Research Article

Cost-benefit analysis of a two similar cold standby space shuttle system with failure caused by vehicle disintegration during launch and failure during re-entry

Ashok Kumar Saini

Associate Professor, Department of Mathematics, B. L. J. S. College, Tosham, Bhiwani, Haryana, INDIA.
Email: drashokksaini2009@gmail.com

Abstract

First U.S. in-flight fatalities. The Space Shuttle Challenger was destroyed 73 seconds after lift-off on STS-51-L. The investigation found that a faulty O-ring seal allowed hot gases from the shuttle solid rocket booster (SRB) to impinge on the external propellant tank and booster strut. The strut and aft end of the tank failed, allowing the top of the SRB to rotate into the top of the tank. Challenger was thrown sideways into the Mach 1.8 wind stream and broke up with the loss of all seven crew members. NASA investigators determined they may have survived the spacecraft disintegration, possibly unconscious from hypoxia; some tried to activate their emergency oxygen. Any survivors of the breakup were killed, however, when the largely intact cockpit hit the water at 320 km/h (200 mph). The vehicle impacted the water about 32 km (20 miles) east of Cape Canaveral. "Tracking reported that the vehicle had exploded and impacted the water in an area approximately located at 28.64 degrees north, 80.28 degrees west", Mission Control, Houston. About half of the vehicle's remains were never recovered, and fragments still wash ashore occasionally on the coast of Brevard County, Florida. The Space Shuttle Columbia was lost as it returned from a two-week mission, STS-107. Damage to the shuttle's thermal protection system (TPS) led to structural failure of the shuttle's left wing and the spacecraft ultimately broke apart. Investigation revealed damage to the reinforced carbon-carbon leading edge wing panel resulted from the impact of a piece of foam insulation that broke away from the external tank during the launch. The vehicle broke up over the southwestern United States and fell in fragments over eastern Texas and central Louisiana. We have taken units- Space Shuttle failure caused due to failure caused by Vehicle disintegration during launch and due to Failure caused by Vehicle disintegration during re-entry with failure time distribution as exponential and repair time distribution as General. We have find out MTSF, Availability analysis, the expected busy period of the server for repair when the failure of Space Shuttle caused by Vehicle disintegration during launch in (0, t], expected busy period of the server for repair in (0,t], the expected busy period of the server for repair when failure of Space Shuttle caused by Vehicle disintegration during re-entry in (0,t], the expected number of visits by the repairman for failure of Space Shuttle caused by Vehicle disintegration during launch in (0,t], the expected number of visits by the repairman for failure of Space Shuttle caused by Vehicle disintegration during re-entry in (0,t] and Cost-Benefit analysis using regenerative point technique. A special case using failure and repair distributions as exponential is derived and graphs have been drawn.

Keyword: Cold Standby, Space Shuttle failure caused by Vehicle disintegration during launch and Failure caused by Vehicle disintegration during re-entry, MTSF, Availability, Busy period, Cost-Benefit Analysis

*__Address for Correspondence:__
Dr. Ashok Kumar Saini, Associate Professor, Department of Mathematics, BLJS College, Tosham, Bhiwani, Haryana, INDIA.
Email: drashokksaini2009@gmail.com

Received Date: 13/12/2014 Accepted Date: 23/12/2014

Access this article online	
Quick Response Code:	Website: www.statperson.com
	DOI: 20 December 2014

How to site this article: Ashok Kumar Saini. Cost-benefit analysis of a two similar cold standby space shuttle system with failure caused by vehicle disintegration during launch and failure during re-entry. *International Journal of Statistika and Mathemtika* Nov 2014 to Jan 2015; 12(2): 126-133. http://www.statperson.com (accessed 23 December 2014).

INTRODUCTION

The **Space Shuttle *Challenger* disaster** occurred on January 28, 1986, when the NASA Space Shuttle orbiter *Challenger* (OV-099) (mission STS-51-L) broke apart 73 seconds into its flight, leading to the deaths of its seven crew members. The spacecraft disintegrated over the Atlantic Ocean, off the coast of Cape Canaveral, Florida at 11:38 EST (16:38 UTC). Disintegration of the vehicle began after an O-ring seal in its right solid rocket booster (SRB) failed at liftoff. The O-ring failure caused a breach in the SRB joint it sealed, allowing pressurized hot gas from within the solid rocket motor to reach the outside and impinge upon the adjacent SRB attachment hardware and external fuel tank. This led to the separation of the right-hand SRB's aft attachment and the structural failure of the external tank. Aerodynamic forces broke up the orbiter. The crew compartment and many other vehicle fragments were eventually recovered from the ocean floor after a lengthy search and recovery operation. The exact timing of the death of the crew is unknown; several crew members are known to have survived the initial breakup of the spacecraft. The shuttle had no escape system, and the impact of the crew compartment with the ocean surface was too violent to be survivable. The disaster resulted in a 32-month hiatus in the shuttle program and the formation of the Rogers Commission, a special commission appointed by United States President Ronald Reagan to investigate the accident. The Rogers Commission found NASA's organizational culture and decision-making processes had been key contributing factors to the accident. NASA managers had known contractor Morton Thiokol's design of the SRBs contained a potentially catastrophic flaw in the O-rings since 1977, but failed to address it properly. They also disregarded warnings (an example of "go fever") from engineers about the dangers of launching, posed by the low temperatures of that morning, and failed to adequately report these technical concerns to their superiors. The **Space Shuttle *Columbia* disaster** occurred on February 1, 2003, when *Columbia* disintegrated over Texas and Louisiana as it re-entered Earth's atmosphere, killing all seven crew members. During the launch of STS-107, *Columbia'*s 28th mission, a piece of foam insulation broke off from the Space Shuttle external tank and struck the left wing. Most previous shuttle launches had seen minor damage from foam shedding, but some engineers suspected that the damage to *Columbia* was more serious. NASA managers limited the investigation, reasoning that the crew could not have fixed the problem if it had been confirmed. When *Columbia* re-entered the atmosphere of Earth, the damage allowed hot atmospheric gases to penetrate and destroy the internal wing structure, which caused the spacecraft to become unstable and slowly break apart. After the disaster, Space Shuttle flight operations were suspended for more than two years, similar to the aftermath of the *Challenger* disaster. Construction of the International Space Station (ISS) was put on hold; the station relied entirely on the Russian Federal Space Agency for resupply for 29 months until Shuttle flights resumed with STS-114 and 41 months for crew rotation until STS-121. Several technical and organizational changes were made, including adding a thorough on-orbit inspection to determine how well the shuttle's thermal protection system had endured the ascent, and keeping a designated rescue mission ready in case irreparable damage was found. Except for one final mission to repair the Hubble Space Telescope, subsequent missions were flown only to the ISS so that the crew could use it as a "safe haven" in case damage to the orbiter prevented safe re-entry. In this paper, we have taken failure of Space Shuttle caused due to Vehicle disintegration during launch and failure caused by Vehicle disintegration during re-entry which is non-instantaneous in nature. Here, we investigate a two identical cold standby –a system in which offline unit cannot fail. When there is failure caused by Vehicle disintegration during re-entry within specified limit, it operates as normal as before but if these are beyond the specified limit the operation of the unit is stopped to avoid excessive damage of the unit and as when there is failure caused by Vehicle disintegration during re-entry continues going on some characteristics of the unit change which we call failure of the unit. After failure of Space Shuttle caused due to Vehicle disintegration during re-entry the failed unit undergoes repair immediately according to first come first served discipline.

ASSUMPTIONS

1. The system consists of two similar cold standby units. The failure time distributions of the operation of the unit stopped automatically, the failure caused by Vehicle disintegration during launch and Failure caused by Vehicle disintegration during re-entry are exponential with rates λ_1, λ_2 and λ_3 whereas the repairing rates for repairing the failed system due to failure caused by Vehicle disintegration during launch and Failure caused by Vehicle disintegration during re-entry are arbitrary with CDF G_1 (t) and G_2 (t) respectively.
2. When there is failure caused by Vehicle disintegration during re-entry within specified limit, it operates as normal as before but if these are beyond the specified limit the operation of the unit is avoided and as the failure caused by Vehicle disintegration during re-entry continues goes on some characteristics of the unit change which we call failure of the unit.

3. The failure caused by Vehicle disintegration during re-entry actually failed the units. The failure caused by Vehicle disintegration during re-entry is non-instantaneous and it cannot occur simultaneously in both the units.
4. The repair facility works on the first fail first repaired (FCFS) basis.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.

Symbols for states of the System

Superscripts: O, CS, SO, VDLF, VDRF

Operative, cold Standby, Stops the operation, space shuttle failure caused due to Vehicle disintegration during launch, due to failure caused by Vehicle disintegration during re-entry respectively

Subscripts: nvdr, uvdr, vdl, ur, wr, uR

No failure caused by Vehicle disintegration during re-entry, under failure caused by Vehicle disintegration during re-entry, failure caused by Vehicle disintegration during launch, under repair, waiting for repair, under repair continued respectively

Up states: 0, 1, 3;

Down states: 2, 4, 5, 6, 7

STATES OF THE SYSTEM

0(O_{nvdr}, CS_{nvdr})

One unit is operative and the other unit is cold standby and there is no failure caused by Vehicle disintegration during re-entry in both the units.

1(SO_{unvdr}, O_{nvdr})

The operation of the first unit stops automatically due to failure caused by Vehicle disintegration during re-entry and cold standby unit starts operating with no failure caused by Vehicle disintegration during re-entry.

2(SO_{unvdr}, $VDRF_{vdl,, ur}$)

The operation of the first unit stops automatically due to failure caused by Vehicle disintegration during re-entry and the other unit fails due to failure caused by Vehicle disintegration during re-entry undergoes repair.

3($VDLF_{ur}$, O_{nvdr})

The first unit fails due to failure caused by Vehicle disintegration during launch undergoes repair and the other unit continues to be operative with no failure caused by Vehicle disintegration during re-entry.

4($VDLF_{uR}$, SO_{unvdr})

The one unit fails due to failure caused by Vehicle disintegration during launch continues to be under repair and the other unit also stops automatically due to failure caused by Vehicle disintegration during re-entry.

5($VDLF_{uR}$, $VDLF_{wr}$)

The repair of the first unit is continued from state 4 and the other unit failed due to failure caused by Vehicle disintegration during launch is waiting for repair.

6($VDLF_{uR}$, SO_{unvdr})

The repair of the first unit is continued from state 3 fails due to failure caused by Vehicle disintegration during launch and operation of other unit stops automatically due to failure caused by Vehicle disintegration during re-entry.

7($VDLF_{wr}$, $VDRF_{vdl, uR}$)

The repair of failed unit due to failure caused by Vehicle disintegration during re-entry is continued from state 2 and the first unit failed due to failure caused by Vehicle disintegration during launch is waiting for repair.

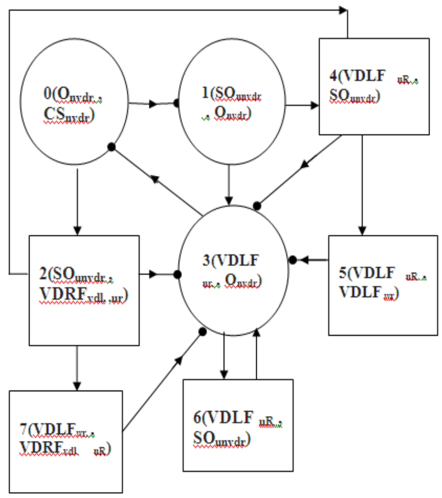

Figure 1: The State Transition Diagram

● Rregeneration point ○ Up State □ Down State

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$p_{01} = \frac{\lambda 1}{\lambda 1+\lambda 3}$, $p_{02} = \frac{\lambda 3}{\lambda 1+\lambda 3}$

$p_{13} = \frac{\lambda 2}{\lambda 1+\lambda 2}$, $p_{14} = \frac{\lambda 1}{\lambda 1+\lambda 2}$

$p_{23} = \lambda_1 G_2^*(\lambda_2)$, $p_{23}^{(7)} = \lambda_2 G_2^*(\lambda_2)$, $p_{24} = \overline{G}_2^*(\lambda_2)$,

$p_{30} = G_1^*(\lambda_1)$, $p_{33}^{(6)} = \overline{G}_1^*(\lambda_1)$

$p_{43} = G_1^*(\lambda_2)$, $p_{43}^{(5)} = G_1^*(\lambda_2)$ (1)

we can easily verify that

$p_{01} + p_{02} = 1$, $p_{13} + p_{14} = 1$,

$p_{23} + p_{23}^{(7)} + p_{24} = 1$, $p_{30} + p_{33}^{(6)} = 1$,

$p_{43} + P_{43}^{(5)} = 1$ (2)

and mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt$

$= -1/\lambda_1$

Similarly

$\mu_1 = 1/\lambda_2$,

$\mu_2 = \int_0^\infty e^{-\lambda 1}\overline{G}1(t)dt$,

$\mu_4 = \int_0^\infty e^{-\lambda 2}\overline{G}1(t)dt$ (3)

Mean Time to System Failure

We can regard the failed state as absorbing

$\theta_0(t) = Q_{01}(t)[s]\theta_1(t) + Q_{02}(t)$

$\theta_1(t) = Q_{13}(t)[s]\theta_3(t) + Q_{14}(t)$, $\theta_3(t) = Q_{30}(t)[s]\theta_0(t) + Q_{33}^{(6)}(t)$ (4-6)

Taking Laplace-Stieltjes transforms of eq. (4-6) and solving, we get

$Q_0^*(s) = N_1(s) / D_1(s)$ (7)

where

$N_1(s) = Q_{01}^*(s)\ \{\ Q_{13}^*(s)\, Q_{33}^{(6)*}(s) + Q_{14}^*(s)\} + Q_{02}^*(s)$

$D_1(s) = 1 - Q_{01}^*(s)\quad Q_{13}^*(s)\, Q_{30}^*(s)$

Making use of relations (1) and (2) it can be shown that $Q_0^*(0)$ =1, which implies that $\theta_1(t)$ is a proper distribution.

$$MTSF = E[T] = \frac{d}{ds}\theta_0(s)\Big|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$= (\mu_0 + p_{01} \mu_1 + p_{01} p_{13} \mu_3) / (1 - p_{01} p_{13} p_{30})$ (8)

where

$\mu_0 = \mu_{01} + \mu_{02}$,
$\mu_1 = \mu_{13} + \mu_{14}$,
$\mu_2 = \mu_{23} + \mu_{23}^{(1)} + \mu_{24}$,
$\mu_3 = \mu_{30} + \mu_{33}^{(6)}$
$\mu_4 = \mu_{43} + \mu_{43}^{(5)}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

The value of $M_0(t) = e^{-\lambda_1 t} e^{-\lambda_3 t}$, $M_1(t) = e^{-\lambda_1 t} e^{-\lambda_2 t}$

$M_3(t) = e^{-\lambda_1} \overline{G}_1(t)$. (9)

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$
$A_1(t) = M_1(t) + q_{13}(t)[c]A_3(t) + q_{14}(t)[c]A_4(t)$,
$A_2(t) = \{q_{23}(t) + q_{23}^{(7)}(t)\}[c]A_3(t) + q_{33}^{(6)}(t)[c]A_3(t)$
$A_3(t) = M_3(t) + \{q_{30}(t) + q_{33}^{(6)}(t)\}[c]A_3(t)$
$A_4(t) = \{q_{43}(t) + q_{43}^{(5)}(t)[c]A_3(t)$ (10 - 14)

Taking Laplace Transform of eq. (10-14) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (15)

where

$N_2(s) = (1 - \hat{q}_{33}^{(6)}(s)) \widehat{M}_0(s) + [\hat{q}_{01}(s) \{ \widehat{M}_1(s) + (\hat{q}_{13}(s) + \hat{q}_{14}(s) (\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s)))\} + \hat{q}_{02}(s)$
$\{\hat{q}_{23}(s)) + \hat{q}_{23}^{(1)}(s)) + \hat{q}_{24}(s)(\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\}] \widehat{M}_3(s)$
$D_2(s) = (1 - \hat{q}_{33}^{(6)}(s)) - \hat{q}_{30}(s) [\hat{q}_{01}(s)$
$\{ \hat{q}_{13}(s) + \hat{q}_{14}(s) (\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s)) \}$
$+ \hat{q}_{20}(s)\{ \hat{q}_{23}(s) + \hat{q}_{23}^{(7)}(s) + \hat{q}_{24}(s)(\hat{q}_{43}(s)) + \hat{q}_{43}^{(5)}(s))\}]$

The steady state availability

$A_0 = \lim_{t\to\infty}[A_0(t)]$
$= \lim_{s\to 0}[s \hat{A}_0(s)] = \lim_{s\to 0} \frac{s N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$A_0 = \lim_{s\to 0} \frac{N_2(s) + s N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)}$ (16)

where

$N_2(0) = p_{30} \widehat{M}_0(0) + p_{01}\widehat{M}_1(0) \widehat{M}_3(0))$
$D_2'(0) = \mu_3 + [\mu_0 + p_{01} (\mu_1 + p_{14} \mu_4 + p_{02}(\mu_2 + p_{24} \mu_4)] p_{30}$

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^{\infty} A_0(z)dz$ So that $\widehat{\lambda_u}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (17)

The expected down time of the system in (0, t] is

$\lambda_d(t) = t - \lambda_u(t)$ So that $\widehat{\lambda_d}(s) = \frac{1}{s^2} - \widehat{\lambda_u}(s)$ (18)

The expected busy period of the server when the operation of the unit stops automatically when there is Failure caused by Vehicle is integration during re-entry in (0,t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$
$R_1(t) = S_1(t) + q_{13}(t)[c]R_3(t) + q_{14}(t)[c]R_4(t)$,
$R_2(t) = S_2(t) + q_{23}(t)[c]R_3(t) + q_{23}^{(7)}(t)[c]R_3(t) + q_{24}(t)[c]R_4(t)$
$R_3(t) = q_{30}(t)[c]R_0(t) + q_{33}^{(6)}(t)[c]R_3(t)$,
$R_4(t) = S_4(t) + (q_{43}(t) + q_{43}^{(5)}(t)) [c]R_3(t)$ (19-23)

where

$S_1(t) = e^{-\lambda_1 t} e^{-\lambda_2 t}$,
$S_2(t) = e^{-\lambda_1 t} \overline{G}_2(t)$,

$S_4(t) = e^{-\lambda_1 t}\, \overline{G}_1(t)$ (24)

Taking Laplace Transform of eq. (19-23) and solving for $\widehat{R_0}(s)$

$\widehat{R_0}(s) = N_3(s) / D_2(s)$ (25)

where

$N_3(s) = (1 - \hat{q}_{33}^{(6)}(s)) [\hat{q}_{01}(s)(\hat{S}_1(s) + \hat{q}_{14}(s)\hat{S}_4(s) + \hat{q}_{02}(s)(\hat{S}_2(s) + \hat{q}_{24}(s)$
$\hat{S}_4(s))]$

and $D_2(s)$ is already defined.

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (26)

where

$N_3(0) = p_{30} [p_{01} (\hat{S}_1(0) + p_{14} \hat{S}_4(0)) + p_{02} (\hat{S}_2(0) + p_{24} \hat{S}_4(0))$

and $D_2'(0)$ is already defined.

The expected period of the system under Failure caused by Vehicle disintegration during re-entry in (0, t] is

$\lambda_{tsp}(t) = \int_0^{\propto} R_0(z)dz$ so that

$\widehat{\lambda_{tsp}}(s) = \frac{\hat{R}_0(s)}{s}$ (27)

The expected Busy period of the server for repair when there is failure caused by Vehicle disintegration during launch in (0, t]

$B_0(t) = q_{01}(t)[c]B_1(t) + q_{02}(t)[c]B_2(t)$

$B_1(t) = q_{13}(t)[c]B_3(t) + q_{14}(t)[c]B_4(t)$,

$B_2(t) = q_{23}(t)[c] B_3(t) + q_{23}^{(7)}(t)[c]B_3(t) + q_{24}(t)[c] B_4(t)$

$B_3(t) = T_3(t) + q_{30}(t)[c] B_0(t) + q_{33}^{(6)}(t)[c]B_3(t)$

$B_4(t) = T_4(t) + \{q_{43}(t) + q_{43}^{(5)}(t)\} [c]B_3(t)$ (28-32)

where

$T_3(t) = e^{-\lambda_2 t}\, \overline{G}_1(t)$,

$T_4(t) = e^{-\lambda_1 t}\, \overline{G}_1(t)$ (33)

Taking Laplace Transform of eq. (28-32) and solving for $\widehat{B_0}(s)$

$\widehat{B_0}(s) = N_4(s) / D_2(s)$ (34)

where

$N_4(s) = \hat{T}_3(s) [\hat{q}_{01}(s) \{\hat{q}_{13}(s) + \hat{q}_{14}(s)$
$(\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\} + \hat{q}_{02}(s) \{\hat{q}_{23}(s) +$
$\hat{q}_{23}^{(7)}(s) + \hat{q}_{24}(s) (\hat{q}_{43}(s) +$
$\hat{q}_{43}^{(5)}(s))\}] + \hat{T}_4(s) [\hat{q}_{01}(s)\hat{q}_{44}(s)(1 - \hat{q}_{33}^{(6)}(s)) + (\hat{q}_{02}(s)\hat{q}_{24}(s)(1-$
$\hat{q}_{33}^{(6)}(s))]$

And $D_2(s)$ is already defined.

In steady state, $B_0 = \frac{N_4(0)}{D_2'(0)}$ (35)

where $N_4(0) = \hat{T}_3(0) + \hat{T}_4(0) \{p_{30} (p_{01}p_{14} + p_{02} p_{24})\}$ and $D_2'(0)$ is already defined.

The expected busy period of the server for repair in (0, t] is

$\lambda_{cf}(t) = \int_0^{\propto} B_0(z)dz$ So that $\widehat{\lambda_{cf}}(s) = \frac{\hat{B}_0(s)}{s}$ (36)

The expected Busy period of the server for repair when there is failure caused by Vehicle disintegration during launch in (o, t]

$P_0(t) = q_{01}(t)[c]P_1(t) + q_{02}(t)[c]P_2(t)$

$P_1(t) = q_{13}(t)[c]P_3(t) + q_{14}(t)[c]P_4(t)$,

$P_2(t) = L_2(t) + q_{23}(t)[c]P_3(t) + q_{23}^{(7)}(t)[c]P_3(t) + q_{24}(t)[c]P_4(t)$

$P_3(t) = q_{30}(t)[c]P_0(t) + q_{33}^{(6)}(t)[c]P_3(t)$,

$P_4(t) = (q_{43}(t) + q_{43}^{(5)}(t)) [c]P_3(t)$ (37-41)

where $L_2(t) = e^{-\lambda_1 t}\, \overline{G}_2(t)$

Taking Laplace Transform of eq. (37-41) and solving for $\widehat{P_0}(s)$

$\widehat{P_0}(s) = N_5(s) / D_2(s)$ (43)

where $N_5(s) = \hat{q}_{02}(s) \hat{L}_2(s) (1 - \hat{q}_{33}^{(6)}(s))$ and $D_2(s)$ is defined earlier.

In the long run, $P_0 = \frac{N_5(0)}{D_2'(0)}$ (44)

where $N_5(0)= p_{30}\, p_{02}\, \hat{L}_2(0)$
and $D_2'(0)$ is already defined.

The expected busy period of the server for repair when failure caused due to failure caused by Vehicle disintegration during launch in (0, t] is

$\lambda_{cf}(t) = \int_0^{\infty} P_0(z)dz$

So that $\widehat{\lambda_{cf}}(s) = \frac{\hat{P}_0(s)}{s}$ (45)

The expected number of visits by the repairman for repairing the units when there is failure caused by Vehicle disintegration during launch in (0, t]

$H_0(t) = Q_{01}(t)[s]H_1(t) + Q_{02}(t)[s]H_2(t)$
$H_1(t) = Q_{13}(t)[s][1+H_3(t)] + Q_{14}(t)[s][1+H_4(t)],$
$H_2(t) = [Q_{23}(t) + Q_{23}^{(7)}(t)]\,[s][1+H_3(t)] + Q_{24}(t)[s][1+ H_4(t)]$
$H_3(t) = Q_{30}(t)[s]H_0(t) + Q_{33}^{(6)}(t)[s]H_3(t),$
$H_4(t) = (Q_{43}(t)+ Q_{43}^{(5)}(t))\,[s]H_3(t)$ (46-50)

Taking Laplace Transform of eq. (46-50) and solving for $H_0^*(s)$

$H_0^*(s) = N_6(s) / D_3(s)$ (51)

where

$N_6(s) = (1 - Q_{33}^{(6)*}(s))\{Q^*_{01}(s)(Q^*_{13}(s) + Q^*_{14}(s)) + Q^*_{02}(s)(Q^*_{24}(s) + Q^*_{23}(s)+Q^*_{23}{}^{(7)}(s))\}$

$D_3(s) = (1 - Q_{33}^{(6)*}(s)) - Q^*_{30}(s)[\,Q^*_{01}(s)\{\, Q^*_{13}(s) + Q^*_{14}(s)\,(Q_{43}^*(s)\,Q^*_{43}{}^{(5)}(s))\} + Q^*_{02}(s)\{Q^*_{23}(s)+ Q^*_{23}{}^{(7)}(s)) + Q^*_{24}(s)(Q^*_{43}(s)) + Q^*_{43}{}^{(5)}(s))\}]$

In the long run,

$H_0 = \frac{N_6(0)}{D_3'(0)}$ (52)

where $N_6(0)= p_{30}$ and $D'_3(0)$ is already defined.

The expected number of visits by the repairman for repairing when there is Failure caused by Vehicle disintegration during re-entry in (0, t]

$V_0(t) = Q_{01}(t)[s]V_1(t) + Q_{02}(t)[s][1+V_2(t)]$
$V_1(t) = Q_{13}(t)[s]V_3(t) + Q_{14}(t)[s]V_4(t),$
$V_2(t) = Q_{24}(t)[s][1+V_4(t)] +[\,Q_{23}(t) + Q_{23}^{(7)}(t)[s][1+V_3(t)]$
$V_3(t) = Q_{30}(t)[s]V_0(t) + Q_{33}^{(6)}(t)[s]V_3(t)$ (53-57)

Taking Laplace-Stieltjes transform of eq. (53-57) and solving for $V_0^*(s)$

$V_0^*(s) = N_7(s) / D_4(s)$ (58)

Where

$N_7(s) = (1 - Q_{33}^{(6)*}(s))\{Q^*_{01}(s)(Q^*_{14}(s) + Q^*_{43}(s)) + Q^*_{02}(s)(Q^*_{24}(s) + Q^*_{02}(s)(Q^*_{23}(s)+Q^*_{23}{}^{(7)}(s))\}$

and $D_4(s)$ is the same as $D_3(s)$

In the long run, $V_0 = \frac{N_7(0)}{D_4'(0)}$ (59)

where $N_7(0)= p_{30}\,[p_{01}\, p_{14}\, p_{43} + p_{02}]$ and $D'_3(0)$ is already defined.

COST BENEFIT ANALYSIS

The cost-benefit function of the system considering mean up-time, expected busy period of the system under Failure caused by Vehicle disintegration during re-entry when the units stops automatically, expected busy period of the server for repair when there is failure caused by Vehicle disintegration during launch, expected total repair cost for repairing the units when there is failure caused by Vehicle disintegration during re-entry, expected number of visits by the repairman when there is failure caused by Vehicle disintegration during launch, expected number of visits by the repairman for failure caused by Vehicle disintegration during re-entry.

The expected total cost-benefit incurred in (0, t] is

C(t) = Expected total revenue in (0,t]

- expected busy period of the system under failure caused by Vehicle disintegration during re-entry when the units automatically stop in (0,t]

- expected total repair cost when there is failure caused by Vehicle disintegration during launch in (0,t]
- expected total repair cost for repairing the units when there is failure caused by Vehicle disintegration during re-entry in (0,t]
- expected number of visits by the repairman for repairing when there is failure caused by Vehicle disintegration during launch in (0,t]
- expected number of visits by the repairman for repairing the units when there is failure caused by Vehicle disintegration during re-entry in (0,t]

The expected total cost per unit time in steady state is

C $=\lim_{t\to\infty}(C(t)/t)$

$= \lim_{s\to 0}(s^2 C(s))$

$= K_1A_0 - K_2 R_0 - K_3B_0 - K_4 P_0$

$- K_5 H_0 - K_6 V_0$

Where

K_1: revenue per unit up-time,

K_2: cost per unit time for which the system is under failure caused by Vehicle disintegration during re-entry when units automatically stop.

K_3: cost per unit time for which the system is under unit repair when there is failure caused by Vehicle disintegration during launch

K_4: cost per unit time for which the system failure due to failure caused by Vehicle disintegration during re-entry

K_5: cost per visit by the repairman when there is failure caused by Vehicle disintegration during launch,

K_6: cost per visit by the repairman when there is failure caused by Vehicle disintegration during re-entry.

CONCLUSION

After studying the system, we have analyzed graphically that when the **failure rate** due to operation of the unit stops automatically, due to failure caused by Vehicle disintegration during launch and due to failure caused by Vehicle disintegration during re-entry increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

REFERENCES

1. Barlow, R.E. and Proschan, F., Mathematical theory of Reliability, 1965; John Wiley, New York.
2. Dhillon, B.S. and Natesen, J, Stochastic Anaysis of outdoor Power Systems in fluctuating environment, Microelectron. Reliab..1983; 23, 867-881.
3. Gnedanke, B.V., Belyayar, Yu.K. and Soloyer, A.D., Mathematical Methods of Relability Theory, 1969 ; Academic Press, New York.
4. Goel, L.R., Sharma,G.C. and Gupta, Rakesh Cost Analysis of a Two-Unit standby system with different weather conditions, Microelectron. Reliab., 1985; 25, 665-659.
5. Cao, Jinhua, Stochastic Behavior of a Man Machine System Operating under Changing Environment Subject to a Markov Process with two States, Microelectron. Reliab.1989, 28, pp. 373-378.

Source of Support: None Declared
Conflict of Interest: None Declared