Research Article

Cost-benefit analysis of two-identical warm standby helicopter system subject to failure due to electrical malfunction and failure due to mechanical malfunction with different repair facilities

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Abstract

Helicopters are complex, sophisticated machines. They consist of engines, rotors, drive shafts, gears, electronics, flight controls and landing gear, all of which must function properly both independently and together for the helicopter to operate safely. These individual systems and their components must be designed, manufactured, maintained and operated with the utmost skill and care if the helicopter is to fly safely. In the present paper we have taken failure due to electrical malfunction and failure due to mechanical malfunction with different repair facilities. When the main unit fails then warm standby system becomes operative. Failure due to mechanical malfunction cannot occur simultaneously in both the units and after failure the unit undergoes type-i or type-ii or type-iii or type iv repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters mtsf, availability, busy period, benefit-function analysis have been evaluated.

Keywords: Warm Standby, failure due to Electrical Malfunction, failure due to Mechanical Malfunction, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

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INTRODUCTION

What causes Helicopter accidents: Recently, government agencies, industry and operators conducted their first International Helicopter Safety Symposium which was held in order to develop means to reduce helicopter accidents. The accident rate in helicopter flight was flat, or perhaps increasing attendees noted. The statistics presented showed that the helicopter accident rate was 7.5 per 100,000 hours of flying, whereas the airplane accident rate was approximately 0.175 per 100,000 flying hours. As the Symposium's chairman noted: "Vertical flight is an exclusive engineering feat that only helicopters can offer. They operate close to the ground, within the earth's boundary layer and are exposed to hazards beyond other flight vehicles. It therefore requires special attention to ensure safety of flight." This is true in general of helicopter operations, and particular types of helicopter operations—military, fire-fighting, law enforcement, medical evacuation—are even more demanding. The causes of helicopter accidents can be grouped into three major

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causal areas: operational Error, mechanical malfunction, and electrical malfunction. Within these broad categories, there are multiple underlying causes.

Operational Error: Although all three categories involve some degree of human error, Operational Error is the one where the human error is most direct and apparent. This human error can occur in flight planning, actual conduct of the flight, in training or in maintenance.

a. Failure to operate the aircraft in accordance with the aircraft's operational limitations.
b. Operating the aircraft in unsafe environmental conditions.
c. Failing to properly plan the flight.
d. Improper maintenance
e. Improper training of flight and maintenance personnel
f. Faulty manuals, training guides, checklists and operational procedures
g. Faulty oversight, auditing and review procedures

Mechanical Malfunction: A component of the aircraft fails or fails to function as intended. This can happen anywhere along the component's life.

a. Improper design
b. Inadequate testing
c. Faulty manufacture
d. Inadequate quality control
e. Inadequate operational monitoring
f. Improper use
g. Poor maintenance
h. Inadequate lubrication or cooling
i. Improper installation

Electrical Malfunction: Here, the electrical source stops working or one of its components has a malfunction.

a. The electrical source malfunctions
b. b. An electrical short occurs
c. An electrical component malfunctions
d. Inadequate design
e. Inadequate testing
f. Inadequate quality control
g. Inadequate operational monitoring

Each of these elements of the three major causal areas contains its own subset of individual factors as to exactly why and how it occurs. Sometimes these factors result in minor or no aircraft damage or injury, but all too frequently they cause great aircraft damage and personal injury, even death. One thing is true as to all causes: they are preventable. In this paper we have taken failure due to Electrical Malfunction and failure due to Mechanical Malfunction with different repair facilities. When the main operative unit fails then warm standby system becomes operative. Failure due to Mechanical Malfunction cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of Type- II by ordinary repairman or Type III, Type IV by multispecialty repairman immediately when failure due to Electrical Malfunction. The repair is done on the basis of first fail first repaired.

Assumptions

1. $\lambda_{1,}$ λ_2 λ_3 are constant failure rates when failure due to Electrical Malfunction and failure due to Mechanical Malfunction respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are $G_1(t)$, $G_2(t)$ and $G_3(t)$ $G_4(t)$.
2. The failure due to Mechanical Malfunction is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after failure due to Electrical Malfunction and failure due to Mechanical Malfunction and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
4. The switches are perfect and instantaneous.
5. All random variables are mutually independent.
6. When both the units fail, we give priority to operative unit for repair.
7. Repairs are perfect and failure of a unit is detected immediately and perfectly.
8. The system is down when both the units are non-operative.

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts: O, WS, EMF, MMF,

Operative, Warm Standby, failure due to Electrical Malfunction, failure due to Mechanical Malfunction respectively.

Subscripts: nemf, emf, mmf, ur, wr, uR

No failure due to Electrical Malfunction, failure due to Electrical Malfunction, failure due to Mechanical Malfunction, under repair, waiting for repair, under repair continued from previous state respectively

Up states: 0, 1, 2, 3, 10;

Down states: 4, 5, 6, 7, 8,9,11,

Regeneration point: 0, 1, 2, 3, 8, 9, 10

States of the System

0(O_{nemf}, CS_{nemf}) One unit is operative and the other unit is warm standby and there is no failure due to Electrical Malfunction of both the units.

1($EMF_{emf, urI}$, O_{nemf}) The operating unit failure due to Electrical Malfunction is under repair immediately of Type- I and standby unit starts operating with no failure due to Electrical Malfunction

2($MMF_{mmf, urII}$, O_{nemf}) The operative unit failure due to Mechanical Malfunction and undergoes repair of type II and the standby unit becomes operative with no failure due to Electrical Malfunction

3($MMF_{mmf, urIII}$, O_{nemf}) The first unit failure due to Mechanical Malfunction and under Type-III multispecialty repairman and the other unit is operative with no failure due to Electrical Malfunction

4($EMF_{emf,uR1}$, $EMF_{emf,wrI}$) The unit failed due to EMF resulting from failure due to Electrical Malfunction under repair of Type- I continued from state 1and the other unit failed due to EMF resulting from failure due to Electrical Malfunction is waiting for repair of Type-I.

5($EMF_{emf,uR1}$, $MMF_{mmf,wrII}$) The unit failed due to EMF resulting from failure due to Electrical Malfunction is under repair of Type- I continued from state 1and the other unit failure due to Mechanical Malfunction is waiting for repair of Type- II.

6($MMF_{mmf, uRII}$, $EMF_{emf ,wrI}$) The operative unit failed due to mechnical malfunction is under repair continues from state 2 of Type –II and the other unit failed due to EMF resulting from failure due to Electrical Malfunction is waiting under repair of Type-I.

7($MMF_{mmf ,uRII}$, $EMF_{emf,wrII}$) The one unit failure due to Mechanical Malfunction is continued to be under repair of Type II and the other unit failed due to EMF resulting from failure due to Electrical Malfunction is waiting for repair of Type-II.

8($EMF_{emf,urIII}$, $MMF_{mmf, wrII}$) The one unit failure due to Electrical Malfunction is under multispecialty repair of Type-III and the other unit failure due to Mechanical Malfunction is waiting for repair of Type-II.

9($EMF_{emf,urIII}$, $MMF_{mmf, wrI}$) The one unit failure due to Electrical Malfunction is under multispecialty repair of Type-III and the other unit failure due to Mechanical Malfunction is waiting for repair of Type-I

10(O_{nemf} $MMF_{mmf, urIV}$)

The one unit is operative with no failure due to Electrical Malfunction and warm standby unit failure due to Mechanical Malfunction and undergoes repair of type IV.

11(O_{nemf} $MMF_{mmf, uRIV}$)

The one unit is operative with no failure due to Electrical Malfunction and warm standby unit failure due to Mechanical Malfunction and repair of type IV continues from state 10.

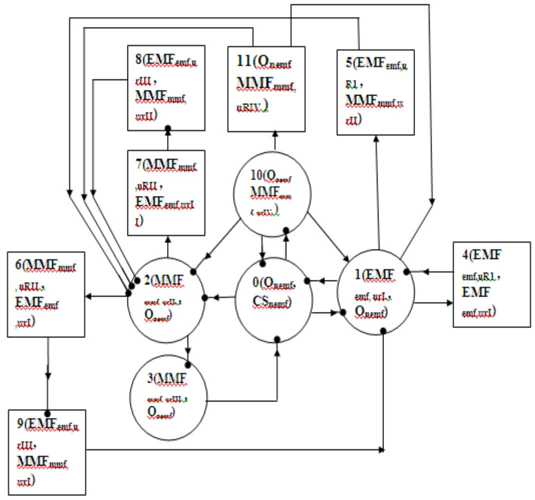

Figure 1: The State Transition Diagram

● Rregeneration point ◯ Up State □ Down State

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$p_{01} = \lambda_1 / \lambda_1 + \lambda_2 + \lambda_3$, $p_{02} = \lambda_2 / \lambda_1 + \lambda_2 + \lambda_3$,
$p_{0,10} = \lambda_3 / \lambda_1 + \lambda_2 + \lambda_3$, $p_{10} = pG_1^*(\lambda_1) + q\, G_2^*(\lambda_2)$,
$p_{14} = p - pG_1^*(\lambda_1) = p_{11}^{(4)}$, $p_{15} = q - q\, G_1^*(\lambda_2) = p_{12}^{(5)}$,
$p_{23} = pG_2^*(\lambda_1) + q\, G_2^*(\lambda_2)$, $p_{26} = p - pG_2^*(\lambda_1) = p_{29}^{(6)}$,
$p_{27} = q - qG_2^*(\lambda_2) = p_{28}^{(7)}$, $p_{30} = p_{82} = p_{91} = 1$
$p_{0,10} = pG_4^*(\lambda_1) + q\, G_4^*(\lambda_2)$, $p_{10,1} = p - pG_4^*(\lambda_1) = p_{10,1}^{(11)}$
$p_{10,2} = q - q\, G_4^*(\lambda_2) = p_{10,2}^{(11)}$ (1)

We can easily verify that

$p_{01} + p_{02} + p_{03} = 1$, $p_{10} + p_{14}\ (= p_{11}^{(4)}) + p_{15}\ (= p_{12}^{(5)}) = 1$,
$p_{23} + p_{26}\ (= p_{29}^{(6)}) + p_{27}\ (= p_{28}^{(7)}) = 1$ $p_{30} = p_{82} = p_{91} = 1$
$p_{10,0} + p_{10,1}^{(11)}\ (= p_{10,1}) + p_{10,2}^{(12)}\ (= p_{10,2}) = 1$ (2)

And mean sojourn time is $\mu_0 = E(T) = \int_0^\infty P|T > t|dt$

Mean Time To System Failure

$Ø_0(t) = Q_{01}(t)[s]\, Ø_1(t) + Q_{02}(t)[s]\, Ø_2(t) + Q_{0,10}(t)[s]\, Ø_{10}(t)$
$Ø_1(t) = Q_{10}(t)[s]\, Ø_0(t) + Q_{14}(t) + Q_{15}(t)$, $Ø_2(t) = Q_{23}(t)[s]\, Ø_3(t) + Q_{26}(t) + Q_{27}(t)$
$Ø_3(t) = Q_{30}(t)[s]\, Ø_0(t)$, $Ø_{10}(t) = Q_{10,0}(t)[s]\, Ø_{10}(t) + Q_{10,2}(t)[s]\, Ø_1(t) + Q_{10,2}(t)[s]\, Ø_2(t)$ (3-6)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-6) and solving for

$ø_0^*(s) = N_1(s) / D_1(s)$ (7)

where

$N_1(s) = \{Q_{01}^* + Q_{0,10}^*\, Q_{10,1}^*\}[Q_{14}^* + Q_{15}^*] + \{Q_{02}^* + Q_{0,10}^*\, Q_{10,2}^*\}[Q_{26}^* + Q_{27}^*]$
$D_1(s) = 1 - \{Q_{01}^* + Q_{0,10}^*\, Q_{10,1}^*\}\, Q_{10}^* - \{Q_{02}^* + Q_{0,10}^*\, Q_{10,2}^*\}\, Q_{23}^*\, Q_{30}^* - Q_{0,10}^*\, Q_{10,0}^*$

(Omitting the arguments s for brevity)

Making use of relations (1) & (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0(t)$ is a proper distribution.

$$MTSF = E[T] = \left. \frac{d}{ds}\, ø_0^*(s) \right|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$= (\mu_0 + \mu_1 (p_{01} + p_{0,10}\, p_{10,1}) + (p_{02} + p_{0,10}\, p_{10,2})(\mu_2 + \mu_3) + \mu_{10}\, p_{0,10} / (1 - (p_{01} + p_{0,10}\, p_{10,1})\, p_{10} - (p_{02} + p_{0,10}\, p_{10,2})\, p_{23}) - p_{0,10}\, p_{10,0}$

where

$\mu_0 = \mu_{01} + \mu_{02} + \mu_{0,10}$, $\mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}$,
$\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}$, $\mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t}$, , $M_1(t) = p\, G_1(t)\, e^{-\lambda_1 t}$

$M_2(t) = q\, \overline{G}_2(t)\, e^{-\lambda_2 t}$, $M_3(t) = G_3(t)$, $\overline{M}_{10}(t) = G_4(t)\, e^{-\lambda_3 t}$

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t)$,

$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]\, A_8(t) + q_{29}^{(6)}(t)]\, [c]A_9(t)$

$A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t)$, $A_8(t) = q_{82}(t)[c]A_2(t)$, $A_9(t) = q_{91}(t)[c]A_1(t)$,

$A_{10}(t) = M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}^{(11)}(t)[c]A_1(t) + q_{10,2}^{(11)}(t)[c]A_2(t)$ (8-14)

Taking Laplace Transform of eq. (8-14) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \qquad (15)$$

where

$N_2(s) = \{ \hat{q}_{0,10} \hat{M}_{10} + \hat{M}_0 \} [\{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7} \hat{q}_{82}\} - \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91}] + \{ \hat{q}_{01} + \hat{q}_{0,10}$
$\hat{q}_{10,1}^{(11)}\}[\hat{M}_1\{1 - \hat{q}_{28}^{(7)} \hat{q}_{82}\} + \hat{q}_{12}^{(5)} \hat{q}_{23} \hat{M}_3 + \hat{M}_2] + \{ \hat{q}_{02} + \hat{q}_{0,10} \hat{q}_{10,2}^{(11)}\} [\{ \hat{q}_{23} \hat{M}_3\}$
$\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)} \hat{q}_{91} \hat{M}_1]$

$D_2(s) = \{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7} \hat{q}_{82}\} - \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91} - \{ \hat{q}_{01} + \hat{q}_{0,10} \hat{q}_{10,1}^{(11)}\}[\hat{q}_{10}$
$\{1 - \hat{q}_{28}^{(7)} \hat{q}_{82}\} + \hat{q}_{12}^{(5)} \hat{q}_{23} \hat{q}_{30}] - \{ \hat{q}_{02} + \hat{q}_{0,10} \hat{q}_{10,2}^{(11)}\}\{[\hat{q}_{23} \hat{q}_{30}\{1 - \hat{q}_{11}^{(4)}\} +$
$\hat{q}_{29}^{(6)} \hat{q}_{91} \hat{q}_{10}]$

(Omitting the arguments s for brevity)

The steady state availability

$$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\,\hat{A}_0(s)] = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0}\frac{N_2(s) + s\,N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \qquad (16)$$

Where

$N_2(0) = \{p_{0,10} \hat{M}_{10}(0) + \hat{M}_0(0)\} [\{1 - p_{11}^{(4)}\}\{1 - p_{28}^{(7)}\} - p_{12}^{(5)} p_{29}^{(6)}] + \{ p_{01} +$
$p_{0,10}\, p_{10,1}^{(11)}\}[\hat{M}_1(0)\{1 - p_{28}^{(7)}\} + p_{12}^{(5)} p_{23} \hat{M}_3(0) + \hat{M}_2(0)] + \{ p_{02} + p_{0,10}\, p_{10,2}^{(11)}\}$
$[\{p_{23} \hat{M}_3(0) + \hat{M}_2(0)\}\{1 - p_{11}^{(4)}\} + p_{29}^{(6)} \hat{M}_1(0)]$

$D_2'(0) = \mu_0[p_{10}(1 - p_{28}^{(7)}\} + p_{12}^{(5)} p_{23}] + \mu_1[p_{29}^{(6)} + p_{01} p_{23} - p_{0,10}\{p_{10,0}\{1 - p_{28}^{(7)}\} +$
$p_{23}\, p_{10,2}^{(11)} p_{23}\}] + \mu_2[(1 - p_{11}^{(4)}) - p_{01} p_{10} - p_{0,10}(p_{10} - p_{10}\, p_{10,2}^{(11)} + p_{12}^{(5)} p_{10,0})]\}$
$+ \mu_3 [p_{23}[p_{12}^{(5)}\{p_{01} + p_{0,10}\, p_{10,1}^{(11)}\} + (1 - p_{11}^{(4)})\{ p_{02} + p_{0,10}\, p_{10,2}^{(11)}\}] + \mu_8 [p_{28}^{(7)}$
$p_{0,10}\, p_{10,0} - p_{10}\{ p_{01} + p_{0,10}\, p_{10,1}^{(11)}\})] + \mu_9 [p_{29}^{(6)}\{ p_{12}^{(5)} (1 - p_{0,10}\, p_{10,0} + (p_{02} +$
$p_{0,10}\, p_{10,2}^{(11)}\})] + \mu_{10} [p_{29}^{(6)}\{ p_{12}^{(5)} (1 - p_{0,10}\, p_{10,0} + (p_{02} + p_{0,10}\, p_{10,2}^{(11)}\})]$

and $\mu_3 = \mu_{30}$, $\mu_9 = \mu_{91}$, $\mu_8 = \mu_{81}$

The expected up time of the system in (0, t] is

$$\lambda_u(t) = \int_0^{\infty} A_0(z)\,dz \text{ So that } \overline{\lambda_u}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)} \qquad (17)$$

The expected down time of the system in (0,t] is

$$\lambda_d(t) = t - \lambda_u(t) \text{ So that } \overline{\lambda_d}(s) = \frac{1}{s^2} - \overline{\lambda_u}(s) \qquad (18)$$

Similarly, we can find out

1. The expected busy period of the server when there is failure due to Electrical Malfunction and failure due to Mechanical Malfunction in (0,t]-R_0
2. The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,t]-H_0
3. The expected number of visits by the multispecialty repairman Type-III, Type-IV for repairing the identical units in (0,t]-W_0 , Y_0 respectively

BENEFIT- FUNCTION ANALYSIS

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to Electrical Malfunction and failure due to Mechanical Malfunction, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0, t] is

$$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s)) = K_1A_0 - K_2R_0 - K_3H_0 - K_4W_0 - K_5Y_0$$

where

K_1: Revenue per unit up-time, K_2 - cost per unit time for which the system is busy under repairing, K_3 - cost per visit by the repairman type- I or type- II for units repair,

K_4: Cost per visit by the multispecialty repairman Type- III for units repair

K_5: Cost per visit by the multispecialty repairman Type- IV for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to Electrical Malfunction and due to Mechanical Malfunction increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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