Research Article

Minimum risk equivariant estimation of the parameter of a scale model based on general progressive right censored sample

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Abstract Considering a general progressive Type II right censored sample, we obtain Minimum Risk Equivariant estimator (MRE) for the parameter of the uniform model in three situations. These generalize the results of Chandrasekar et.al. (2002) for progressive Type II right censored sample. The paper is organized as follows: Section 2 deals with the problem of equivariant estimation under squared error loss function. Section 3 discusses the problem of equivariant estimation under Absolute error loss function. In the last Section, we consider the problem of equivariant estimation of the parameter under Linex loss function (Varian, 1975).
Keywords: Absolute Error Loss function, General Progressive Censored Sample, Linex Loss function, MRE, Squared Error Loss function, Uniform

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INTRODUCTION

Progressive Type II right censored sampling is an important method of obtaining data in life-testing studies. As pointed out by Aggarwala and Balakrishnan (1998), the scheme of progressive censoring enables us to use live units, removed early, in other tests. Balakrishnan and Sandhu (1996), by assuming a general progressive Type II right censored sample, derived the BLUE's for the parameters of one-and two-parameter exponential distributions. For the later, they also derived MLE's and shown that they are simply the BLUE's, adjusted for their bias. Let us consider the following general progressive Type II right censoring scheme (Balakrishnan and Sandhu, 1996) : Suppose N randomly selected units were placed on a life test; the failure times of the first r units to fail were not observed ; at the time of the $(r+1)^{th}$ failure, R_{r+1} number of surviving units are withdrawn from the test randomly, and so on; at the time of the $(r+i)^{th}$ failure, R_{r+i} number of surviving units are randomly withdrawn from the test ; finally, at the time of the n-th failure, the remaining $R_n = N - n - R_{r+1} - R_{r+2} - \ldots - R_{n-1}$ are withdrawn from the test. Suppose $X_{r+1:N} \leq X_{r+2:N} \leq \ldots \leq X_{n:N}$ are the life-times of the completely observed units to fail, and $R_{r+1}, R_{r+2}, \ldots, R_n$ are the number of units withdrawn from the test at these failure times, respectively. It follows that $N = n + \sum_{i=r+1}^{n} Ri$

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If the failure times are from a continuous population with the pdf f and the distribution function F, then the joint pdf of ($X_{r+1:N}$, $X_{r+2:N}$, ... , $X_{n:N}$) is given by

$$g_\theta(x_{r+1},...,x_n)=\mathbf{c}\mathbf{[}\{F_\theta(x_{r+1})\}^r X \prod_{i=r+1}^{n} f_\theta(x_i)\{1-F_\theta(x_i)\}^{R_i} \;] \ldots (1.1)$$

where $c=\binom{N}{r}(N-r)\prod_{j=r+2}^{n}\left(N-\sum_{i=r+1}^{j-1}R_i-j+1\right).$

UNIFORM SCALE MODEL

In this case the common pdf is taken as

$$f_\tau(x)=\begin{cases}1/\tau,\ 0\le x\le\tau\,;\,\tau>o\\ 0\quad,\ \text{otherwise}.\end{cases}$$

Thus (1.1) reduces to

$$g_\tau(x_{r+1},...x_n)=c(x_{r+1}/\tau)^r(1/\tau^n)\underset{i=1}{\overset{n}{\pi}}(1-x_i/\tau)^{R_i},\qquad 0\le x_{r+1}\le x_n\le\tau:\tau>0. \qquad (2.1)$$

Note that the joint distribution of $\left(X_{r+1:N},...,X_{n:N}\right)$ belongs to a scale family with the scale parameter τ . We are interested in estimating τ^m , m fixed, by considering three loss functions. Following Lehmann and Casella (1998) the MRE estimator of τ^m is given by $\delta^*(X)=\frac{\delta_0(X)}{w*(Z)}$ where δ_0 is a scale equivariant estimator and $w(z)=w*(z)$ minimizes $E_1[\gamma\{\frac{\delta_0(X)}{w(z)}\}\,|\,z]$ .HereE_1denotes E_τ when $\tau=1; Z_i=X_{i:N}/X_{n:N},\ i=r+1,r+2,...,n-1,$ $Z=\left(Z_{r+1},Z_{r+2},...,Z_n\right)$ and γ is an invariant loss function.

SQUARED ERROR LOSS FUNCTION

If the loss function is of the form $\gamma\left(\delta/\tau^m\right)=\left(\delta/\tau^m-1\right)^2$ then $w^*=\frac{E_1(\delta_0^2\,|\,z)}{E_1(\delta_0\,|\,z)}$. Take $\delta_0(X)=X_{n:N}^m$. Clearly $\delta_0(X)$ is a scale equivalent estimator but not complete sufficient statistic. Since we are interested in the evaluation of conditional distribution under $\tau=1,$ we take $\tau=1$ in (2.1). In order to find w^*, consider the transformation $Z_n=X_{n:N}$ and $Z_i=X_{i:N}/X_{n:N}, i=r+1,r+2,...,n-1.$ Then $X_{n:N}=Z_n$ and $X_{i:N}=Z_nZ_i, i=r+1,r+2,...,n-1$ and the Jacobian of the transformation is $J=Z_n^{n-r-1}$. Thus the joint pdf of $\left(Z_{r+1},Z_{r+2},...,Z_n\right)$ is given by

$$h_1\left(z_{r+1},...,z_{n-1}\right)=\mathbf{c}\,\mathbf{[}\int_0^1 z_{r+1}^r z_n^{n-1}\left(1-z_n\right)^{R_n}\times \underset{i=r+1}{\overset{n-1}{\pi}}\left(1-z_nz_i\right)^{R_i}dz_n]\,.$$

Thus the conditional pdf of Z_n given $\left(Z_{r+1},...,Z_{n-1}\right)$ is given by

$$h_2(z_n \mid z_{r+1},\dots,z_{n-1}) = \frac{z_{r+1}^r z_n^{n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}} (1-z_n z_i)^{R_i}}{\int_0^1 z_{r+1}^r z_n^{n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}} (1-z_n z_i)^{R_i}\, dz_n},$$ Now $$w^* = \frac{E_1(\delta_0^2 \mid z)}{E_1(\delta_0 \mid z)},$$

$$0 < z_n < 1 \ldots (3.1)$$

where $$E_1(\delta_0^2 \mid z) = \frac{\int_0^1 z_{r+1}^r z_n^{2m+n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}} (1-z_n z_i)^{R_i}\, dz_n}{\int_0^1 z_{r+1}^r z_n^{n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}} (1-z_n z_i)^{R_i}\, dz_n}, \quad (3.2)$$

in view of (3.1) and

$$E_1(\delta_0 \mid z) = \frac{\int_0^1 z_{r+1}^r z_n^{m+n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}} (1-z_n z_i)^{R_i}\, dz_n}{\int_0^1 z_{r+1}^r z_n^{n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}} (1-z_n z_i)^{R_i}\, dz_n}, \quad (3.3)$$

in view of (3.1).

Therefore the MRE estimator of τ^m is given by

$$\delta^*(X) = X_{n:N}^m \times \left[\frac{\int_0^1 z_{r+1}^r z_n^{m+n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}} (1-z_n z_i)^{R_i}\, dz_n}{\int_0^1 z_{r+1}^r z_n^{2m+n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}} (1-z_n z_i)^{R_i}\, dz_n}\right], \quad (3.4)$$

in view of (3.2) and (3.3).

Remark 3.1: The Pittman form of the MRE estimator of τ^m with respect to the squared error loss function is

$$\delta^*(X) = X_{n:N}^m \frac{\int_0^1 z_{r+1}^r z_n^{m+n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}} (1-z_n z_i)^{R_i}\, dz_n}{\int_0^1 z_{r+1}^r z_n^{2m+n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}} (1-z_n z_i)^{R_i}\, dz_n},$$ which coincides with the one given in (3.4).

Remark 3.2: If r=0 and $R_i = r_i$, then the above estimator reduced to

$$\delta^*(X) = X_{n:N}^m \frac{\int_0^1 z_n^{m+n-1}(1-z_n)^{r_n} \underset{i=1}{\overset{n-1}{\pi}} (1-z_n z_i)^{r_i}\, dz_n}{\int_0^1 z_n^{2m+n-1}(1-z_n)^{r_n} \underset{i=1}{\overset{n-1}{\pi}} (1-z_n z_i)^{r_i}\, dz_n},$$ which is the same as the Type-II progressive right censored sample case of (Leo Alexander, 2000).

ABSOLUTE ERROR LOSS FUNCTION

If the loss function in of the form $\gamma(\delta/\tau) = |\delta - \tau|/\tau$, then $w^* = c$ is obtained by solving the following equation $N\int_0^c x_{n:N}^N dx = N\int_c^1 x_{n:N}^N dx$. Here $\delta_0(X) = X_{n:N}$ and its pdf $f(x_{n:N}) = NX_{n:N}^{N-1}$, $0 < x_{n:N} < 1$.

Thus we get $c = (1/2)^{1/(N+1)}$ by solving the above equation.

Therefore the MRE estimator of τ is given by $\delta^*(X) = 2^{1/(N+1)} X_{n:N}$.

LINEX LOSS FUNCTION

The scale invariant Linear loss function (Varian, 1975) given by $L(\tau;\delta) = e^{a(\delta/\tau - 1)} - a(\delta/\tau - 1) - 1$. In order to find w^*, take $\delta_0(X) = X_{n:N}$, consider

$$R(\delta\,|\,z) = E_1\{(e^{a(\delta_0/w-1)} - a(\delta_0/w - 1) - 1)\,|\,z\}$$

$$= e^{-a} E_1(e^{a\delta_0/w}\,|\,z) - a/wE_1(\delta_0\,|\,z) + a - 1$$

$$= e^{-a}\frac{\int_0^1 z_{r+1}^r e^{a/w.z_n} z_n^{n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}}(1-z_n z_i)^{R_i} dz_n}{\int_0^1 z_{r+1}^r z_n^{n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}}(1-z_n z_i)^{R_i} dz_n} - a/w\frac{\int_0^1 z_{r+1}^r z_n^{n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}}(1-z_n z_i)^{R_i} dz_n}{\int_0^1 z_{r+1}^r z_n^{n-1}(1-z_n)^{R_n} \underset{i=r+1}{\overset{n-1}{\pi}}(1-z_n z_i)^{R_i} dz_n}$$

$+ a - 1$,

in view of the following equations,

$$h(z_{r+1}\,|\,z_{r+2},\ldots,z_n) = \frac{z_{r+1}^{n-r-1}(1-e^{-z_{r+1}})^r\, e^{-z_{r+1}[(R_{r+1}+1)+\sum_{i=r+2}^{n}(R_i+1)z_i]}}{E_1(-\log U)^{n-r-1} B((R_{r+1}+1)+\sum_{i=r+2}^{n}(R_i+1)z_i, r+1)}$$

and $E_1(z_{r+1}^m\,|\,z_{r+2},\ldots,z_n) = \dfrac{E_1(-\log U)^{n-r+m-1}}{E_1(-\log U)^{n-r-1}}$,when m=1.

Thus w^* is to be obtained as the value of w minimizes $R(\delta\,|\,z)$.

Therefore the MRE estimator of τ is given by $\delta^*(\mathbf{X}) = X_{n:N}/w^*$.

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