke, B.V., Belyayar, Yu.K. and Soloyer, A.D., Mathematical Methods of Relability Theory, 1969 ; Academic Press, New York.

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