Research Article

Benefit-function of two similar warm standby navy vessel ship system subject to failure due to typhoon and failure due to mediterranean storm with different repair facilities

Ashok Kumar Saini

Associate Professor, Department of Mathematics, B. L. J. S. College, Tosham, Bhiwani, Haryana, INDIA.
Email: drashokksaini2009@gmail.com

Abstract

a **maritime disaster** is an event which usually involves a ship or ships and can involve military action. Because of the nature of maritime travel, there is often a substantial loss of life. **Notable disasters:** the sinking of RMS titanic in 1912, with 1,517 fatalities, is probably the most famous shipwreck, but not the biggest in terms of life lost. The wartime sinking of the wilhelm gustl off in january 1945 in world war ii by a soviet navy submarine, with an estimated loss of about 9,400 people, remains the greatest maritime disaster ever. In peacetime, the 1987 loss of the ferry doña paz, with an estimated 4,386 dead, is the largest non-military loss recorded.in the present paper we have taken failure due to typhoon and failure due to mediterranean storm with different repair facilities. When the main unit fails then warm standby system becomes operative. Failure due to mediterranean storm cannot occur simultaneously in both the units and after failure the unit undergoes type-i or type-ii or type-iii or type iv repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters mtsf, availability, busy period, benefit-function analysis have been evaluated.

Keywords: warm standby, failure due to typhoon, failure due to mediterranean storm, first come first serve, MTSF, availability, busy period, benefit -function.

***Address for Correspondence:**
Dr. Ashok Kumar Saini, Associate Professor, Department of Mathematics, BLJS College, Tosham, Bhiwani, Haryana, INDIA.
Email: drashokksaini2009@gmail.com

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INTRODUCTION

Many maritime disasters happen outside the realms of war. All ships, including those of the military, are vulnerable to problems from weather conditions, faulty design or human error. Some of the disasters below occurred in periods of conflict, although their losses were unrelated to any military action. although their losses were unrelated to any military action.

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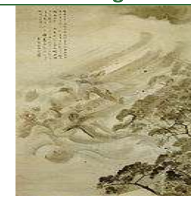
Table 1:

Year	Country	Description	Lives lost	Image
1274 1281	Mongol Empire	Kamikaze - The Mongol fleet destroyed in a typhoon.	100,000+	
256 BCE 253 BCE	Roman Republic	First Punic War – In the First Punic War, between the Roman Republic and Carthage, a Roman fleet that had just rescued a Roman army from Africa was caught in a Mediterranean storm. Rome may have lost more than 90,000 men.	90,000+	
1588	 Spain	Spanish Armada – On 8 August 1588, Philip II of Spain sent the Armada to invade England. Spain lost 15,000–20,000 soldiers and sailors, mainly in storms rather than battle.	15,000-20,000	

Stochastic behavior of systems operating under changing environments has widely been studied. Dhillon, B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment. Kan Cheng (1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken failure due to Typhoon and failure due to Mediterranean storm with different repair facilities. When the main operative unit fails then warm standby system becomes operative. Failure due to Mediterranean storm cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of Type- II by ordinary repairman or Type III, Type IV by multispecialty repairman immediately when failure due to Typhoon. The repair is done on the basis of first fail first repaired.

Assumptions

1. $\lambda_1, \lambda_2\ \lambda_3$ are constant failure rates when failure due to Typhoon and failure due to Mediterranean storm respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are $G_1(t)$, $G_2(t)$ and $G_3(t)$ $G_4(t)$.
2. The failure due to Mediterranean storm is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after failure due to Typhoon and failure due to Mediterranean storm and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
4. The switches are perfect and instantaneous.
5. All random variables are mutually independent.
6. When both the units fail, we give priority to operative unit for repair.
7. Repairs are perfect and failure of a unit is detected immediately and perfectly.
8. The system is down when both the units are non-operative.

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts: O, WS, TF, MSF,
Operative, Warm Standby, failure due to Typhoon, failure due to Mediterranean storm respectively.
Subscripts: ntf, tf, msf, ur, wr, uR
No failure due to Typhoon, failure due to Typhoon, failure due to Mediterranean storm, under repair, waiting for repair, under repair continued from previous state respectively
Up states: 0, 1, 2, 3, 10; **Down states:** 4, 5, 6, 7, 8,9,11
regeneration point: 0,1,2, 3, 8, 9,10

States of the System

0(O_{ntf}, WS_{ntf}) One unit is operative and the other unit is warm standby and there is no failure due to Typhoon of both the units.
1($TF_{tf,\,urI}$, O_{ntf}) The operating unit failure due to Typhoon is under repair immediately of Type- I and standby unit starts operating with no failure due to Typhoon
2($MSF_{msf,\,urII}$, O_{ntf}) The operative unit failure due to Mediterranean storm and undergoes repair of type II and the standby unit becomes operative with no failure due to Typhoon
3($MSF_{msf,\,urIII}$, O_{ntf}) The first unit failure due to Mediterranean storm and under Type-III multispecialty repairman and the other unit is operative with no failure due to Typhoon
4($TF_{tf,uR1}$, $TF_{tf,wrI}$) The unit failed due to TF resulting from failure due to Typhoon under repair of Type- I continued from state 1and the other unit failed due to TF resulting from failure due to Typhoon is waiting for repair of Type-I.
5($TF_{tf,uR1}$, $MSF_{msf,\,wrII}$) The unit failed due to TF resulting from failure due to Typhoon is under repair of Type- I continued from state 1and the other unit failure due to Mediterranean storm is waiting for repair of Type- II.
6($MSF_{msf,\,uRII}$, $TF_{tf,\,wrI}$) The operative unit failed due to controls damaged by structural failure is under repair continues from state 2 of Type –II and the other unit failed due to TF resulting from failure due to Typhoon is waiting under repair of Type-I.
7($MSF_{msf,\,uRII}$, $TF_{tf,wrII}$) The one unit failure due to Mediterranean storm is continued to be under repair of Type II and the other unit failed due to TF resulting from failure due to Typhoon is waiting for repair of Type-II.
8($TF_{tf,urIII}$, $MSF_{msf,\,wrII}$) The one unit failure due to Typhoon is under multispecialty repair of Type-III and the other unit failure due to Mediterranean storm is waiting for repair of Type-II.
9($TF_{tf,urIII}$, $MSF_{msf,\,wrI}$) The one unit failure due to Typhoon is under multispecialty repair of Type-III and the other unit failure due to Mediterranean storm is waiting for repair of Type-I
10(O_{ntf} $MSF_{msf,\,urIV}$)
The one unit is operative with no failure due to Typhoon and warm standby unit failure due to Mediterranean storm and undergoes repair of type IV.
11(O_{ntf} $MSF_{msf,\,uRIV}$)
The one unit is operative with no failure due to Typhoon and warm standby unit failure due to Mediterranean storm and repair of type IV continues from state 10.

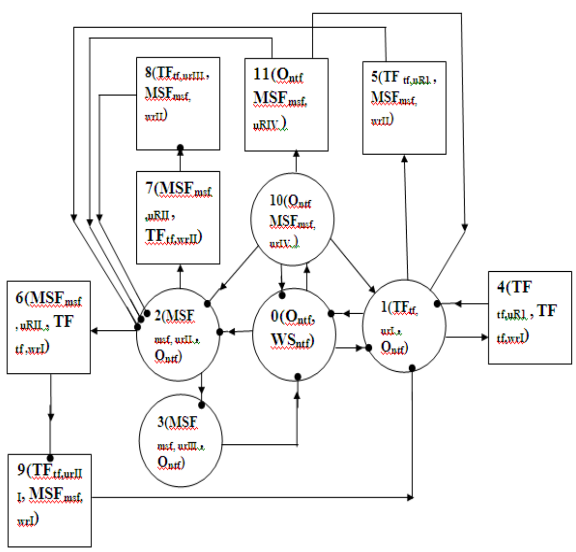


Figure 1: The State Transition Diagram

● Rregeneration point ◯ Up State ☐ Down State

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$p_{01} = \lambda_1 / \lambda_1 + \lambda_2 + \lambda_3$, $p_{02} = \lambda_2 / \lambda_1 + \lambda_2 + \lambda_3$,
$p_{0,10} = \lambda_3 / \lambda_1 + \lambda_2 + \lambda_3$, $p_{10} = pG_1^*(\lambda_1) + q\,G_2^*(\lambda_2)$,
$p_{14} = p - pG_1^*(\lambda_1) = p_{11}^{(4)}$, $p_{15} = q - q\,G_1^*(\lambda_2) = p_{12}^{(5)}$,
$p_{23} = pG_2^*(\lambda_1) + q\,G_2^*(\lambda_2)$, $p_{26} = p - pG_2^*(\lambda_1) = p_{29}^{(6)}$,

$p_{27} = q - qG_2^*(\lambda_2) = p_{28}^{(7)}$, $p_{30} = p_{82} = p_{91} = 1$
$p_{0,10} = pG_4^*(\lambda_1) + q\,G_4^*(\lambda_2)$,
$p_{10,1} = p - pG_4^*(\lambda_1) = p_{10,1}^{(11)}$, $p_{10,2} = q - q\,G_4^*(\lambda_2) = p_{10,2}^{(11)}$ (1)
We can easily verify that
$p_{01} + p_{02} + p_{03} = 1$, $p_{10} + p_{14}\,(=p_{11}^{(4)}) + p_{15}\,(=p_{12}^{(5)}) = 1$,
$p_{23} + p_{26}\,(=p_{29}^{(6)}) + p_{27}\,(=p_{28}^{(7)}) = 1$ $p_{30} = p_{82} = p_{91} = 1$
$p_{10,0} + p_{10,1}^{(11)}\,(=p_{10,1}) + p_{10,2}^{(12)}\,(=p_{10,2}) = 1$ (2)
And mean sojourn time is
$\mu_0 = E(T) = \int_0^\infty P[T > t]\,dt$

Mean Time To System Failure

$\varnothing_0(t) = Q_{01}(t)[s]\,\varnothing_1(t) + Q_{02}(t)[s]\,\varnothing_2(t) + Q_{0,10}(t)[s]\,\varnothing_{10}(t)$
$\varnothing_1(t) = Q_{10}(t)[s]\,\varnothing_0(t) + Q_{14}(t) + Q_{15}(t)$
$\varnothing_2(t) = Q_{23}(t)[s]\,\varnothing_3(t) + Q_{26}(t) + Q_{27}(t)$
$\varnothing_3(t) = Q_{30}(t)[s]\,\varnothing_0(t)$
$\varnothing_{10}(t) = Q_{10,0}(t)[s]\,\varnothing_{10}(t) + Q_{10,2}(t)[s]\,\varnothing_1(t) + Q_{10,2}(t)[s]\,\varnothing_2(t)$ (3-6)
We can regard the failed state as absorbing
Taking Laplace-Stiljes transform of eq. (3-6) and solving for
$\varnothing_0^*(s) = N_1(s) / D_1(s)$ (7)
where
$N_1(s) = \{Q_{01}^* + Q_{0,10}^*\,Q_{10,1}^*\}\,[\,Q_{14}^*(s) + Q_{15}^*(s)\,] + \{Q_{02}^* + Q_{0,10}^*\,Q_{10,2}^*\}\,[\,Q_{26}^*(s) + Q_{27}^*(s)\,]$
$D_1(s) = 1 - \{Q_{01}^* + Q_{0,10}^*\,Q_{10,1}^*\}\,Q_{10}^* - \{Q_{02}^* + Q_{0,10}^*\,Q_{10,2}^*\}\,Q_{23}^*\,Q_{30}^* - Q_{0,10}^*\,Q_{10,0}^*$
Making use of relations (1) and (2) it can be shown that $\varnothing_0^*(0) = 1$, which implies that $\varnothing_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \left.\frac{d}{ds}\varnothing_0^*(s)\right|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$= (\mu_0 + \mu_1\,(p_{01} + p_{0,10}\,p_{10,1}) + (p_{02} + p_{0,10}\,p_{10,2})(\mu_2 + \mu_3) + \mu_{10}\,p_{0,10} / (1 - (p_{01} + p_{0,10}\,p_{10,1})\,p_{10} - (p_{02} + p_{0,10}\,p_{10,2})\,p_{23}) - p_{0,10}\,p_{10,0}$
where
$\mu_0 = \mu_{01} + \mu_{02} + \mu_{0,10}$, $\mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}$,
$\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}$, $\mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have
$M_0(t) = e^{-\lambda_1 t}\,e^{-\lambda_2 t}\,e^{-\lambda_3 t}$
$M_1(t) = p\,G_1(t)\,\overline{e}^{-\lambda_1 t}$
$M_2(t) = q\,G_2(t)\,\overline{e}^{-\lambda_2 t}$,
$M_3(t) = G_3(t)$, $\overline{M}_{10}(t) = G_4(t)\,e^{-\lambda_3 t}$ –
The point wise availability $A_i(t)$ have the following recursive relations
$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t)$
$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t)$,
$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]\,A_8(t) + q_{29}^{(6)}(t)]\,[c]A_9(t)$ $A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t)$
$A_8(t) = q_{82}(t)[c]A_2(t)$
$A_9(t) = q_{91}(t)[c]A_1(t)$
$A_{10}(t) = M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}^{(11)}(t)[c]A_1(t) + q_{10,2}^{(11)}(t)[c]A_2(t)$ (8-14)
Taking Laplace Transform of eq. (8-14) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (15)
where
$N_2(s) = \{\hat{q}_{0,10}\,\hat{M}_{10} + \hat{M}_0\}\,[\{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7)}\,\hat{q}_{82}\} - \hat{q}_{12}^{(5)}\,\hat{q}_{29}^{(6)}$
$\hat{q}_{91}] + \{\hat{q}_{01} + \hat{q}_{0,10}\,\hat{q}_{10,1}^{(11)}\}[\hat{M}_1\{1 - \hat{q}_{28}^{(7)}\,\hat{q}_{82}\} + \hat{q}_{12}^{(5)}\,\hat{q}_{23}\,\hat{M}_3 +$

$\tilde{M}_2]+\{\hat{q}_{02}+\hat{q}_{0,10}\,\hat{q}_{10,2}^{(11)}\}\,[\{\hat{q}_{23}\,\tilde{M}_3\}\{1-\hat{q}_{11}^{(4)}\}+\hat{q}_{29}^{(6)}\,\hat{q}_{91}\,\tilde{M}_1]$

$D_2(s) = \{1-\hat{q}_{11}^{(4)}\}\{1-\hat{q}_{28}^{(7}\,\hat{q}_{82}\}-\hat{q}_{12}^{(5)}\,\hat{q}_{29}^{(6)}\,\hat{q}_{91}-\{\hat{q}_{01}+\hat{q}_{0,10}$
$\hat{q}_{10,1}^{(11)}\}[\hat{q}_{10}\,\{1-\hat{q}_{28}^{(7)}\,\hat{q}_{82}\}+\hat{q}_{12}^{(5)}\,\hat{q}_{23}\,\hat{q}_{30}\,]-\{\hat{q}_{02}+\hat{q}_{0,10}$
$\hat{q}_{10,2}^{(11)}\}\{[\hat{q}_{23}\,\hat{q}_{30}\,\{1-\hat{q}_{11}^{(4)}\}+\hat{q}_{29}^{(6)}\,\hat{q}_{91}\,\hat{q}_{10}]$

(Omitting the arguments s for brevity)

The steady state availability

$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\,\bar{A}_0(s)] = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0}\frac{N_2(s)+s\,N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \tag{16}$$

Where

$N_2(0) = \{p_{0,10}\,\tilde{M}_{10}(0)+\tilde{M}_0(0)\}\,[\{1-p_{11}^{(4)}\}\{1-p_{28}^{(7)}\}-p_{12}^{(5)}\,p_{29}^{(6)}]$
$+\{p_{01}+p_{0,10}\,p_{10,1}^{(11)}\}[\tilde{M}_1(0)\{1-p_{28}^{(7)}\}+p_{12}^{(5)}\,p_{23}\,\tilde{M}_3(0)+$
$\tilde{M}_2(0)]+\{p_{02}+p_{0,10}\,p_{10,2}^{(11)}\}\,[\{p_{23}\,\tilde{M}_3(0)+\tilde{M}_2(0)\}\{1-p_{11}^{(4)}\}+p_{29}^{(6)}\,\tilde{M}_1(0)]$

$D_2'(0) = \mu_0[p_{10}(1-p_{28}^{(7)}\}+p_{12}^{(5)}\,p_{23}]+\mu_1[p_{29}^{(6)}+p_{01}\,p_{23}-p_{0,10}$
$\{p_{10,0}\{1-p_{28}^{(7)}\}+p_{23}\,p_{10,2}^{(11)}\,p_{23}\}]+\mu_2[(1-p_{11}^{(4)})-p_{01}\,p_{10}-p_{0,10}\,(p_{10}-p_{10}$
$p_{10,2}^{(11)}+p_{12}^{(5)}\,p_{10,0})]\}+\mu_3\,[p_{23}[p_{12}^{(5)}\{p_{01}+p_{0,10}\,p_{10,1}^{(11)}\}+(1-p_{11}^{(4)})\{p_{02}+p_{0,10}\,p_{10,2}^{(11)}\}]+\mu_8\,[p_{28}^{(7)}(1-p_{0,10}$
$p_{10,0}-p_{10}\{p_{01}+p_{0,10}\,p_{10,1}^{(11)}\})]+\mu_9\,[p_{29}^{(6)}\{p_{12}^{(5)}\,(1-p_{0,10}\,p_{10,0}+(p_{02}+p_{0,10}\,p_{10,2}^{(11)}\})]+\mu_{10}\,[p_{29}^{(6)}\{p_{12}^{(5)}\,(1-p_{0,10}$
$p_{10,0}+(p_{02}+p_{0,10}\,p_{10,2}^{(11)}\})]$

and

$\mu_3=\mu_{30},\ \mu_9=\mu_{91},\ \mu_8=\mu_{81}$

The expected up time of the system in (0, t] is

$$\lambda_u(t) = \int_0^{\infty} A_0(z)\,dz \text{ So that } \bar{\lambda}_u(s) = \frac{\bar{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)} \tag{17}$$

The expected down time of the system in (0, t] is

$$\lambda_d(t) = t-\lambda_u(t) \text{ So that } \bar{\lambda}_d(s) = \frac{1}{s^2}-\bar{\lambda}_u(s) \tag{18}$$

The expected busy period of the server when there is failure due to Typhoon and failure due to Mediterranean storm in (0,t]-R_0

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t) + q_{0,10}(t)[c]R_{10}(t)$
$R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) + q_{12}^{(5)}(t)[c]\,R_2(t) + q_{11}^{(4)}(t)[c]R_1(t)$
$R_2(t) = S_2(t) + q_{23}(t)[c]R_3(t) + q_{28}^{(7)}(t)\,R_8(t) + q_{29}^{(6)}(t)][c]R_9(t)$
$R_3(t) = S_3(t) + q_{30}(t)[c]R_0(t)$
$R_8(t) = S_8(t) + q_{82}(t)[c]R_2(t)$
$R_9(t) = S_9(t) + q_{91}(t)[c]R_1(t)$
$R_{10}(t) = S_{10}(t) + q_{10,0}(t)[c]R_0(t) + q_{10,1}^{(11)}(t)[c]R_1(t) + q_{10,2}^{(11)}(t)[c]R_2(t)$ (19-25)

where

$S_1(t) = p\,G_1\overline{(t)}\,e^{-\lambda_1 t},$
$S_2(t) = q\,G_2\overline{(t)}\,e^{-\lambda_2 t}$
$S_3(t) = S_8(t) = S_9(t) = \overline{G_3(t)}$
$S_{10}(t) = \overline{G_4(t)}$ (26)

Taking Laplace Transform of eq. (19-25) and solving for $\bar{R}_0(s)$

$$\bar{R}_0(s) = N_3(s)\,/\,D_2(s) \tag{27}$$

where

$N_3(s) = \{\hat{q}_{01}+\hat{q}_{0,10}\,\hat{q}_{10,1}^{(11)}\}[\hat{S}_1(1-\hat{q}_{28}^{(7)}\,\hat{q}_{82}\}+\hat{q}_{12}^{(5)}[\hat{S}_2+\hat{q}_{23}\,\hat{S}3+$
$\hat{q}_{28}^{(7)}\,\hat{S}8+\hat{q}_{29}^{(6)}\,\hat{S}9)]]+\{\hat{q}_{02}+\hat{q}_{0,10}\,\hat{q}_{10,2}^{(11)}\}\,[\,\{\hat{S}_2+\hat{q}_{23}\hat{S}_3+\hat{q}_{28}^{(7)}\,\hat{S}_8+\hat{S}_9\,\hat{q}_{29}^{(6)}\,)(1-\hat{q}_{11}^{(4)})+\hat{S}_1\,\hat{q}_{29}^{(6)}\,\hat{q}_{91}]+\hat{q}_{0,10}$
$\hat{S}_{10}\,[\{1-\hat{q}_{28}^{(7)}\,\hat{q}_{82}\}\{1-\hat{q}_{11}^{(4)}\}-\hat{q}_{29}^{(6)}\,\hat{q}_{91}\,\hat{q}_{12}^{(5)}]$

and $D_2(s)$ is already defined.
(Omitting the arguments s for brevity)

In the long run, $R_0 = \dfrac{N_3(0)}{D_2'(0)}$ (28)

Where

$N_3(0) = \{p_{01} + p_{0,10}\, p_{10,1}^{(11)}\}[\hat{S}_1(1 - p_{28}^{(7)}\} + p_{12}^{(5)}[\hat{S}_2 + p_{23}\hat{S}3 + p_{28}^{(7)}\hat{S}8 + p_{29}^{(6)}\hat{S}9)]] + \{p_{02} + p_{0,10}\, p_{10,2}^{(11)}\}[\{\hat{S}_2 + p_{23}\hat{S}_3 + p_{28}^{(7)}\hat{S}_8 + \hat{S}_9\, p_{29}^{(6)})(1 - p_{11}^{(4)}) + \hat{S}_1 p_{29}^{(6)}] + p_{0,10}\,\hat{S}_{10}[\{1 - p_{28}^{(7)}\}\{1 - p_{11}^{(4)}\} - p_{29}^{(6)}\, p_{12}^{(5)}]$

and $D_2'(0)$ is already defined.

The expected busy period of the server when there is failure due to Typhoon and failure due to Mediterranean storm in (0,t] is

$\lambda_{rv}(t) = \int_0^\infty R_0(z)dz$ So that $\overline{\lambda_{rv}}(s) = \dfrac{\bar{R}_0(s)}{s}$

The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0, t]-H_0

$H_0(t) = Q_{01}(t)[s][1 + H_1(t)] + Q_{02}(t)[s][1 + H_2(t)] + Q_{0,10}(t)[s]\, H_{10}(t)]$
$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{12}^{(5)}(t)[s]\, H_8(t) + Q_{11}^{(4)}(t)]\,[s]H_1(t),$
$H_2(t) = Q_{23}(t)[s]H_3(t) + Q_{28}^{(7)}(t)\,[s]\, H_8(t) + Q_{29}^{(6)}(t)]\,[c]H_9(t)$
$H_3(t) = Q_{30}(t)[s]H_0(t)$
$H_8(t) = Q_{82}(t)[s]H_2(t)$
$H_9(t) = Q_{91}(t)[s]H_1(t)$
$H_{10}(t) = Q_{10,0}(t)[s]H_{10}(t)] + Q_{10,1}^{(11)}(t)[s]H_1(t)] + Q_{10,2}^{(11)}(t)[s]\, H_2(t)]$ (29-35)

Taking Laplace Transform of eq. (29-35) and solving for $H_0^*(s)$

$H_0^*(s) = N_4(s) / D_3(s)$ (36)

$N_4(s) = \{Q_{01}^* + Q_{02}^*\}[\{1 - Q_{11}^{(4)*}\}\{1 - Q_{28}^{(7)*}\, Q_{82}^*\} - Q_{12}^{(5)*}\, Q_{29}^{(6)*}\, Q_{91}^*]$

And

$D_3(s) = \{1 - Q_{11}^{(4)*}\}\{1 - Q_{28}^{(7)*}\, Q_{82}^*\} - Q_{12}^{(5)*}\, Q_{29}^{(6)*}\, Q_{91}^*](1 - Q_{0,10}^*\, Q_{10,0}^*) - \{Q_{01}^* + Q_{0,10}^*\, Q_{10,1}^{(11)*}\}[Q_{10}^*\{1 - Q_{28}^{(7)*}\, Q_{82}^*\} + Q_{12}^{(5)*}\, Q_{23}^*\, Q_{30}^*] - \{Q_{02}^* + Q_{0,10}^*\, Q_{10,2}^{(11)*}\}[Q_{23}^*\, Q_{30}^*\{1 - Q_{11}^{(4)*}\} + Q_{29}^{(6)*}\, Q_{91}^*\, Q_{10}^*]$

(Omitting the arguments s for brevity)

In the long run,

$H_0 = N_4(0) / D_3'(0)$ (37)

where

$N_4(0) = \{1 - p_{0,10}\}[\{1 - p_{11}^{(4)}\}\{1 - p_{28}^{(7)}\} - p_{12}^{(5)}\, p_{29}^{(6)}]$

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0,t]-W_0

$W_0(t) = Q_{01}(t)[s]W_1(t) + Q_{02}(t)[s]\, W_2(t) + Q_{10,0}(t)[s]\, W_{10}(t)$
$W_1(t) = Q_{10}(t)[s]W_0(t)] + Q_{12}^{(5)}(t)[s]W_2(t) + Q_{11}^{(4)}(t)]\,[s]W_1(t),$
$W_2(t) = Q_{23}(t)[s]W_3(t) + Q_{28}^{(7)}(t)\,[s]\, W_8(t) + Q_{29}^{(6)}(t)]\,[c]W_9(t)$
$W_3(t) = Q_{30}(t)[s][1 + W_0(t)]$
$W_8(t) = Q_{82}(t)[s][1 + W_2(t)]$
$W_9(t) = Q_{91}(t)[s][1 + W_1(t)]$
$W_{10}(t) = Q_{10,0}(t)[s]W_0(t) + Q_{10,1}^{(11)}(t)[s]\, W_1(t) + Q_{10,2}^{(12)}(t)[s]\, W_2(t)$ (38-44)

Taking Laplace Transform of eq. (33-39) and solving for $H_0^*(s)$

$H_0^*(s) = N_5(s) / D_3(s)$ (45)

$N_5(s) = \{Q_{01}^* + Q_{0,10}^* Q_{0,10}^{(11)*}\}[Q_{12}^{(5)*}[Q_{23}^*\, Q_{30}^* + Q_{28}^{(5)*}\, Q_{82}^* + Q_{29}^{(6)*}\, Q_{91}^*] + \{Q_{02}^* + Q_{0,10}^* Q_{10,2}^{(11)*}\}[[Q_{23}^*\, Q_{30}^* + Q_{28}^{(5)*}\, Q_{82}^* + Q_{29}^{(6)*}\, Q_{91}^*\{1 - Q_{11}^{(4)*}\}]$

(Omitting the arguments s for brevity)

In the long run,

$W_0 = N_5(0) / D_3'(0)$ (46)

where $N_5(0) = \{p_{01} + p_{0,10}\, p_{10,1}^{(11)}\}$
$p_{12}^{(5)} + \{p_{02} + p_{0,10}\, p_{10,2}^{(11)}\}\{1 - p_{11}^{(4)}\}]$

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0, t]-Y_0

$Y_0(t)=Q_{01}(t)[s]Y_1(t)+ Q_{02}(t)[s]\ Y_2(t) + Q_{0,10}(t)[s]\ [1+Y_{10}(t)]$
$Y_1(t) = Q_{10}(t)[s]Y_0(t) + Q_{12}^{(5)}(t)[s]\ Y_2(t) + Q_{11}^{(4)}(t)]\ [s]Y_1(t),$
$Y_2(t) = Q_{23}(t)[s]Y_3(t) + Q_{28}^{(7)}(t)\ [s]Y_8(t) +Q_{29}^{(6)}(t)]\ [c]Y_9(t)$
$Y_3(t) = Q_{30}(t)[s][1+Y_0(t)\ ]$
$Y_8(t) = Q_{82}(t)[s]Y_2(t)$
$Y_9(t) = Q_{91}(t)[s]Y_1(t)$
$Y_{10}(t)=Q_{10,0}(t)[s]Y_0(t)+ Q_{10,1}^{(11)}(t)[s]\ Y_1(t) + Q_{10,2}^{(12)}(t)[s]\ Y_2(t)$ (47-53)
Taking Laplace Transform of eq. (47-53) and solving for$Y_0^*(s)$, we get
$Y_0^*(s) = N_6(s) / D_3(s)$ (54)
$N_6(s) = Q_{0,10}^*\ [\{1 – Q_{11}^{(4)*}\}(1- Q_{28}^{(5)*}\ Q_{82}^*\ \}\ - Q_{12}^{(5)*}Q_{29}^{(6)*}\ Q_{91}^*\{1- Q_{0,10}^*Q_{,10,0}^*\ \}+\{Q_{02}^* + Q_{0,10}^*Q_{10,2}^{(11)*}\}[\ [\ Q_{23}^*\ Q_{30}^*\ \{1 – Q_{11}^{(4)*}\}+ Q_{10}^*\ Q_{29}^{(6)*}\ Q_{91}^*\ ]$
(Omitting the arguments s for brevity)
In the long run,
$W_0 = N_6(0) / D_3'(0)$ (55)
where $N_6(0) = p_{0,10}[\{1-p_{11}^{(4)}\}\{1- p_{28}^{(7)}\}- p_{12}^{(5)}\ p_{29}^{(6)}]$
$p_{12}^{(5)} + \{\ p_{02}+ p_{0,10}\ p_{10,2}^{(11)}\}\ \{1 – p_{11}^{(4)}\}]$

BENEFIT- FUNCTION ANALYSIS

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to Typhoon and failure due to Mediterranean storm, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0, t] is
C (t) = Expected total revenue in (0, t]

- expected busy period of the server when there is failure due to Typhoon and failure due to Mediterranean storm in (0,t]
- expected number of visits by the repairman Type- I or Type- II for repairing of identical the units in (0,t]
- expected number of visits by the multispecialty repairman Type- III for repairing of identical the units in (0,t]
- expected number of visits by the multispecialty repairman Type- IV for repairing of identical the units in (0,t]

$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2C(s))$
$= K_1A_0 - K_2R_0 - K_3H_0 - K_4W_0 – K_5Y_0$
where
K_1: Revenue per unit up-time,
K_2: Cost per unit time for which the system is busy under repairing,
K_3: Cost per visit by the repairman type- I or type- II for units repair,
K_4: Cost per visit by the multispecialty repairman Type- III for units repair
K_5: Cost per visit by the multispecialty repairman Type- IV for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to Typhoon and due to Mediterranean storm increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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