Research Article

Stochastic analysis of a two non-identical unit parallel system model with preparation time for repair and proviso of rest

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Abstract

The present paper deals with stochastic analysis of reliability characteristics of two non identical unit parallel system, each unit having two modes normal (N) and total failure (F). A failed unit needs some preparation time for starting its repair which is taken to be a random variable having some probability distribution. After a random period of operation the second unit goes for a rest and after rest for a random period of time; it again starts working without affecting its efficiency. A single repairman is always available with the system for the preparation of repair of failed unit and to repair a failed unit as well. All the failure time distributions and preparation time distributions are taken to be negative exponential with different parameters. The repair time distribution and completion of preparation time distribution are taken as general. The operating time after which the second unit goes for rest is also taken as negative exponential and completion time of rest period of second unit is taken as general.

Keywords: Reliability; Availability; Busy period; Expected number of Repairs; Profit Analysis; Graphical study of Model.

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INTRODUCTION

Two unit parallel system models have been studied extensively by various authors [1-6] in the field of reliability theory under different sets of assumptions, different modes of operative unit, different types of failures and having different failure and repair time distributions. In most of these models it is assumed that the repair of the failed unit starts immediately as soon as unit fails, but sometimes due to some unavoidable circumstances the repair of the failed unit may not be started immediately upon its failure and it requires some preparation time for repair to begin with. Gupta et.al[1]made comparison of two stochastic models each having two units and a failed unit required a fixed preparation time for repair. Recently Gupta et.al[2] carried out the analysis of a two unit standby system model each unit having three modes and each failed unit requiring preparation time for repair which is considered to be a random variable. The present paper deals with stochastic analysis of reliability characteristics of a two non identical unit parallel system, each unit having two modes normal (N) and total failure (F). Upon failure of a unit some preparation time is needed to start its repair which is taken to be a random variable having some probability distribution. The second unit goes for rest after a random period of operation and after rest for a random period of time; it again starts its working without affecting its efficiency.

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Using the regenerative point technique the following important reliability characteristics of interest are obtained:
[1] Transition probabilities and mean sojourn times.
[2] Reliability and Mean time to system failure.
[3] Point wise and steady-state availabilities of the system.
[4] Expected busy time of the repairman during (0, t] and in the steady-state.
[5] Expected number of repairs by repairman during (0, t] and in the steady-state.
[6] Net expected profit incurred by the system during (0, t] and in the steady-state.

2. SYSTEM DESCRIPTION AND ASSUMPTIONS

[1] The system comprises of two non-identical units (N_1, N_2), connected in parallel configuration.
[2] Each unit has two mode- Normal (N) and Total failure (F).
[3] The second unit goes for rest after a random period of operation and after rest for a random period of time; it again starts working without affecting its efficiency.
[4] Whenever a unit fails, some preparation time is needed to start its repair. The preparation time for repair is assumed to be random variable.
[5] A single repairman is always available with the system for preparation of repair and to repair the failed unit.
[6] The repair discipline is first come first serve (FCFS).
[7] A repaired unit is as good as new and is immediately reconnected to the system.
[8] All the failure time distributions and preparation time distribution are taken to be negative exponential with different parameters. The repair time distribution and completion of preparation time distribution are taken as general.
[9] The operating time after which the second unit goes for rest is taken as negative exponential and completion time of rest period of second unit is taken as general.

3. NOTATIONS AND SYMBOLS FOR THE SYSTEM STATES

α_1/α_2 : Constant failure rate of $1^{st}/2^{nd}$ unit.
α_3 : Constant rate of rest time of 2^{nd} unit.
μ_1/μ_2 : Constant rate of preparation time for repair of $1^{st}/2^{nd}$ unit.
$G_1(.)$: C.d.f. of repair time of 1^{st} unit.
$G_2(.)$: C.d.f. of repair time of 2^{nd} unit.
$G_3(.)$: C.d.f. of completion of rest time of 2^{nd} unit.
m_i : Mean repair time of i^{th} unit.

3.1. Symbols for the states of the system

N_{1O}/N_{2O} : $1^{st}/2^{nd}$ unit is operative.
N_{rest} : 2^{nd} unit is under rest.
F_{1p}/F_{2p} : $1^{st}/2^{nd}$ unit is under preparation for repair.
F_{1wp}/F_{2wp} : $1^{st}/2^{nd}$ unit is waiting for preparation for repair.
F_{1r}/F_{2r} : $1^{st}/2^{nd}$ unit is under repair.
With the help of the above symbols, the possible states of the system are:

$S_0 = [N_{1O}, N_{2O}]$ $\quad S_1 = [F_{1p}, N_{2O}]$ $\quad S_2 = [F_{1p}, F_{2wp}]$

$S_3 = [F_{1r}, N_{2O}]$ $\quad S_4 = [F_{1p}, N_{rest}]$ $\quad S_5 = [F_{1r}, F_{2wp}]$

$S_6 = [F_{1r}, N_{rest}]$ $\quad S_7 = [N_{1O}, F_{2p}]$ $\quad S_8 = [F_{1wp}, F_{2p}]$

$S_9 = [F_{1wp}, F_{2r}]$ $\quad S_{10} = [N_{1O}, N_{rest}]$ $\quad S_{11} = [N_{1O}, F_{2r}]$

The transition diagram along with all the transitions is shown in figure 1.

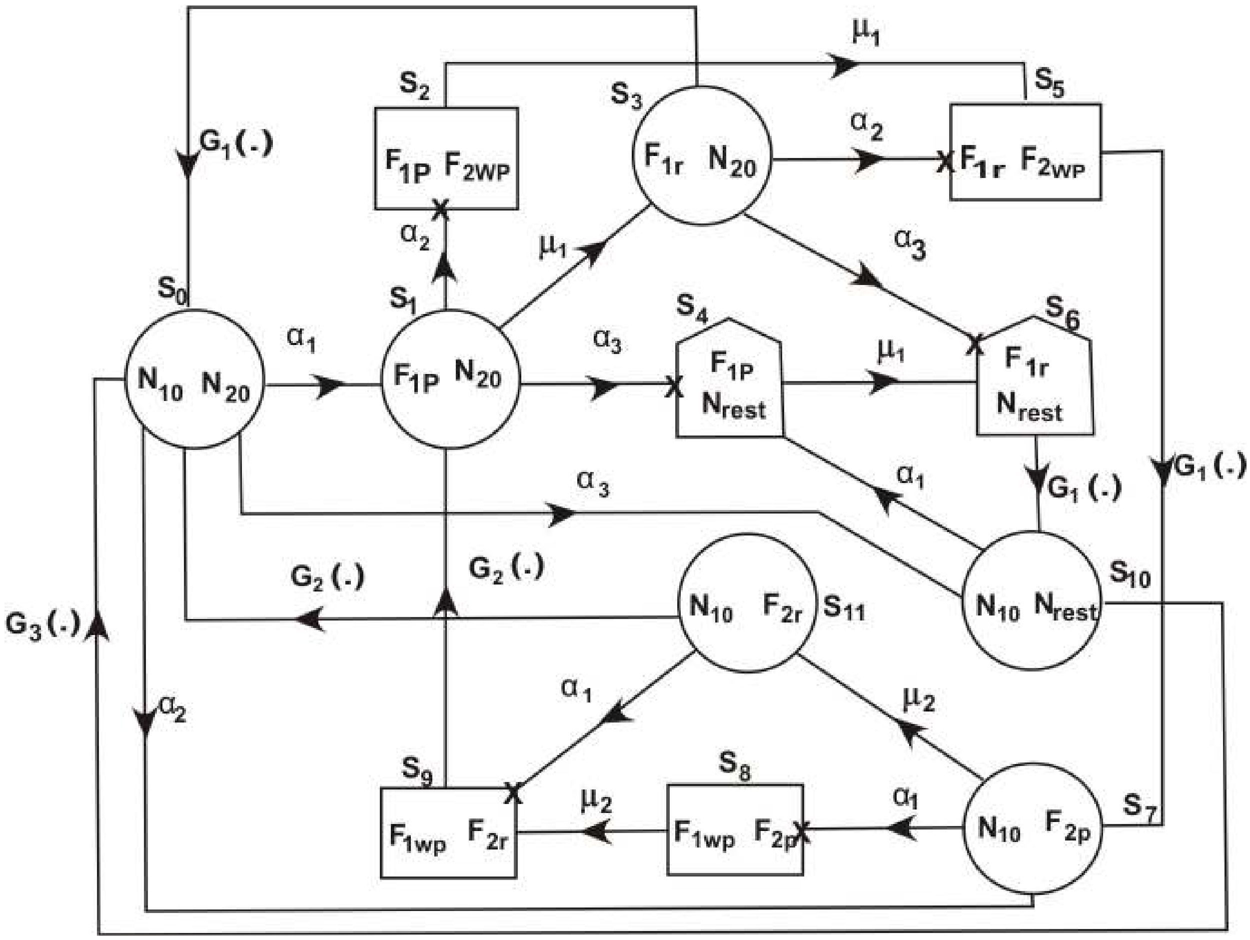

Figure 1: Transition Diagram

4. TRANSITION PROBABILITIES AND SOJOURN TIMES

4.1. STEADY STATE PROBABILITIES

First we find the following steady-state probabilities of transition:

$$p_{01} = \alpha_1 \int_0^\infty e^{-(\alpha_1+\alpha_2+\alpha_3)u}\, du = \frac{\alpha_1}{(\alpha_1+\alpha_2+\alpha_3)}$$

Similarly,

$$p_{07} = \frac{\alpha_2}{(\alpha_1+\alpha_2+\alpha_3)} \qquad p_{0,10} = \frac{\alpha_3}{((\alpha_1+\alpha_2+\alpha_3)}$$

$$p_{13} = \frac{\mu_1}{(\alpha_2+\alpha_3+\mu_1)} \qquad p_{15}^{(2)} = \frac{\alpha_2}{(\alpha_2+\alpha_3+\mu_1)}$$

$$p_{16}^{(4)} = \frac{\alpha_3}{(\alpha_2+\alpha_3+\mu_1)} \qquad p_{79}^{(8)} = \frac{\alpha_1}{(\alpha_1+\mu_2)}$$

$$p_{7,11} = \frac{\mu_2}{(\alpha_1+\mu_2)} \qquad p_{10,0} = \widetilde{G}_3(\alpha_1)$$

$$p_{10,4} = \left[1-\widetilde{G}_3(\alpha_1)\right] \qquad p_{11,0} = \widetilde{G}_2(\alpha_1)$$

$$p_{11,1}^{(9)} = \left[1-\widetilde{G}_2(\alpha_1)\right] \qquad p_{30} = \widetilde{G}_1(\alpha_2+\alpha_3)$$

$$p_{37}^{(5)} = \frac{\alpha_2}{(\alpha_2+\alpha_3)}\left[1-\widetilde{G}_1(\alpha_2+\alpha_3)\right] \qquad p_{3,10}^{(6)} = \frac{\alpha_3}{(\alpha_2+\alpha_3)}\left[1-\widetilde{G}_1(\alpha_2+\alpha_3)\right]$$

$$p_{25} = p_{46} = p_{57} = p_{6,10} = p_{89} = p_{91} = 1 \qquad (1\text{-}17)$$

It can be easily seen that the following results hold good:

$$p_{01} + p_{07} + p_{0,10} = 1 \qquad p_{13} + p_{15}^{(2)} + p_{16}^{(4)} = 1$$

$p_{30} + p_{37}^{(5)} + p_{3,10}^{(6)} = 1$ $\quad$ $p_{79}^{(8)} + p_{7,11} = 1$

$p_{10,0} + p_{10,4} = 1$ $\quad$ $p_{11,0} + p_{11,1}^{(9)} = 1$

$p_{25} = p_{46} = p_{57} = p_{6,10} = p_{89} = p_{91} = 1$ (18-24)

4.2. Mean sojourn times

The mean sojourn time in state S_i denoted by Ψ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time Ψ_i, in state S_i, we observe that as long as the system is in state S_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time Ψ_i in state S_i is:

$\Psi_i = E[T_i] = \int P(T_i > t)dt$

Thus

$$\Psi_0 = \int_0^\infty e^{-(\alpha_1+\alpha_2+\alpha_3)t}\,dt = \frac{1}{(\alpha_1+\alpha_2+\alpha_3)}$$

$$\Psi_1 = \int_0^\infty e^{-(\alpha_2+\alpha_3+\mu_1)t}\,dt = \frac{1}{(\alpha_2+\alpha_3+\mu_1)}$$

$$\Psi_2 = \Psi_4 = \int_0^\infty e^{-\mu_1 t}\,dt = \frac{1}{\mu_1}$$

$$\Psi_3 = \int_0^\infty e^{-(\alpha_2+\alpha_3)t}\,\bar{G}_1(t)dt = \frac{1}{(\alpha_2+\alpha_3)}\left[1 - \tilde{G}_1(\alpha_2+\alpha_3)\right]$$

$$\Psi_5 = \Psi_6 = \int_0^\infty \bar{G}_1(t)dt = m_1$$

$$\Psi_7 = \int_0^\infty e^{-(\alpha_1+\mu_2)t}dt = \frac{1}{(\alpha_1+\mu_2)}$$

$$\Psi_8 = \int_0^\infty e^{-\mu_2 t}\,dt = \frac{1}{\mu_2}$$

$$\Psi_9 = \int_0^\infty \bar{G}_2(t)dt = m_2$$

$$\Psi_{10} = \int_0^\infty e^{-\alpha_1 t}\,\bar{G}_3(t)dt = \frac{1}{\alpha_1}\left[1 - \tilde{G}_3(\alpha_1)\right]$$

$$\Psi_{11} = \int_0^\infty e^{-\alpha_1 t}\,\bar{G}_2(t)dt = \frac{1}{\alpha_1}\left[1 - \tilde{G}_2(\alpha_1)\right] \quad (25\text{-}36)$$

5. ANALYSIS OF RELIABILITY AND MTSF

Let the random variable T_i be the time to system failure when system starts up from state $S_i \in E_i$, then the reliability of the system is given by

$R_i(t) = P[T_i > t]$

Using the definition of $R_i(t)$, relations among $R_i(t)$ can be developed, taking their Laplace transforms and solving the resultant set of equations for $R_0^*(s)$, we get

$$R_0^*(s) = N_1(s)/D_1(s) \quad (37)$$

Where,

$$N_1(s) = \left(Z_0^* + q_{01}^* Z_1^* + q_{01}^* q_{13}^* Z_3^* + q_{07}^* Z_7^* + q_{07}^* q_{7,11}^* Z_{11}^* + q_{0,10}^* Z_{10}^*\right) \quad (38)$$

and

$$D_1(s) = \left[1 - \left(q_{01}^* q_{13}^* q_{30}^* + q_{07}^* q_{7,11}^* q_{11,0}^* + q_{0,10}^* q_{10,0}^*\right)\right] \quad (39)$$

To get MTSF, we use the well known formula

$$E(T_0) = \int R_0(t) = \lim_{s\to 0} R_0^*(s) = N_1(0)/D_1(0) \quad (40)$$

Where,

$$N_1(0) = \left(\Psi_0 + p_{01}\Psi_1 + p_{01}p_{13}\Psi_3 + p_{07}\Psi_7 + p_{07}p_{7,11}\Psi_{11} + p_{0,10}\Psi_{10}\right) \quad (41)$$

and

$$D_1(0) = \left[1 - p_{01}p_{13}p_{30} - p_{07}p_{7,11}p_{11,0} + p_{0,10}p_{10,0}\right] \quad (42)$$

since we have,

$q_{ij}^*(0) = p_{ij}$ *and* $\lim_{s\to 0} Z_i^*(s) = \int Z_i(t)dt = \Psi_i$.

6. AVAILABILITY ANALYSIS

Define $A_i(t)$ as the probability that the system is up at epoch 't' when it initially starts from regenerative state S_i . To obtain recurrence relations among pointwise availabilities $A_i(t)$ we use the simple probabilistic arguments. Taking the Laplace transform and solving the resultant set of equations for $A_0^*(s)$, we have

$$A_0^*(s) = N_2(s)/D_2(s) \quad (43)$$

Where,

$$N_2(s) = \Big[\big(Z_0^* D + q_{01}^* Z_1^* + q_{07}^* Z_7^* + q_{01}^* q_{13}^* Z_3^* + q_{07}^* q_{7,11}^* Z_{11}^*\big) + q_{07}^*(Z_1^* + q_{13}^* Z_3^*)\left(q_{79}^{(8)*} q_{91}^* + q_{7,11}^* q_{11,1}^{(9)*}\right) + q_{01}^*(Z_7^* + q_{7,11}^* Z_{11}^*)\left(q_{15}^{(2)*} q_{57}^* + q_{13}^* q_{37}^{(5)*}\right)\Big]\left(1 - q_{10,4}^* q_{46}^* q_{6,10}^*\right) + q_{0,10}^* Z_{10}^* D + \left[q_{01}^* + q_{07}^*\left(q_{79}^{(8)*} q_{91}^* + q_{7,11}^* q_{11,1}^{(9)*}\right)\right]\left(q_{13}^* q_{3,10}^{(6)*} + q_{16}^{(4)*} q_{6,10}^*\right) Z_{10}^* \quad (44)$$

and

$$D_2(s) = \left(1 - q_{10,4}^* q_{46}^* q_{6,10}^* - q_{0,10}^* q_{10,0}^*\right) D - \left\{q_{7,11}^* q_{11,0}^* \left[q_{07}^* + q_{01}^*\left(q_{15}^{(2)*} q_{57}^* + q_{13}^* q_{37}^{(5)*}\right)\right] + q_{13}^* q_{30}^* \left[q_{01}^* + q_{07}^*\left(q_{79}^{(8)*} q_{91}^* + q_{7,11}^* q_{11,1}^{(9)*}\right)\right]\right\}\left(1 - q_{10,4}^* q_{46}^* q_{6,10}^*\right) - q_{10,0}^* \left[q_{01}^* + q_{07}^*\left(q_{79}^{(8)*} q_{91}^* + q_{7,11}^* q_{11,1}^{(9)*}\right)\right]\left(q_{13}^* q_{3,10}^{(6)*} + q_{16}^{(4)*} q_{6,10}^*\right) \quad (45)$$

Where,

$$D = 1 - \left(q_{15}^{(2)*} q_{57}^* + q_{13}^* q_{37}^{(5)*}\right)\left(q_{79}^{(8)*} q_{91}^* + q_{7,11}^* q_{11,1}^{(9)*}\right)$$

The steady state availability will be given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\, A_0^*(s) = N_2(0)/D_2(0)$$

since

$$q_{ij}^*(s)|_{s=0} = q_{ij}^*(0) = p_{ij}$$

and

$$\lim_{s\to 0} Z_i^*(s) = \int_0^\infty Z_i(t)dt = \Psi_i$$

Therefore,

$$N_2(0) = \Big[\big(\Psi_0 D + p_{01}\Psi_1 + p_{07}\Psi_7 + p_{01}p_{13}\Psi_3 + p_{07}p_{7,11}\Psi_{11}\big) + p_{07}(\Psi_1 + p_{13}\Psi_3)\left(p_{79}^{(8)} + p_{7,11}p_{11,1}^{(9)}\right) + p_{01}\big(\Psi_7 + p_{7,11}\Psi_{11}\big)\left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right)\Big]\left(1 - p_{10,4}\right) + p_{0,10}\Psi_{10} D\left[p_{01} + p_{07}\left(p_{79}^{(8)} + p_{7,11}p_{11,1}^{(9)}\right)\right]\left(p_{13}p_{3,10}^{(6)} + p_{16}^{(4)}\right)\Psi_{10} \quad (46)$$

Where,

$$D = 1 - \left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right)\left(p_{79}^{(8)} + p_{7,11}p_{11,1}^{(9)}\right)$$

$$= 1 - \left(1 - p_{7,11}p_{11,0}\right)\left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right)$$

and $D_2(0) = 0$

Hence on using L'Hospital's rule, A_0 becomes

$$A_0 = N_2(0)/D_2'(0) \quad (47)$$

where

$$D_2'(0) = \left(m_{01} + m_{07} + m_{0,10}\right)A + \left(m_{13} + m_{15}^{(4)} + m_{16}^{(4)}\right)B + \left(m_{30} + m_{37}^{(5)} + m_{3,10}^{(6)}\right)p_{13}B + m_{41}C + m_{57}p_{15}^{(2)}B + m_{6,10}C + \left(m_{79}^{(8)} + m_{7,11}\right)D + m_{91}p_{79}^{(8)}D + \left(m_{10,0} + m_{10,4}\right)E + \left(m_{11,0} + m_{11,1}^{(9)}\right)p_{7,11}D$$

Using the relation $\sum_j m_{ij} = \Psi_i$, we get

$$D_2'(0) = \Psi_0 A + \left(\Psi_1 + p_{13}\Psi_3 + p_{15}^{(2)} m_1\right)B + (\Psi_4 + m_1)C + \left(\Psi_7 + p_{79}^{(8)} m_2 + p_{7,11}\Psi_{11}\right)D + \Psi_3 E \quad (48)$$

Where,

$$A = p_{10,0}\left[1 - \left(1 - p_{7,11}p_{11,0}\right)\left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right)\right]$$

$$B = p_{10,0}\left[p_{01} + p_{07}\left(1 - p_{7,11}p_{11,0}\right)\right]$$

$$C = p_{10,4}\left\{1 - \left(1 - p_{7,11}p_{11,0}\right)\left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right) - p_{13}p_{30}\left[p_{01} + p_{07}\left(1 - p_{7,11}p_{11,0}\right)\right] - p_{7,11}p_{11,0}\left[p_{07} + p_{01}\left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right)\right]\right\}$$

$$D = p_{10,0}\left[p_{07} + p_{01}\left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right)\right]$$

$E = \left\{1 - (1 - p_{7,11}p_{11,0})\left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right) - p_{13}p_{30}[p_{01} + (1 - p_{7,11}p_{11,0})] - p_{7,11}p_{11,0}\left[p_{07} + p_{01}\left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right)\right]\right\}$

using (46) and (48) in (47), we get the expression for A_0.

The expected up time of the system during (0, t] is given by

$\mu_{up}(t) = \int_0^t A_0(u)\,du$

So that, $\mu_{up}^*(s) = A_0^*(s)/s$

7. BUSY PERIOD ANALYSIS

Define $B_i(t)$ as the probability that the repairman is busy in the repair of the failed unit when the system initially starts from state $S_i \in E$. Using probabilistic arguments, the value of $B_0(t)$ can be obtained in its L.T. as:

$B_0^*(s) = N_3(s)/D_2(s)$ (49)

Where,

$N_3(s) = \left\{\left(q_{07}^*Z_7^* + q_{07}^*q_{79}^{(8)*}Z_9^* + q_{07}^*q_{7,11}^*Z_{11}^*\right) + q_{01}^*(Z_7^* + q_{79}^{(8)*}Z_9^* + q_{7,11}^*Z_{11}^*)\left(q_{15}^{(2)*}q_{57}^* + q_{13}^*q_{37}^{(5)*}\right) + (Z_1^* + q_{13}^*Z_3^* + q_{15}^{(2)*}Z_5^* + q_{16}^{(4)*}Z_6^*)\left[q_{01}^* + q_{07}^*\left(q_{79}^{(8)*}q_{91}^* + q_{7,11}^*q_{11,1}^{(9)*}\right)\right]\right\}(1 - q_{10,4}^*q_{46}^*q_{6,10}^*) + \left\{q_{0,10}^*D + \left[q_{01}^* + q_{07}^*\left(q_{79}^{(8)*}q_{91}^* + q_{7,11}^*q_{11,1}^{(9)*}\right)\right]\left(q_{13}^*q_{3,10}^{(6)*} + q_{16}^{(4)*}q_{6,10}^*\right)\right\}(q_{10,4}^* + q_{10,4}^*q_{46}^*Z_6^*)$ (50)

Where,

$D = 1 - \left(q_{15}^{(2)*}q_{57}^* + q_{13}^*q_{37}^{(5)*}\right)\left(q_{79}^{(8)*}q_{91}^* + q_{7,11}^*q_{11,1}^{(9)*}\right)$

In the steady state, the probability that the repairman will be busy is given by

$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} s\,B_0^*(s) = N_3(0)/D_2'(0)$ (51)

Where,

$N_3(0) = \left\{\left(p_{07}\Psi_7 + p_{07}p_{79}^{(8)}m_2 + p_{07}p_{7,11}\Psi_{11}\right) + p_{01}\left(\Psi_7 + p_{79}^{(8)}m_2 + p_{7,11}\Psi_{11}\right)\left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right) + \left[p_{01} + p_{07}\left(p_{79}^{(8)} + p_{7,11}p_{11,1}^{(9)}\right)\right]\left(\Psi_1 + p_{13}\Psi_3 + p_{15}^{(2)}m_1 + p_{16}^{(4)}m_1\right)\right\}(1 - p_{10,4}) + \left\{p_{0,10}D + \left[p_{01} + p_{07}\left(p_{79}^{(8)} + p_{7,11}p_{11,1}^{(9)}\right)\right]\left(p_{13}p_{3,10}^{(6)} + p_{16}^{(4)}\right)\right\}(p_{10,4} + p_{10,4}m_1)$ (52)

and

$D = 1 - \left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right)\left(p_{79}^{(8)} + p_{7,11}p_{11,1}^{(9)}\right)$

The expected busy period of the repairman during (0, t] is given by

$\mu_b(t) = \int_0^t B_0(u)\,du$

so that, $\mu_b^*(s) = B_0^*(s)/s$

8. EXPECTED NUMBER OF REPAIRS

Let us define $V_i(t)$ as the expected number of repairs by repairman during the time interval $(0, t]$ when the system initially starts from regenerative state S_i.Using the definition of $V_i(t)$ the recursive relations among $V_i(t)$ can be easily developed, taking their L.S.T. and solving the resultant set of equations for $\tilde{V}_0(s)$, we get

$\tilde{V}_0(s) = N_4(s)/D_2(s)$ (53)

Where,

$N_4(s) = \tilde{Q}_{0,10}\tilde{Q}_{10,4}\tilde{Q}_{46}\tilde{Q}_{6,10}D + \left[\tilde{Q}_{01} + \tilde{Q}_{07}\left(\tilde{Q}_{79}^{(8)}\tilde{Q}_{91} + \tilde{Q}_{7,11}\tilde{Q}_{11,1}^{(9)}\right)\right]\left(\tilde{Q}_{13}\tilde{Q}_{3,10}^{(6)} + \tilde{Q}_{16}^{(4)}\tilde{Q}_{6,10}\right) + \left\{\left[\tilde{Q}_{07} + \tilde{Q}_{01}\left(\tilde{Q}_{15}^{(2)}\tilde{Q}_{57} + \tilde{Q}_{13}\tilde{Q}_{37}^{(5)}\right)\right]\left(\tilde{Q}_{79}^{(8)}\tilde{Q}_{91} + \tilde{Q}_{7,11}\tilde{Q}_{11,1}^{(9)} + \tilde{Q}_{7,11}\tilde{Q}_{11,0}\right) + \left[\tilde{Q}_{01} + \tilde{Q}_{07}\left(\tilde{Q}_{79}^{(8)}\tilde{Q}_{91} + \tilde{Q}_{7,11}\tilde{Q}_{11,1}^{(9)}\right)\right]\left(\tilde{Q}_{15}^{(2)}\tilde{Q}_{57} + \tilde{Q}_{13}\tilde{Q}_{37}^{(5)} + \tilde{Q}_{13}\tilde{Q}_{30}\right)\right\}(1 - \tilde{Q}_{10,4}\tilde{Q}_{46}\tilde{Q}_{6,10})$ (54)

and

$D = 1 - \left(\tilde{Q}_{15}^{(2)}\tilde{Q}_{57} + \tilde{Q}_{13}\tilde{Q}_{37}^{(5)}\right)\left(\tilde{Q}_{79}^{(8)}\tilde{Q}_{91} + \tilde{Q}_{7,11}\tilde{Q}_{11,1}^{(9)}\right)$

In the long run the expected number of repairs per unit of time by the repairman is given by

$V_0 = \lim_{t\to\infty} \frac{[V_0(t)]}{t} = \lim_{s\to 0} s^2 V_0^*(s) = N_4(0)/D_2'(0)$ (55)

Where,

$$N_4(0) = p_{10,4}p_{0,10}D + \left[p_{01} + p_{07}\left(p_{79}^{(8)} + p_{7,11}p_{11,1}^{(9)}\right)\right]\left(p_{16}^{(4)} + p_{13}p_{3,10}^{(6)}\right) + \left\{\left[p_{07} + p_{01}\left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right)\right]\left(p_{79}^{(8)} + p_{7,11}p_{11,1}^{(9)} + p_{7,11}p_{11,0}\right) + \left[p_{01} + p_{07}\left(p_{79}^{(8)} + p_{7,11}p_{11,1}^{(9)}\right)\right]\left(p_{15}^{(2)} + p_{13}p_{37}^{(5)} + p_{13}p_{30}\right)\right\}\left(1 - p_{10,4}\right) \quad (56)$$

and $D = 1 - \left(p_{15}^{(2)} + p_{13}p_{37}^{(5)}\right)\left(p_{79}^{(8)} + p_{7,11}p_{11,1}^{(9)}\right)$

9. PROFIT FUNCTION ANALYSIS

Two profit functions $P_1(t)$ and $P_2(t)$ can easily be obtained for the system model under study with the help of characteristics obtained earlier. The expected total profits incurred during (0, t] are:

$P_1(t) =$ Expected total revenue in (0,t]− Expected total expenditure in (0,t]

$$= K_0\mu_{up}(t) - K_1\mu_b(t) \quad (57)$$

Similarly,

$$P_2(t) = K_0\mu_{up}(t) - K_2V_0(t) \quad (58)$$

Where,

K_0 is revenue per unit up time.

K_1 is the cost per unit time for which repair man is busy in repair of the failed unit.

K_2 is per unit repair cost.

The expected total profits per unit time, in steady state, is

$$P_1 = \lim_{t\to\infty}[P_1(t)/t] = \lim_{s\to 0} s^2P_1^*(s)$$

So that,

$$P_1 = K_0A_0 - K_1B_0 \quad (59)$$

and

$$P_2 = K_0A_0 - K_2V_0 \quad (60)$$

10. GRAPHICAL STUDY OF THE SYSTEM MODEL

For more concrete study of system behaviour, we plot MTSF and Profit functions with respect to α_1 (failure rate of 1st unit) for different values of λ_1(repair rate of 1st unit).

Fig.2 shows the variations in MTSF in respect of α_1 for three different values of λ_1 as 0.25, 0.50 and 0.75 while the other parameters are kept fixed as α_2=0.30, α_3=0.03, μ_1=0.90, μ_2=0.80, λ_2=0.70, λ_3=0.08. It is observed from the graph that MTSF decreases with the increase in the failure parameter α_1 and for higher values of λ_1, the MTSF is higher i.e., the repair facility has a better understanding of failure phenomenon resulting in longer lifetime of the system.

Fig.3 represents the change in profit functions P_1 and P_2 w.r.t. α_1 for different values of λ_1 as 0.25, 0.50 and 0.75 while the other parameters are fixed as α_2=0.30, α_3=0.03, μ_1=0.90, μ_2=0.80, λ_2=0.70, λ_3=0.08, k_0=1000, k_1=500, k_2=450. From the graph it is seen that both profit functions decrease with the increase in failure rate α_1 and increase with the increase in λ_1. It is also observed that profit function P_2 is always higher as compared to profit function P_1 for fixed values of α_1 and λ_1. Thus the better understanding of failure phenomenon by the repairman results in better system performance.

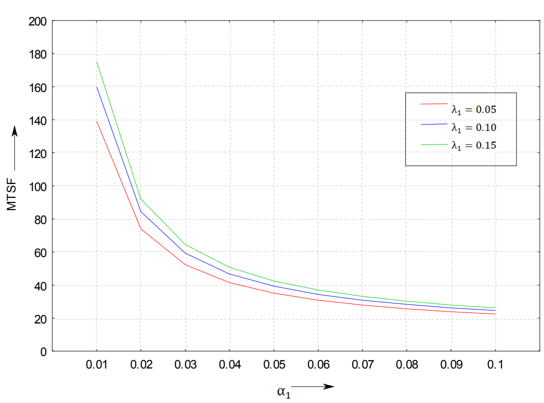

Figure 2

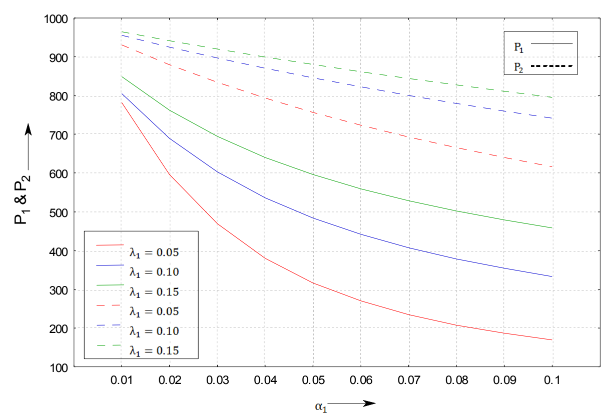

Figure 3

Legend:

Figure 2: Behaviour of MTSF w.r.t. α_1 for different values of λ_1

Figure 3: Behaviour of P_1 and P_2 w.r.t. α_1 for different values of λ_1

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