Research Article

Stochastic analysis of a two non-identical unit parallel system model with preparation time for operation and proviso of rest

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Abstract

The present paper deals with stochastic analysis of reliability characteristics of a two non identical unit in standby configuration, each unit having two modes normal (N) and total failure (F). The non-priority unit goes for rest after a random period of operation and after rest for a random period of time; it again starts working without affecting its efficiency. Some preparation time is required to put a standby unit, a repaired unit or a unit from rest into operation which is taken to be a random variable having some probability distribution A single repairman is always available with the system for preparation of operation and to repair a failed unit. The priority unit gets priority in repair over non- priority unit but when non- priority unit completes its rest period and priority unit is under repair then non-priority unit gets priority in operation. All the failure time distributions and preparation time distribution are taken to be negative exponential with different parameters. The repair time distribution are taken as general. The rest period distribution and completion of rest period distribution of non-priority unit are also taken as negative exponential.

Keywords: Reliability; Availability; Busy period; Expected number of Repairs; Profit Analysis; Graphical study of Model.

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INTRODUCTION

Incorporation of redundancies, preventive maintenance, repair maintenance and corrective maintenance are some of the techniques used to increase the system reliability/availability. Introduction of redundancies is most common technique adopted for this purpose, provided cost constraints are not affected to a great extent. Two types of redundancies i.e. active and passive (standby) may be used as per situations demand. In active redundancy a redundant unit immediately starts its operation but in standby redundancy the redundant unit may need some significant time to start. Two unit active/passive redundant system models have been studied widely by various authors in the field of reliability theory under different sets of assumptions, different modes of operative unit, different types of failures and having different failure and repair time distributions. Gupta and Chaudhary (1994); Gupta and Shivakar (2003); Mogha et.al (2003) have analysed two unit redundant system models under different sets of assumptions. In most of these papers it is assumed that the redundant unit or a unit after repair starts its operation immediately. But in practice sometimes a redundant or

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repaired unit requires some preparation time to start its functioning. Keeping this fact in view in the present study a stochastic analysis of reliability characteristics of a two non identical unit system with proviso of rest is carried out. Each unit of the system has two modes normal (N) and total failure (F). The units are taken as priority (P) unit and non priority or ordinary (O) unit. The non-priority unit goes for rest after a random period of operation and after rest for a random period of time; it again starts working without affecting its efficiency. Some preparation time is required to put a standby and repaired unit or a unit from rest into operation. This preparation time for operation is taken to be a random variable having some probability distribution. A single repairman is always available with the system for preparation of operation and to repair a failed unit. The priority unit gets preference in repair over non- priority unit but when the rest period of non- priority unit is completed and priority unit is under repair then non-priority unit gets priority in operation. The preparation time distribution and all the failure time distributions are taken to be negative exponential with different parameters. The repair time distributions are taken as general. The distributions of operation time and completion of rest period of non-priority unit are also taken as negative exponential.

SYSTEM DESCRIPTION AND ASSUMPTIONS

1. The system comprises of two non-identical units (N_1, N_2), connected in parallel configuration. Initially one unit of the system is operative called the priority (P) unit and the other is kept as standby called the non-priority or ordinary (O) unit.
2. Each unit has two mode- Normal (N) and Total failure (F).
3. The O-unit goes for rest after a random period of operation and after rest for a random period of time; it again starts working without affecting its efficiency.
4. A random preparation time is required for standby unit or repaired unit or a unit from rest to be put into operation.
5. The P-unit gets priority for both operation and repair over the O-unit. But if P-unit is under repair and rest period of O-unit is over then priority in operation will be given to the O-unit, and priority will be given to preparation for operation over repair
6. A single repairman is always available with the system for preparation of operation and to repair a failed unit.
7. A repaired unit is as good as new and is immediately reconnected to the system.
8. All the failure time distributions, rest period distributions and preparation time distributions are taken to be negative exponential with different parameters.
9. The repair time distribution are taken as general.

NOTATIONS AND SYMBOLS FOR THE SYSTEM STATES

α_1/α_2 : Constant failure rate of P /O- unit.
α_3 : Constant rate of rest time of O-unit.
α_4 : Constant rate of completion of rest time of O-unit.
μ_1/μ_2 : Constant rate of preparation time for operation of P /O- unit.
$G_1(.)/G_2(.)$: C.d.f. of repair time of P/O- unit.

Symbols for the states of the system

N_{1O}/N_{2O} : P/O-unit is operative.
N_{rest}/N_{2S} : O-unit is under rest/standby.
N_{1p}/N_{2p} : P/O-unit is under preparation for operation.
F_{1w}/F_{2w} : P/O-unit is waiting for repair.
F_{1r}/F_{2r} : P/O-unit is under repair.
N_{2pw} : O-unit is waiting for preparation for operation.

With the help of the above symbols, the possible states of the system are:

$S_0 = [N_{1O}, N_{2S}]$ $S_1 = [F_{1w}, N_{2p}]$
$S_2 = [F_{1r}, N_{2O}]$ $S_3 = [N_{1p}, N_{2O}]$
$S_4 = [F_{1r}, N_{rest}]$ $S_5 = [F_{1r}, F_{2w}]$
$S_6 = [N_{1p}, F_{2w}]$ $S_7 = [N_{1p}, N_{rest}]$
$S_8 = [N_{1p}, N_{2pw}]$ $S_9 = [N_{1O}, F_{2r}]$
$S_{10} = [N_{1O}, N_{rest}]$

The transition diagram along with all the transitions is shown in fig.1.

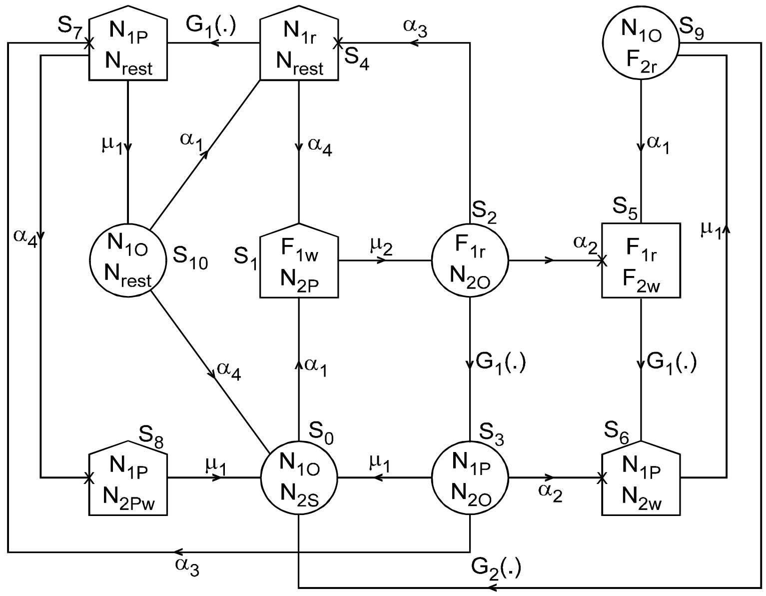

Figure 1

TRANSITION PROBABILITIES AND SOJOURN TIMES

Let $T_0(\equiv 0), T_1, T_2$ .... denotes the regenerative epochs and X_n denotes the state visited at epoch T_{n^+} i.e., just after the transition at T_n. Then $\{X_n, T_n\}$ constitute a Markov-Renewal process with state space E, set of regenerative states and $Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i]$ is the semi Markov kernel over E. Then the transition probability matrix of the embedded Markov chain is

$P = (p_{ij}) = \left(Q_{ij}(\infty)\right) = (Q(\infty))$

Therefore, the steady-state probabilities of transition are as follows:

$p_{01} = \alpha_1 \int_0^\infty e^{-\alpha_1 u}\, du = 1$

$p_{12} = \mu_2 \int_0^\infty e^{-\mu_2 u}\, du = 1$

$p_{23} = \int_0^\infty e^{-(\alpha_2+\alpha_3)u}\, dG_1(u) = \widetilde{G}_1(\alpha_2 + \alpha_3)$

Similarly,

$p_{26}^{(5)} = \frac{\alpha_2}{(\alpha_2+\alpha_3)}\left[1 - \widetilde{G}_1(\alpha_2 + \alpha_3)\right]$

$p_{27}^{(4)} = \frac{\alpha_3}{(\alpha_2+\alpha_3-\alpha_4)}\left[\widetilde{G}_1(\alpha_4) - \widetilde{G}_1(\alpha_2 + \alpha_3)\right]$

$p_{21}^{(4)} = \frac{\alpha_3}{(\alpha_2+\alpha_3-\alpha_4)}\left[1 - \widetilde{G}_1(\alpha_4)\right] - \frac{\alpha_3\alpha_4}{(\alpha_2+\alpha_3)(\alpha_2+\alpha_3-\alpha_4)}\left[1 - \widetilde{G}_1(\alpha_2 + \alpha_3)\right]$

$p_{30} = \frac{\mu_1}{(\alpha_2+\alpha_3+\mu_1)}$ $\qquad p_{39}^{(6)} = \frac{\alpha_2}{(\alpha_2+\alpha_3+\mu_1)}$

$p_{3,10}^{(7)} = \frac{\alpha_3\mu_1}{(\alpha_2+\alpha_3+\mu_1)(\alpha_4+\mu_1)}$ $\qquad p_{30}^{(7,8)} = \frac{\alpha_3\alpha_4}{(\alpha_2+\alpha_3+\mu_1)(\alpha_4+\mu_1)}$

$p_{41} = \left[1 - \widetilde{G}_1(\alpha_4)\right]$ $\qquad p_{47} = \widetilde{G}_1(\alpha_4)$

$p_{70}^{(8)} = \frac{\alpha_4}{(\alpha_4+\mu_1)}$ $\qquad p_{7,10} = \frac{\mu_1}{(\alpha_4+\mu_1)}$

$p_{90} = \widetilde{G}_2(\alpha_1)$ $\qquad p_{95} = \left[1 - \widetilde{G}_2(\alpha_1)\right]$

$p_{10,0} = \frac{\alpha_4}{(\alpha_1+\alpha_4)}$ $\qquad p_{10,4} = \frac{\alpha_1}{(\alpha_1+\alpha_4)}$

$p_{56} = p_{69} = p_{80} = 1$ (1-19)

It can be easily seen that the following results hold good:

$p_{23} + p_{21}^{(4)} + p_{26}^{(5)} + p_{27}^{(4)} = 1$

$p_{30} + p_{39}^{(6)} + p_{3,10}^{(7)} + p_{30}^{(7,8)} = 1$

$p_{41} + p_{47} = 1$ $\quad$ $p_{70}^{(8)} + p_{7,10} = 1$

$p_{90} + p_{95} = 1$ $\quad$ $p_{10,0} + p_{10,4} = 1$

$p_{01} = p_{12} = p_{56} = p_{69} = p_{80} = 1$ (20-26)

Mean sojourn times

The mean sojourn time in state S_i denoted by Ψ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time Ψ_i, in state S_i, we observe that as long as the system is in state S_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time Ψ_i in state S_i is:

$\Psi_i = E[T_i] = \int P(T_i > t)dt$

Thus

$$\Psi_0 = \int_0^\infty e^{-\alpha_1 t}\, dt = \frac{1}{\alpha_1}$$
$$\Psi_1 = \int_0^\infty e^{-\mu_2 t}\, dt = \frac{1}{\mu_2}$$
$$\Psi_2 = \int_0^\infty e^{-(\alpha_2+\alpha_3)t}\, \overline{G}_1(t)dt = \frac{1}{(\alpha_2+\alpha_3)}\left[1 - \widetilde{G}_1(\alpha_2+\alpha_3)\right]$$
$$\Psi_3 = \int_0^\infty e^{-(\alpha_2+\alpha_3+\mu_1)t}\, dt = \frac{1}{(\alpha_2+\alpha_3+\mu_1)}$$
$$\Psi_4 = \int_0^\infty e^{-\alpha_4 t}\, \overline{G}_1(t)dt = \frac{1}{\alpha_4}\left[1 - \widetilde{G}_1(\alpha_4)\right]$$
$$\Psi_5 = \int_0^\infty \overline{G}_1(t)dt$$
$$\Psi_6 = \Psi_8 = \int_0^\infty e^{-\mu_1 t}\, dt = \frac{1}{\mu_1}$$
$$\Psi_7 = \int_0^\infty e^{-(\alpha_4+\mu_1)t}dt = \frac{1}{(\alpha_4+\mu_1)}$$
$$\Psi_9 = \int_0^\infty e^{-\alpha_1 t}\, \overline{G}_2(t)dt = \frac{1}{\alpha_1}\left[1 - \widetilde{G}_2(\alpha_1)\right]$$
$$\Psi_{10} = \int_0^\infty e^{-(\alpha_1+\alpha_4)t}\, dt = \frac{1}{(\alpha_1+\alpha_4)} \qquad (27\text{-}36)$$

EXPECTED UP TIME OF UNITS DURING (0, t]

Define $A_i(t)$ as the probability that the system is up at epoch 't' when it initially starts from regenerative state S_i. To obtain recurrence relations among pointwise availabilities $A_i(t)$, we use the simple probabilistic arguments. Taking the Laplace transform and solving the resultant set of equations for $A_0^*(s)$, we have

$$A_0^*(s) = N_1(s)/D_1(s) \qquad (37)$$

where,

$$N_1(s) = \left[Z_0^*\left(1 - q_{12}^* q_{21}^{(4)*}\right) + q_{01}^* q_{12}^*(Z_2^* + q_{23}^* Z_3^*)\right](1 - q_{95}^* q_{56}^* q_{69}^*)\left(1 - q_{10,4}^* q_{47}^* q_{7,10}^*\right) + \left(q_{01}^* q_{12}^* Z_{10}^* - q_{10,4}^* q_{41}^* q_{12}^* Z_0^*\right)\left(q_{23}^* q_{3,10}^{(7)*} + q_{27}^{(4)*} q_{7,10}^*\right)(1 - q_{95}^* q_{56}^* q_{69}^*) + q_{01}^* q_{12}^* Z_9^*\left(q_{23}^* q_{39}^{(6)*} + q_{26}^{(5)*} q_{69}\right)\left(1 - q_{10,4}^* q_{47}^* q_{7,10}^*\right) \qquad (38)$$

and

$$D_1(s) = \left[1 - q_{12}^* q_{21}^{(4)*} - q_{01}^*\, q_{12}^*\left(q_{23}^*\, q_{30}^* + q_{23}^* q_{30}^{(7,8)*} + q_{27}^{(4)*} q_{70}^{(8)*}\right)\right](1 - q_{95}^* q_{56}^* q_{69}^*)\left(1 - q_{10,4}^* q_{47}^* q_{7,10}^*\right) - \left[q_{10,4}^* q_{41}^* q_{12}^* + q_{01}^* q_{12}^*\left(q_{10,0}^* + q_{10,4}^* q_{47}^* q_{70}^{(8)*}\right)\right]\left(q_{23}^* q_{3,10}^{(7)*} + q_{27}^{(4)*} q_{7,10}^*\right)(1 - q_{95}^* q_{56}^* q_{69}^*) - q_{01}^* q_{12}^* q_{90}^*\left(q_{23}^* q_{39}^{(6)*} + q_{26}^{(5)*} q_{69}^*\right)\left(1 - q_{10,4}^* q_{47}^* q_{7,10}^*\right) \qquad (39)$$

In steady state the up time of the system will be given by

$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\, A_0^*(s) = N_1(0)/D_1(0)$

Where,

$$N_1(0) = p_{90}\left[\Psi_0\left(1 - p_{21}^{(4)}\right) + \Psi_2 + p_{23}\,\Psi_3\right](1 - p_{10,4}\, p_{47}\, p_{7,10}) + p_{90}\left(\Psi_{10} - p_{10,4} p_{41} \Psi_0\right)\left(p_{23} p_{3,10}^{(7)} + p_{27}^{(4)} p_{7,10}\right) + \Psi_9\left(p_{23} p_{39}^{(6)} + p_{26}^{(5)}\right)(1 - p_{10,4}\, p_{47} p_{7,10}) \qquad (40)$$

and

$\mathrm{D}_1(0)=\left[1-\mathrm{p}_{28}^{(4)}\mathrm{p}_{82}-\mathrm{p}_{01}\mathrm{p}_{12}\left(\mathrm{p}_{23}\mathrm{p}_{30}+\mathrm{p}_{27}^{(4)}\mathrm{p}_{70}^{(11)}\right)\right](1-\mathrm{p}_{95}\mathrm{p}_{56}\mathrm{p}_{69})(1-\mathrm{p}_{10,4}\mathrm{p}_{47}\mathrm{p}_{7,10})-\left[\mathrm{p}_{10,4}\mathrm{p}_{48}\mathrm{p}_{82}+\mathrm{p}_{01}\mathrm{p}_{12}\left(\mathrm{p}_{10,0}+\mathrm{p}_{10,4}\mathrm{p}_{47}\mathrm{p}_{70}^{(11)}\right)\right]\left(\mathrm{p}_{23}\mathrm{p}_{3,10}^{(7)}+\mathrm{p}_{27}^{(4)}\mathrm{p}_{7,10}\right)(1-\mathrm{p}_{95}\mathrm{p}_{56}\,\mathrm{p}_{69})-\mathrm{p}_{01}\mathrm{p}_{12}\mathrm{p}_{90}\left(\mathrm{p}_{23}\mathrm{p}_{39}^{(6)}+\mathrm{p}_{26}^{(5)}\mathrm{p}_{69}\right)(1-\mathrm{p}_{10,4}\mathrm{p}_{47}\mathrm{p}_{7,10})$ (41)

Since $D_1(0)=0$

Hence on using L'Hospital's rule, A_0 becomes

$A_0 = N_1(0)/D_1'(0)$ (42)

Where,

$D_1'(0)=m_{01}A+m_{12}B+\left(m_{21}^{(4)}+m_{23}+m_{26}^{(5)}+m_{27}^{(4)}\right)B+\left(m_{30}+m_{30}^{(7,8)}+m_{39}^{(6)}+m_{3,10}^{(7)}\right)p_{23}B+(m_{41}+m_{47})p_{10,4}F+m_{56}m_{95}\mathrm{E}+m_{69}C+\left(m_{70}^{(8)}+m_{7,10}\right)D+(m_{90}+m_{95})E+(m_{10,0}+m_{10,4})F$

Using the relation $\sum_j \mathrm{m_{ij}}=\Psi_\mathrm{i}$, we get

$\mathrm{D}_1'(0)=\Psi_0\mathrm{A}+(\Psi_1+\Psi_2+\mathrm{p}_{23}\Psi_3)\mathrm{B}+\Psi_6\mathrm{C}+\Psi_7\mathrm{D}+(\Psi_9+\mathrm{p}_{95}\Psi_5)\mathrm{E}+(\Psi_{10}+\mathrm{p}_{10,4}\Psi_4)\mathrm{F}$ (43)

Where,

$\mathrm{A}=\mathrm{p}_{90}\left[\left(1-\mathrm{p}_{21}^{(4)}\right)(1-\mathrm{p}_{10,4}\,\mathrm{p}_{47}\,\mathrm{p}_{7,10})+\mathrm{p}_{10,4}\,\mathrm{p}_{41}\left(\mathrm{p}_{23}\,\mathrm{p}_{3,10}^{(7)}+\mathrm{p}_{27}^{(4)}\mathrm{p}_{7,10}\right)\right]$

$\mathrm{B}=\mathrm{p}_{90}(1-\mathrm{p}_{10,4}\mathrm{p}_{47}\mathrm{p}_{7,10})$

$\mathrm{C}=\left(\mathrm{p}_{26}^{(5)}+\mathrm{p}_{23}\mathrm{p}_{39}^{(6)}\mathrm{p}_{95}\right)(1-\mathrm{p}_{10,4}\mathrm{p}_{47}\mathrm{p}_{7,10})$

$\mathrm{D}=\mathrm{p}_{90}\left(\mathrm{p}_{27}^{(4)}+\mathrm{p}_{23}\mathrm{p}_{10,4}\mathrm{p}_{47}\mathrm{p}_{3,10}^{(7)}\right)$

$\mathrm{E}=\left(\mathrm{p}_{23}\mathrm{p}_{39}^{(6)}+\mathrm{p}_{26}^{(5)}\right)(1-\mathrm{p}_{10,4}\mathrm{p}_{47}\mathrm{p}_{7,10})$

$\mathrm{F}=\left(\mathrm{p}_{23}\mathrm{p}_{39}^{(6)}+\mathrm{p}_{26}^{(5)}\right)(1-\mathrm{p}_{10,4}\mathrm{p}_{47}\mathrm{p}_{7,10})$

Using (40) and (43) in (42), we get the expression for A_0.

The expected up time of the system during (0, t] is given by

$\mu_{\mathrm{up}}(\mathrm{t})=\int_0^{\mathrm{t}}\mathrm{A}_0(\mathrm{u})\,\mathrm{du}$

so that, $\mu_{\mathrm{up}}^*(\mathrm{s})=\mathrm{A}_0^*(\mathrm{s})/\mathrm{s}$

BUSY PERIOD ANALYSIS

Define $B_i(t)$ as the probability that the repairman is busy in the repair of the failed unit when the system initially starts from state $S_i \in E$. Using probabilistic arguments, the value of $B_0(t)$ can be obtained in its L.T. as:

$\mathrm{B}_0^*(\mathrm{s})=\mathrm{N}_2(\mathrm{s})/\mathrm{D}_1(\mathrm{s})$ (44)

Where,

$\mathrm{N}_2(\mathrm{s})=\mathrm{q}_{01}^*\left[\mathrm{Z}_1^*+\mathrm{q}_{12}^*(\mathrm{Z}_2^*+\mathrm{q}_{23}^*\mathrm{Z}_3^*+\mathrm{q}_{26}^{(5)*}\mathrm{Z}_6^*+\mathrm{q}_{27}^{(4)*}\mathrm{Z}_7^*)\right](1-\mathrm{q}_{95}^*\mathrm{q}_{56}^*\mathrm{q}_{69}^*)(1-\mathrm{q}_{10,4}^*\mathrm{q}_{47}^*\mathrm{q}_{7,10}^*)+\mathrm{q}_{01}^*\mathrm{q}_{12}^*(\mathrm{q}_{10,4}^*\mathrm{Z}_4^*+\mathrm{q}_{10,4}^*\mathrm{q}_{47}^*\mathrm{Z}_7^*)\left(\mathrm{q}_{23}^*\mathrm{q}_{3,10}^{(7)*}+\mathrm{q}_{27}^{(4)*}\mathrm{q}_{7,10}^*\right)(1-\mathrm{q}_{95}^*\mathrm{q}_{56}^*\mathrm{q}_{69}^*)+\mathrm{q}_{01}^*\mathrm{q}_{12}^*(\mathrm{Z}_9^*+\mathrm{q}_{95}^*\mathrm{Z}_5^*+\mathrm{q}_{95}^*\mathrm{q}_{56}^*\mathrm{Z}_6^*)\left(\mathrm{q}_{23}^*\mathrm{q}_{39}^{(6)*}+\mathrm{q}_{26}^{(5)*}\mathrm{q}_{69}^*\right)(1-\mathrm{q}_{10,4}^*\mathrm{q}_{47}^*\mathrm{q}_{7,10}^*)$ (45)

In the steady state, the probability that the repairman will be busy is given by

$\mathrm{B}_0=\lim_{\mathrm{t}\to\infty}\mathrm{B}_0(\mathrm{t})=\lim_{\mathrm{s}\to0}\mathrm{s}\,\mathrm{B}_0^*(\mathrm{s})=\lim_{\mathrm{s}\to0}\frac{\mathrm{sN}_2(\mathrm{s})}{\mathrm{D}_1(\mathrm{s})}=\lim_{\mathrm{s}\to0}\mathrm{N}_2(\mathrm{s})\lim_{\mathrm{s}\to0}\frac{\mathrm{s}}{\mathrm{D}_1(\mathrm{s})}$ (46)

and

$\mathrm{B}_0 = \mathrm{N}_2(0)/\mathrm{D}_1'(0)$ (47)

Where,

$\mathrm{N}_2(0)=\mathrm{p}_{90}\left[\Psi_1+\Psi_2+\mathrm{p}_{23}\,\Psi_3+\mathrm{p}_{26}^{(5)}\Psi_6+\mathrm{p}_{27}^{(4)}\Psi_7\right](1-\mathrm{p}_{10,4}\,\mathrm{p}_{47}\mathrm{p}_{7,10})+\mathrm{p}_{90}\left[\mathrm{p}_{10,4}\Psi_4+\mathrm{p}_{10,4}\,\mathrm{p}_{47}\Psi_7\right]\left(\mathrm{p}_{23}\mathrm{p}_{3,10}^{(7)}+\mathrm{p}_{27}^{(4)}\mathrm{p}_{7,10}\right)+[\Psi_9+\mathrm{p}_{95}(\Psi_5+\Psi_6)]\left(\mathrm{p}_{23}\mathrm{p}_{39}^{(6)}+\mathrm{p}_{26}^{(5)}\right)(1-\mathrm{p}_{10,4}\,\mathrm{p}_{47}\mathrm{p}_{7,10})$ (48)

The expected busy period of the repairman during (0, t] is given by

$\mu_\mathrm{b}(\mathrm{t})=\int_0^{\mathrm{t}}\mathrm{B}_0(\mathrm{u})\,\mathrm{du}$

so that, $\mu_\mathrm{b}^*(\mathrm{s})=\mathrm{B}_0^*(\mathrm{s})/\mathrm{s}$

EXPECTED NUMBER OF REPAIRS

Let us define $V_i(t)$ as the expected number of repairs by repairman during the time interval $(0,t]$ when the system initially starts from regenerative state S_i.Using the definition of $V_i(t)$ the recursive relations among $V_i(t)$ can be easily developed, taking their L.S.T. and solving the resultant set of equations for $\tilde{V}_0(s)$, we get

$$\tilde{V}_0(s) = N_3(s)/D_1(s) \quad (49)$$

Where,

$$N_3(s) = \tilde{Q}_{01}\tilde{Q}_{12}\left\{\left[\left(\tilde{Q}_{23} + \tilde{Q}_{26}^{(5)} + \tilde{Q}_{27}^{(4)}\right)(1 - \tilde{Q}_{95}\tilde{Q}_{56}\tilde{Q}_{69}) + (\tilde{Q}_{90} + \tilde{Q}_{95}\tilde{Q}_{56})\left(\tilde{Q}_{23}\tilde{Q}_{39}^{(6)} + \tilde{Q}_{26}^{(5)}\tilde{Q}_{69}\right)\right](1 - \tilde{Q}_{10,4}\tilde{Q}_{47}\tilde{Q}_{7,10}) + \tilde{Q}_{10,4}\tilde{Q}_{47}\left(\tilde{Q}_{23}\tilde{Q}_{3,10}^{(7)} + \tilde{Q}_{27}^{(4)}\tilde{Q}_{7,10}\right)(1 - \tilde{Q}_{95}\tilde{Q}_{56}\tilde{Q}_{69})\right\} \quad (50)$$

In the steady state, the probability that the repairman will be busy is given by

$$V_0 = \lim_{t\to\infty} V_0(t) = \lim_{s\to 0} s\, V_0^*(s) = \lim_{s\to 0} \frac{sN_3(s)}{D_1(s)} = \lim_{s\to 0} N_3(s) \lim_{s\to 0} \frac{s}{D_1(s)} \quad (51)$$

using L'Hospital's rule, V_0 becomes

$$V_0 = N_3(0)/D_1'(0) \quad (52)$$

Where,

$$N_3(0) = \left[p_{90}\left(p_{23} + p_{26}^{(5)} + p_{27}^{(4)}\right) + (p_{90} + p_{95})\left(p_{23}p_{39}^{(6)} + p_{26}^{(5)}\right)\right](1 - p_{10,4}p_{47}p_{7,10}) + p_{90}\, p_{10,4}p_{47}\left(p_{23}p_{3,10}^{(7)} + p_{27}^{(4)}p_{7,10}\right) \quad (53)$$

EXPECTED PREPARATION TIME OF UNIT DURING (0, t]

Let us define $D_i(t)$ as the expected preparation time for repair taken by the units during the time interval $(0,t]$ when the system initially starts from regenerative state S_i.Using the definition of $D_i(t)$, the recursive relations among $D_i(t)$ can be easily developed, taking their L.S.T. and solving the resultant set of equations for $\tilde{V}_0(s)$, we get

$$D_0^*(s) = N_4(s)/D_1(s) \quad (54)$$

Where,

$$N_4(s) = q_{01}^*\left[Z_1^* + q_{12}^*\left(q_{23}^*Z_3^* + q_{26}^{(5)*}Z_6^* + q_{27}^{(4)*}Z_7^*\right)\right](1 - q_{95}^*q_{56}^*q_{69}^*)(1 - q_{10,4}^*q_{47}^*q_{7,10}^*) + q_{01}^*\, q_{12}^*q_{10,4}^*\, q_{47}^*\, Z_7^*\left(q_{23}^*q_{3,10}^{(7)*} + q_{27}^{(4)*}q_{7,10}^*\right)(1 - q_{95}^*q_{56}^*q_{69}^*) + q_{01}^*q_{12}^*\, q_{95}^*q_{56}^*Z_6^*\left(q_{23}^*q_{39}^{(6)*} + q_{26}^{(5)*}q_{69}^*\right)(1 - q_{10,4}^*q_{47}^*q_{7,10}^*) \quad (55)$$

In the steady state, the probability that the Expected preparation time is given by

$$D_0 = \lim_{t\to\infty} D_0(t) = \lim_{s\to 0} s\, D_0^*(s) = \lim_{s\to 0} \frac{sN_4(s)}{D_1(s)} = \lim_{s\to 0} N_4(s) \lim_{s\to 0} \frac{s}{D_1(s)} \quad (56)$$

and D_0 becomes

$$D_0 = N_4(0)/D_1'(0) \quad (57)$$

Where,

$$N_4(0) = p_{90}\left[\Psi_1 + p_{23}\Psi_3 + p_{26}^{(5)}\Psi_6 + p_{27}^{(4)}\Psi_7\right](1 - p_{10,4}\, p_{47}p_{7,10}) + p_{90}\, p_{10,4}\, p_{47}\Psi_7\left(p_{23}p_{3,10}^{(7)} + p_{27}^{(4)}p_{7,10}\right) + p_{95}\Psi_6\left(p_{23}p_{39}^{(6)} + p_{26}^{(5)}\right)(1 - p_{10,4}\, p_{47}p_{7,10}) \quad (58)$$

PROFIT FUNCTION ANALYSIS

Two profit functions $P_1(t)$ and $P_2(t)$ can easily be obtained for the system model under study with the help of characteristics obtained earlier. The expected total profit incurred during (0, t] is

$P(t) =$ Expected total revenue in (0, t]− Expected total expenditure in (0, t]

Hence

$$P_1(t) = K_0\mu_{up}(t) - K_1\mu_b(t) - K_2D_0(t) \quad (59)$$

Similarly,

$$P_2(t) = K_0\mu_{up}(t) - K_2D_0(t) - K_3V_0(t) \quad (60)$$

Where,

K_0 is revenue per unit up time.

K_1 is the cost per unit time for which repair man is busy in repair of the failed unit.

K_2 is the cost of preparation per unit.

K_3 is per unit repair cost.

The expected total profits per unit time, in steady state, is

$$P_1 = \lim_{t\to\infty}[P_1(t)/t] = \lim_{s\to 0} s^2P_1^*(s)$$

So that,

$$P_1 = K_0A_0 - K_1B_0 - K_2D_0 \quad (61)$$

And

$$P_2 = K_0A_0 - K_2D_0 - K_3V_0 \quad (62)$$

GRAPHICAL STUDY OF THE SYSTEM MODEL

For more concrete study of system behaviour, we take repair time distribution of P/O-unit as exponential distributions with different parameters as λ_1& λ_2 respectively. Then we plot Profit functions with respect to α_1 (failure rate of P-unit) for different values of λ_1(repair rate of P-unit). Fig.2 represents the change in profit function P_1 and P_2 w.r.t. α_1 for different values of λ_1 as 0.25, 0.50 and 0.75 while the other parameters are fixed as α_2=0.35, α_3=0.50, α_4=0.20, μ_1=0.40, μ_2=0.35, λ_2=0.70, k_0=1000, k_1=500, k_2=350, k_3=250. From the graph it is seen that both profit functions decrease with the increase in failure rate α_1 and increase with the increase in λ_1. It is also observed that profit function P_2 is always higher as compared to profit function P_1 for fixed values of α_1 and λ_1. Thus the better understanding of failure phenomenon by the repairman results in better system performance.

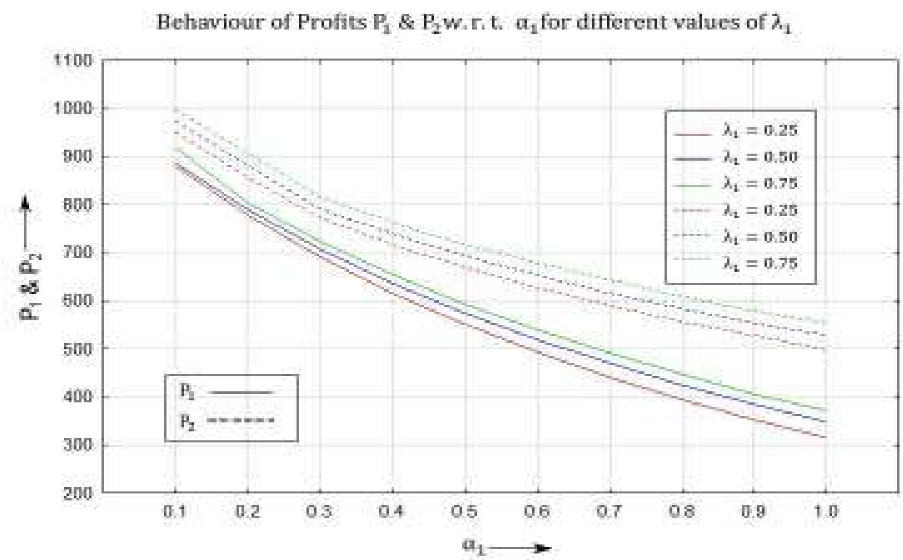

Fig.2

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