Original Research Article

Analysis of wave propagation between solid-solid interfaces

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Abstract

A study of reflection and refraction phenomena of plane wave from a plane interface between micropolar elastic solid half space and micropolar porous elastic solid half space is made in this present paper. Using the method potentials and the appropriate boundary conditions at the interface, the amplitudes of various reflected and refracted waves are obtained. It has been noticed that these amplitudes are complex valued and their absolute values depend upon the angle of incidence, frequency of incident wave and characteristics of medium. The variation of modulus of amplitudes ratios of various reflected and refracted waves against the angle of incidence are computed numerically for obliquely incident wave which is travelling at high frequency as well as at low frequency. A particular case of reflection at free surface of micropolar elastic solid has been deduced and discussed with the help of graphs. Results thus are obtained and depicted graphically with angle of incidence of incident wave.

Key Words: Amplitude ratio, longitudinal wave, reflection, refraction, micropolar elastic solid, porous, frequency.

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INTRODUCTION

The micropolar theory of elasticity constructed by Eringen and his co-workers are intended to be applied on such materials for which the ordinary theory of elasticity fails because of microstructure in the materials. Micropolar elastic materials, roughly speaking, are the classical elastic materials with an extra independent degree of freedom for local rotations. These materials respond to spin inertia, body and surface couples as a consequence they exhibit certain new static and dynamic effects, e.g. new types of waves and couples stresses. From a continuum mechanical point of view, micropolar elastic solids may be characterized by a set of constitutive equations which define the elastic properties of such materials. A linear theory as a special case of the nonlinear theory of micro-elastic solids was first constructed by Eringen and Suhubi (1964a, b). Later, Eringen (1965) and (1966) recognized and extended this theory. Eringen's theory of micropolar elasticity keeps importance because of its applications in many physical substances for example material particles having rigid directors, chopped fibers composites, platelet composites, aluminium epoxy, liquid crystal with side chains, a large class of substance like liquid crystal with rigid molecules, rigid suspensions, animal blood with rigid cells, foams, porous materials, bones, magnetic fields, clouds with dust, concrete with sand and muddy fluids are example of micropolar materials. Cowin and co-workers developed the theories of non-linear and linear elastic material with voids. The linear theory of elastic material with voids is a special class of the nonlinear theory in which the change in void volume fraction and the strain are taken as independent kinematic variables. Material may be called porous material which has the properties of small distributed pores. Many problems of waves and vibrations concerning the micropolar elasticity and material with voids have been discussed by many researchers in the past, e.g., chandersekhariah (1987), Wright (1998), Golamhossen (2000), Iesan and Nappa (2003), Tomar and Singh (2005, 06). Recently, such problems discussed by Tomar and Khurana (2011), Kumari (2013), Shekhar and Parvez (2015), Zhang *et al.* (2016) and Merkel and

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Luding (2017). The present paper is concerned with reflection and refraction of longitudinal waves at the interface between micropolar elastic solid half space and micropolar elastic solid half space with porous. The particular cases are deduced and discussed with the help of graphs.

BASIC EQUATIONS AND CONSTITUTIVE RELATIONS

For medium M_1 (Micropolar elastic solid) The equation of motion in micropolar elastic medium are given by Eringen (1968) as

$$(c_1{}^2 + c_3{}^2)\nabla^2\phi = \frac{\partial^2\phi}{\partial t^2}, \tag{1}$$

$$(c_2{}^2 + c_3{}^2)\nabla^2 U + c_3{}^2\nabla \times \Phi = \frac{\partial^2 U}{\partial t^2}, \tag{2}$$

$$(c_4{}^2\nabla^2 - 2\omega_0{}^2)\Phi + \omega_0{}^2\nabla \times U = \frac{\partial^2\Phi}{\partial t^2}, \tag{3}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, c_2^2 = \frac{\mu}{\rho}, c_3^2 = \frac{\kappa}{\rho}, c_4^2 = \frac{\gamma}{\rho j}, \omega_0^2 = \frac{\kappa}{\rho j}, \tag{4}$$

Parfitt and Eringen (1969) have shown that eq. (1) corresponds to longitudinal wave propagating with velocityV_1, given by $V_1{}^2 = c_1{}^2 + c_3{}^2$and equations. (2) - (3) are coupled equations in vector potentials U and Φ and these correspond to coupled transverse and micro-rotation waves. If $\frac{\omega^2}{\omega_0{}^2} > 20$, there exist two sets of coupled-wave propagating with velocities $1/\lambda_1$ and $1/\lambda_2$.

where

$$\lambda_1{}^2 = \frac{1}{2}\left[B - \sqrt{B^2 - 4C}\right], \lambda_2{}^2 = \frac{1}{2}\left[B + \sqrt{B^2 - 4C}\right], \tag{5}$$

and

$$B = \frac{q(p-2)}{\omega^2} + \frac{1}{(c_2{}^2 + c_3{}^2)} + \frac{1}{c_4{}^2},$$

$$C = \left(\frac{1}{c_4{}^2} - \frac{2q}{\omega^2}\right)\frac{1}{(c_2{}^2 + c_3{}^2)},$$

$$p = \frac{\kappa}{\mu + \kappa}, q = \frac{\kappa}{\gamma}. \tag{6}$$

where symbols $\lambda, \mu, \gamma, \kappa$, j. have their usual meaning. We consider a two dimensional problem by taking the following components of displacement and micro-rotation as

$$U = (u, 0, w), \Phi = (0, \Phi_2, 0), \tag{7}$$

where

$$u = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial z}, w = \frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x}, \tag{8}$$

and components of stresses as

$$t_{zz} = (\lambda + 2\mu + \kappa)\frac{\partial^2\phi}{\partial z^2} + \lambda\frac{\partial^2\phi}{\partial x^2} + (2\mu + \kappa)\frac{\partial^2\psi}{\partial x\,\partial z}, \tag{9}$$

$$t_{zx} = (2\mu + \kappa)\frac{\partial^2\phi}{\partial x\,\partial z} - (\mu + \kappa)\frac{\partial^2\psi}{\partial z^2} + \mu\frac{\partial^2\psi}{\partial x^2} - \kappa\Phi_2, \tag{10}$$

$$m_{zy} = \gamma\frac{\partial\Phi_2}{\partial z}. \tag{11}$$

For medium M_2 (Micropolar elastic solid with porous)

Following Iesan (1985), the constitutive and field equations of micropolar porous elastic solid (see figure 1), in the absence of body force density and body couple density, can be written as

$$(\bar\lambda + 2\bar\mu + \bar\kappa)\nabla(\nabla.\mathbf{u}) - (\bar\mu + \bar\kappa)\nabla \times (\nabla \times \mathbf{u}) + \bar\kappa\,\nabla \times \bar\Phi + \beta^*\nabla\bar\psi = \bar\rho\,\ddot{\mathbf{u}}, \tag{12}$$

$$(\bar\alpha + \bar\beta + \bar\gamma)\nabla(\nabla.\bar\Phi) - \bar\gamma\,\nabla \times (\nabla \times \bar\Phi) + \bar\kappa\,\nabla \times \mathbf{u} - 2\,\bar\kappa\,\bar\Phi = \bar\rho\,\bar{j}\,\ddot{\bar\Phi}, \tag{13}$$

$$\alpha^*\nabla^2\bar\psi - \xi^*\bar\psi - \omega^*\dot{\bar\psi} - \beta^*\nabla.\mathbf{u} = \bar\rho\,\kappa^*\,\ddot{\bar\psi}. \tag{14}$$

where $\bar\lambda$ and $\bar\mu$; $\bar\kappa, \bar\alpha, \bar\beta$ and $\bar\gamma$; $\alpha^*, \beta^*, \xi^*, \omega^*$ and κ^* are Lame's constant; elastic constants of micropolarity and elastic constants due to presence of voids, respectively; $\mathbf{u}$ (x, t) and $\bar\Phi$(x, t) are the displacement and microrotation vectors, $\bar\psi$ is the change in the void volume fraction from that of in the reference state; $\bar{j}$ is the micro-inertia and $\bar\rho$ is the density of the medium. The superposed dots on the right hand side of these equations second ordered partial derivatives with respect to time. For time harmonic plane wave propagation (i.e., $\propto \exp\{-\bar\omega t\}$), the equations of motion (12) – (14) reduced to

$$(\bar c_1^2 + \bar c_3^2)\nabla(\nabla.\mathbf{u}) - (\bar c_2^2 + \bar c_3^2)\nabla \times (\nabla \times \mathbf{u}) + \bar c_3^2\,\nabla \times \bar\Phi + \bar c_6^2\nabla\bar\psi + \bar\omega^2\mathbf{u} = 0, \tag{15}$$

$$(\bar c_4^2 + \bar c_5^2)\nabla(\nabla.\bar\Phi) - \bar c_4^2\nabla \times (\nabla \times \bar\Phi) + \bar\omega_0^2\,\nabla \times \mathbf{u} - 2\bar\omega_0^2\bar\Phi + \bar\omega^2\bar\Phi = 0, \tag{16}$$

$$(\alpha^*\nabla^2 - \xi^* + i\bar\omega\omega^* + \bar\rho\,\kappa^*\,\bar\omega^2)\bar\psi - \beta^*\nabla.\mathbf{u} = 0 \tag{17}$$

where

$$\bar c_1^2 = \frac{\bar\lambda + 2\bar\mu}{\bar\rho}, \bar c_2^2 = \frac{\bar\mu}{\bar\rho}, \bar c_3^2 = \frac{\bar\kappa}{\bar\rho}, \bar c_4^2 = \frac{\bar\gamma}{\bar\rho\,\bar{j}}, \bar c_5^2 = \frac{\bar\alpha + \bar\beta}{\bar\rho\,\bar{j}},$$

$$\bar\omega_0^2 = \frac{\bar c_3^2}{\bar{j}} = \frac{\bar\kappa}{\bar\rho\,\bar{j}}, \ \bar c_6^2 = \frac{\beta^*}{\bar\rho}. \tag{18}$$

The constitutive relations for the micropolar porous elastic solid are given by (see Iesan, 1985)

$$\bar t_{kl} = \bar\lambda\bar u_{r,r}\delta_{kl} + \bar\mu(\bar u_{k,l} + \bar u_{l,k}) + \bar\kappa(\bar u_{l,k} - \epsilon_{klr}\bar\phi_r) + \delta_{kl}\beta^*\bar\psi, \tag{19}$$

$$\bar m_{kl} = \bar\alpha\bar\phi_{r,r}\delta_{kl} + \bar\beta\bar\phi_{k,l} + \bar\gamma\bar\phi_{l,k}, \tag{20}$$

$$\bar h_k = \alpha^*\,\bar\psi_{,k}. \tag{21}$$

where $\bar t_{kl}$, $\bar m_{kl}$, and $\bar h_k$ are the force stress tensor, couple stress tensor and equilibrated force vector, respectively.

Introducing the scalar potentials $\bar q$ and ξ, the vector potentials $\bar U$ and Π, through the Helmholtz's decomposition of vectors as

$$\mathbf{u} = \nabla\,\bar q + \nabla \times \bar U, \bar\Phi = \nabla\xi + \nabla \times \Pi, \nabla.\bar U = \nabla.\Pi = 0 \tag{22}$$

and employing these relations into equations of motion (15) – (17), we obtain the following system of equations

$$\left[(\bar c_1^2 + \bar c_3^2)\nabla^2 + \bar\omega^2\right]\bar q + \bar c_6^2\bar\psi = 0, \tag{23}$$

$$\left[(\bar c_2^2 + \bar c_3^2)\nabla^2 + \bar\omega^2\right]\bar U + \bar c_3^2\nabla^2 \times \Pi = 0, \tag{24}$$

$$\left[(\bar c_4^2 + \bar c_5^2)\nabla^2 - 2\bar\omega_0^2 + \bar\omega^2\right]\xi = 0, \tag{25}$$

$$\left[\bar{c}_4^2\nabla^2 - 2\bar{\varpi}_0^2 + \bar{\varpi}^2\right]\Pi + \bar{\varpi}_0^2\nabla \times \bar{U} = 0, \quad (26)$$

$$(\alpha^*\nabla^2 - \xi^* + i\bar{\varpi}\omega^* + \bar{\rho}\,\kappa^*\,\bar{\varpi}^2)\bar{\psi} - \beta^*\nabla^2\bar{q} = 0. \quad (27)$$

Following the procedure adopted by Tomar and Singh (2006) for plane waves advancing along the positive direction of a unit vector, we can obtain the dispersion relations giving the phase speeds of an independent longitudinal microrotational wave and two sets of coupled transverse waves along with the following dispersion equation giving the phase speeds of two longitudinal waves

$$\left(\tau^2 - \frac{\bar{\varpi}^2}{S^2}\right)\left(\tau^2 - \frac{\bar{\varpi}^2}{T^2} + \frac{1}{l_2^2} - \frac{i\bar{\varpi}\omega^*}{\alpha^*}\right) - \frac{H^*}{l_1^2}\tau^2 = 0, \quad (28)$$

where

$$S^2 = \bar{c}_1^2 + \bar{c}_3^2, T^2 = \frac{\alpha^*}{\bar{\rho}\,\kappa^*}, l_1^2 = \frac{\alpha^*}{\beta^*}, l_2^2 = \frac{\alpha^*}{\xi^*} \text{ and}$$

$$H^* = \frac{\beta^*}{(\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})}.$$

The quantity S is velocity of longitudinal displacement wave discussed by (Parfitt and Eringen 1969), T is the velocity of wave carrying a change in void volume discussed by (Puri and Cowin 1985)and H^* is a dimensionless number similar to that introduced by (Puri and Cowin 1985) and reduces to it in the absence of micropolarity. Also, equation (28) can be written as

$$\left(\tau^2 - \frac{\bar{\varpi}^2}{S^2}\right)\left(\tau^2 - \frac{\bar{\varpi}^2}{T^2} + \frac{1}{l_2^2} - \frac{i\bar{\varpi}\omega^*}{\alpha^*}\right) - \frac{N^*}{l_1^2}\tau^2 = 0,$$

$$N^* = \frac{l_2^2\,H^*}{l_1^2} \quad (29)$$

where 0 □ N^* □ 1. It can be seen that in the absence of micropolarity, the dispersion relation (29) match with the dispersion relation (24) of (Ciarletta and Sumbatyan 2003). If we put the void parameter $\beta^* = 0$, then the dispersion relation (19) yields $\tau^2 = \frac{\bar{\varpi}^2}{S^2}$, which gives the velocity of longitudinal displacement wave.

The general solution of the dispersion relation (29) is complex valued, but it admits the real solutions for high limit and low limit frequencies. Rewriting the equation (29) as given below:

$$\tau^4 - \left(\frac{\bar{\varpi}^2}{S^2} + \frac{\bar{\varpi}^2}{T^2} - \frac{1}{l_2^2} + \frac{N^*}{l_2^2} + \frac{i\bar{\varpi}\omega^*}{\alpha^*}\right)\tau^2 + \frac{\bar{\varpi}^2}{S^2}\left(\frac{\omega^2}{T^2} - \frac{1}{l_2^2} + \frac{i\bar{\varpi}\omega^*}{\alpha^*}\right) = 0 \quad (30)$$

For high frequency case ($l_2\bar{\varpi} \gg 1$), we obtain the following two roots of the equation (30) given as

$$\tau_1 = \frac{\bar{\varpi}}{S}, \tau_2 = \frac{\bar{\varpi}}{T} + \frac{\omega^*\,T}{2\alpha^*}, \quad (31)$$

Similarly for low limit frequency case ($l_2\bar{\varpi} \ll 1$), we obtain the following two roots of the equation (30) given as

$$\tau_1 = \frac{\bar{\varpi}}{S\sqrt{1 - N^*}}, \tau_2 = \frac{\bar{\varpi}}{c_4^*} + \frac{i\sqrt{1 - N^*}}{l_2}, \quad (32)$$

where

$$c_4^* = \frac{2\alpha^*\sqrt{1 - N^*}}{\omega^*\,l_2}.$$

c_4^* is the phase speed of volume fractional wave discussed by (Puri and Cowin 1985).

FORMULATION OF THE PROBLEM

Consider a two dimensional problem by taking the z-axis pointing into the lower half-space and the plane interface z=0 separating the uniform micropolar elastic solid half space M_1 [z>0] and the micropolar elastic solid half space with porous M_2 [z<0]. A longitudinal wave propagates through the medium M_1 and incident at the plane z=0 and making an angle θ_0 with normal to the surface. Corresponding to incident longitudinal wave, we get three reflected waves in the medium M_1 and four refracted waves in medium M_2.

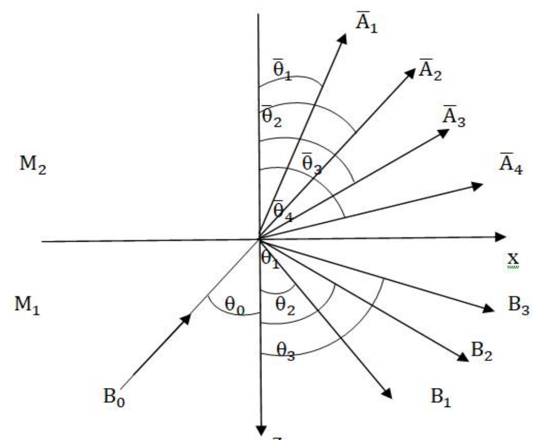

Figure 1: Geometry of the problem.

In medium M_1

$$\phi = B_0 \exp\{ik_0\,(x\sin\theta_0 - z\cos\theta_0) - i\omega_1 t\} + B_1 \exp\{ik_0\,(x\sin\theta_1 + z\cos\theta_1) - i\omega_1 t\}, \quad (33)$$

$$\psi = B_2 \exp\{ik_1(x\sin\theta_2 + z\cos\theta_2) - i\omega_2 t\} + B_3 \exp\{ik_2(x\sin\theta_3 + z\cos\theta_3) - i\omega_3 t\}, \quad (34)$$

$$\Phi_2 = EB_2 \exp\{ik_1(x\sin\theta_2 + z\cos\theta_2) - i\omega_2 t\} + FB_3 \exp\{ik_2(x\sin\theta_3 + z\cos\theta_3) - i\omega_3 t\}, \quad (35)$$

where

$$E = \frac{k_1^2\left(k_1^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno.}}, \quad (36)$$

$$F = \frac{k_2^2\left(k_2^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno.}}, \quad (37)$$

and

$$\text{deno.} = p\left(2q - \frac{\omega^2}{c_4{}^2}\right), k_1^2 = \lambda_1^2\omega^2, k_2^2 = \lambda_2^2\omega^2. \qquad (38)$$

where B_0, B_1, B_2, B_3 are amplitudes of incident longitudinal displacement wave, reflected longitudinal wave, reflected coupled transverse and reflected micro-rotation waves respectively.

In medium M_2

$$\bar{q} = \sum_{i=1,2} \bar{A}_i \bar{P}_i, \bar{U}_2 = \sum_{i=3,4} \bar{A}_i \bar{P}_i, \bar{\phi}_2 = \sum_{i=3,4} \bar{\eta}_{3,4} \bar{A}_i \bar{P}_i, \qquad (39)$$

where $\bar{P}_i = \exp\{i\bar{k}_i(x\sin\bar{\theta}_i - z\cos\bar{\theta}_i) - i\bar{\omega}_1 t\}$ and $\bar{A}_i\ (i = 1,2,3,4)$ are the amplitudes ratios, can be determined using boundary conditions at the interface z = 0. The quantities $\bar{\eta}_{3,4}$ are the coupling parameters between $\bar{U}_2$ (the y-component of vector $\bar{U}$) and $\bar{\phi}_2$, are given by Parfitt and Eringen (1969) and can be rewritten as below

$$\bar{\eta}_{3,4} = \bar{\omega}_0^2\left[\bar{V}_{3,4}^2 - 2\frac{\bar{\omega}_0^2}{\bar{k}_{3,4}^2} - \bar{c}_4^2\right]^{-1}.$$

Using equations from (22) into equations (19)-(21), we can write the requisite components of stresses and displacements into potential form. The requisite components of stresses are given by

$$\bar{t}_{zz} = (\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})\,\bar{q}_{,zz} + (2\bar{\mu} + \bar{\kappa})\,\bar{U}_{2,xz} + \bar{\lambda}\,\bar{q}_{,xx} + \beta^*\,\bar{\psi},$$

$$\bar{t}_{zx} = (2\bar{\mu} + \bar{\kappa})\,\bar{q}_{,xz} - (\bar{\mu} + \bar{\kappa})\,\bar{U}_{2,zz} + \bar{\mu}\,\bar{U}_{2,xx} - \bar{\kappa}\,\bar{\phi}_2,$$

$$\bar{m}_{zy} = \bar{\gamma}\,\bar{\phi}_{2,z}, \bar{h}_k = \alpha^*\,\bar{\psi}_{,z}. \qquad (40)$$

where $\bar{\psi} = (\nabla^2 + \bar{\xi}_2^2)\bar{q}$,

$$\bar{\xi}_1^2 = \frac{\omega^2}{\bar{c}_3^2 + 2\bar{c}_2^2}, \bar{\xi}_2^2 = \frac{\omega^2}{\bar{c}_1^2 + \bar{c}_3^2}.$$

and we consider a two dimensional problem in x-z plane by taking

$$\mathbf{u} = (u_1, 0, u_3), \Phi = (0, \bar{\phi}_2, 0), \bar{\psi} = \bar{\psi}(x, z).$$

and the requisite components of displacements are given by

$$u_1 = \bar{q}_{,x} - \bar{U}_{2,z}, u_3 = \bar{q}_{,z} - \bar{U}_{2,x}.$$

BOUNDARY CONDITIONS

For welded contact interface between micropolar elastic solid half-space and micropolar porous elastic solid half-space, the appropriate boundary conditions are continuity of force stresses, couple stresses, force vector, displacements and microrotation. Mathematically, these boundary conditions can be written as:

At the interface z = 0,

$$t_{zz} = \bar{t}_{zz}, t_{zx} = \bar{t}_{zx}, m_{zy} = \bar{m}_{zy}, \bar{h}_z = 0,$$

$$u = u_1, w = u_3, \Phi_2 = \bar{\phi}_2. \qquad (41)$$

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{\lambda_1^{-1}} = \frac{\sin\theta_3}{\lambda_2^{-1}} = \frac{\sin\bar{\theta}_1}{\bar{V}_1} = \frac{\sin\bar{\theta}_2}{\bar{V}_2} = \frac{\sin\bar{\theta}_3}{\bar{V}_3} = \frac{\sin\bar{\theta}_4}{\bar{V}_4}, \qquad (42)$$

For longitudinal wave,

$$V_0 = V_1, \theta_0 = \theta_1, \qquad (43)$$

Also at $z = 0$

$$k_0V_0 = k_1V_1 = k_2\lambda_1^{-1} = k_3\lambda_2^{-1} = \bar{k}_1\bar{V}_1 = \bar{k}_2\bar{V}_2 = \bar{k}_3\bar{V}_3 = \bar{k}_4\bar{V}_4 = \omega. \qquad (44)$$

Making the use of potentials given by equations (33)-(35) and (39) in the boundary conditions given by (41) and using (42)-(44), we get a system of seven non homogeneous equations which can be written as

$$\sum_{j=1}^{7} a_{ij}\,Z_j = Y_i, (i = 1,2,3,4,5,6,7) \qquad (45)$$

where

$$Z_1 = \frac{B_1}{B_0}, Z_2 = \frac{B_2}{B_0}, Z_3 = \frac{B_3}{B_0}, Z_4 = \frac{\bar{A}_1}{B_0}, Z_5 = \frac{\bar{A}_2}{B_0}, Z_6 = \frac{\bar{A}_3}{B_0}, Z_7 = \frac{\bar{A}_4}{B_0}, \qquad (46)$$

where Z_1 to Z_7 are the amplitude ratios of reflected longitudinal wave, reflected coupled wave (CD I)at an angle θ_2, reflected coupled-wave (CD II) at an angle θ_3, refracted longitudinal displacement wave (LD) at an angle, refracted longitudinal volume fractional wave (LVM) at an angle and two refracted sets of two coupled waves at an angle respectively. Also a_{ij} and Y_i in non-dimensional form are as

$$a_{11} = -\left\{\frac{\lambda}{\mu} + D_2\cos^2\theta_1\right\}, a_{12} = -D_2\sin\theta_2\cos\theta_2\,\frac{k_2^2}{k_0^2},$$

$$a_{13} = -D_2\sin\theta_3\cos\theta_3\,\frac{k_3^2}{k_0^2},$$

$$a_{14} = \left(\frac{2\bar{\mu} + \bar{\kappa}}{\mu}\right)\left\{\frac{\bar{k}_1^2}{k_0^2}\sin^2\bar{\theta}_1 - \frac{\bar{\xi}_1^2}{k_0^2}\right\},$$

$$a_{15} = \left(\frac{2\bar{\mu} + \bar{\kappa}}{\mu}\right)\left\{\frac{\bar{k}_2^2}{k_0^2}\sin^2\bar{\theta}_2 - \frac{\bar{\xi}_1^2}{k_0^2}\right\},$$

$$a_{16} = \left(\frac{2\bar{\mu} + \bar{\kappa}}{\mu}\right)\frac{\bar{k}_3^2}{k_0^2}\sin\bar{\theta}_3\cos\bar{\theta}_3,$$

$$a_{17} = \left(\frac{2\bar{\mu} + \bar{\kappa}}{\mu}\right)\frac{\bar{k}_4^2}{k_0^2}\sin\bar{\theta}_4\cos\bar{\theta}_4.$$

$$a_{21} = D_2\sin\theta_1\cos\theta_1,$$

$$a_{22} = -\left\{(D_1\cos^2\theta_2 - \sin^2\theta_2) - \frac{\kappa}{\mu}\frac{E}{k_2^2}\right\}\frac{k_2^2}{k_0^2},$$

$$a_{23} = -\left\{(D_1\cos^2\theta_3 - \sin^2\theta_3) - \frac{\kappa}{\mu}\frac{F}{k_3^2}\right\}\frac{k_3^2}{k_0^2},$$

$$a_{24} = \left(\frac{2\bar{\mu} + \bar{\kappa}}{\mu}\right)\frac{\bar{k}_1^2}{k_0^2}\sin\bar{\theta}_1\cos\bar{\theta}_1,$$
$$a_{25} = \left(\frac{2\bar{\mu} + \bar{\kappa}}{\mu}\right)\frac{\bar{k}_2^2}{k_0^2}\sin\bar{\theta}_2\cos\bar{\theta}_2,$$
$$a_{26} = -\left(\frac{\bar{\mu} + \bar{\kappa}}{\mu}\right)\frac{\bar{k}_3^2}{k_0^2}\sin\bar{\theta}_3\cos\bar{\theta}_3 - \frac{\bar{\kappa}\,\bar{\eta}_3}{\mu\,k_0^2},$$
$$a_{27} = -\left(\frac{\bar{\mu} + \bar{\kappa}}{\mu}\right)\frac{\bar{k}_4^2}{k_0^2}\sin\bar{\theta}_4\cos\bar{\theta}_4 - \frac{\bar{\kappa}\,\bar{\eta}_4}{\mu\,k_0^2}.$$
$$a_{31} = 0, a_{32} == E\gamma\cos\theta_2\frac{\delta_1}{k_0}, a_{33} = F\gamma\cos\theta_3\frac{\delta_2}{k_0},$$
$$a_{34} = a_{35} = 0,$$
$$a_{36} = \bar{\eta}_3\,\bar{\gamma}\cos\bar{\theta}_3\frac{\bar{k}_3}{k_0}, a_{37} = \bar{\eta}_4\,\bar{\gamma}\cos\bar{\theta}_4\frac{\bar{k}_4}{k_0}.$$
$$a_{41} = a_{42} = a_{43} = a_{46} = a_{47} = 0,$$
$$a_{44} = \frac{\bar{k}_1}{k_0}\left(\bar{k}_1^2 - \bar{\xi}_2^2\right)\cos\bar{\theta}_1,$$
$$a_{45} = \frac{\bar{k}_2}{k_0}\left(\bar{k}_2^2 - \bar{\xi}_2^2\right)\cos\bar{\theta}_2.$$
$$a_{51} = -\sin\theta_1, a_{52} = \frac{k_2}{k_0}\cos\theta_2, a_{53} = \frac{k_3}{k_0}\cos\theta_3,$$
$$a_{54} = \frac{\bar{k}_1}{k_0}\sin\bar{\theta}_1, a_{55} = \frac{\bar{k}_2}{k_0}\sin\bar{\theta}_2,$$
$$a_{56} = \frac{\bar{k}_3}{k_0}\cos\bar{\theta}_3, a_{57} = \frac{\bar{k}_4}{k_0}\cos\bar{\theta}_4$$
$$a_{61} = -\cos\theta_1, a_{62} = \frac{k_2}{k_0}\sin\theta_2,$$
$$a_{63} = \frac{k_3}{k_0}\sin\theta_3, a_{64} = \frac{\bar{k}_1}{k_0}\cos\bar{\theta}_1,$$
$$a_{65} = \frac{\bar{k}_2}{k_0}\cos\bar{\theta}_2, a_{66} = -\frac{\bar{k}_3}{k_0}\sin\bar{\theta}_3,$$
$$a_{67} = -\frac{\bar{k}_4}{k_0}\sin\bar{\theta}_4.$$
$$a_{71} = 0, a_{72} = -E, a_{73} = -F, a_{74} = a_{75} = 0,$$
$$a_{76} = \bar{\eta}_3,$$
$$a_{77} = \bar{\eta}_4, Y_7 = a_{71}. \tag{47}$$
$$Y_1 = a_{11}, Y_2 = a_{12}, Y_3 = a_{31}, Y_4 = a_{41}, Y_5 = -a_{51}.$$
$$Y_6 = a_{61}$$
where $D_1 = 1 + \frac{\lambda}{\mu}, D_2 = 1 + D_1$.

NUMERICAL RESULTS AND DISCUSSION

The theoretical results obtained above indicate that the amplitudes ratios $Z_i(i = 1,2,3,4,5,6,7)$ depend on the angle of incidence of incident wave and elastic properties of half spaces. In order to study in more detail the behavior of various amplitudes ratios. Following Gauthier (1982), the physical constants for micropolar elastic solid are

$$\lambda = 7.59 \times 10^{11}\text{dyne}/\text{cm}^2, \mu = 1.89 \times 10^{11}\text{dyne}/\text{cm}^2,$$
$$\kappa = 0.0149 \times 10^{11}\text{dyne}/\text{cm}^2, \rho = 2.19\text{gm}/\text{cm}^3$$
$$\gamma = 0.0268 \times 10^{11}\text{ dyne}, j = 0.0196\text{ cm}^2,$$
$$\frac{\omega^2}{{\omega_0}^2} = 20. \tag{48}$$

For a particular modal microplar elastic solid with porous, the physical constants are given as

$$\bar{\lambda} = 5.5 \times 10^{11}\text{dyne}/\text{cm}^2, \bar{\mu} = 2.14 \times 10^{11}\text{dyne}/\text{cm}^2,$$
$$\bar{\kappa} = 0.129 \times 10^{11}\text{dyne}/\text{cm}^2, \bar{\gamma} = 1.88 \times 10^{11}\text{dyne}/\text{cm}^2.$$
$$\bar{j} = 0.0166\text{ cm}^2, \bar{\rho} = 2.2\frac{\text{gm}}{\text{cm}^3},$$
$$\xi^* = 10 \times 10^{11}\text{dyne}/\text{cm}^2,$$
$$\beta^* = 8 \times 10^{11}\text{dyne}/\text{cm}^2,\ \omega^* = 0.01 \times 10^{11}\text{dyne}/\text{cm}^2,$$
$$\alpha^* = 0.002 \times \frac{10^{11}\text{dyne}}{\text{cm}^2}, \tag{49}$$

A computer programme in MATLAB has been developed to calculate the modulus of amplitude ratios of various reflected and refracted waves for the particular model and to depict graphically. In figures (2) – (15), solid lines shows the variations of amplitude ratios of reflected and refracted waves when medium-I is micropolar elastic solid (MPES) and medium-II micropolar elastic solid with porous. Figures (16) – (17) solid lines shows the variations of amplitude ratios of reflected waves when medium-I micropolar elastic solid becomes a free surface.

For the case of high frequency:

In figures (2), we have plotted the modulus of amplitude ratios ($|Z_i|$, i = 1, 3) of reflected waves against the angle of incidence. The amplitude ratio $|Z_3|$ corresponding to the set of coupled transverse (CD II) wave propagating with speed λ_2^{-1} are quite small in comparison to the amplitude $|Z_1|$ with the frequency $\bar{\omega}$=500rad/s and thus amplitude ratios sharply increase at the angle 18^o and then sharply decrease with the angle of incidence.

In figures (3), we have plotted the modulus of amplitude ratios $|Z_2|$ of the reflected (CD I) wave against the angle of incidence. The amplitude ratio $|Z_2|$ corresponding to the reflected (CD I) wave propagating with speed λ_1^{-1} and with the frequency $\bar{\omega}$=500rad/s. Amplitude ratios sharply increase at the angle 18^o and then sharply decrease with the angle of incidence.

Figures (4) – (6), shows the variation of modulus of amplitude ratios ($|Z_i|$, i = 4, 5, 6, 7) of refracted waves against the angle of incidence, when a (LD) wave is incident obliquely at the interface with the frequency $\bar{\omega}$=500rad/s. The variation of modulus of amplitude ratios ($|Z_i|$, i = 4, 5, 6, 7) of refracted waves is similar as in the later case.

For the case of low Frequency:

In figures (7), we have plotted the modulus of amplitude ratios ($|Z_i|$, i = 1, 3) of reflected waves against the angle of incidence. The amplitude ratio $|Z_3|$ corresponding to the set of coupled transverse (CD II) wave propagating with speed λ_2^{-1} are quite small in comparison to the amplitude $|Z_1|$ with the frequency $\bar{\varpi}$=10rad/s and thus amplitude ratios sharply increase at the angle 18^o and then sharply decrease with the angle of incidence.

In figures (8), we have plotted the modulus of amplitude ratios $|Z_2|$ of the reflected (CD I) wave against the angle of incidence. The amplitude ratio $|Z_2|$ corresponding to the reflected (CD I) wave propagating with speed λ_1^{-1} and with the frequency $\bar{\varpi}$=10rad/s. Amplitude ratios sharply increase at the angle 18^o and then sharply decrease with the angle of incidence. Figures (9) – (11), shows the variation of modulus of amplitude ratios ($|Z_i|$, i = 4, 5, 6, 7) of refracted waves against the angle of incidence, when a (LD) wave is incident obliquely at the interface with the frequency $\bar{\varpi}$=10rad/s. The variation of modulus of amplitude ratios ($|Z_i|$, i = 4, 5, 6, 7)) of refracted waves is similar as in the later case.

In figures (12), depicts the variations of modulus of amplitude ratios ($|Z_i|$, i = 1, 3) of reflected waves against the angle of incidence. When the micropolarity approaches to zero then the amplitude ratio $|Z_3|$ corresponding to the set of coupled transverse wave propagating with speed λ_2^{-1} are quite small in comparison to the amplitude ratio $|Z_1|$ with the frequency $\bar{\varpi}$=10rad/s and thus amplitude ratios sharply increase at the angle 18^o and then sharply decrease with the angle of incidence. In figure (13), depicts the variations of modulus of amplitude ratio $|Z_2|$ of reflected (CD I) wave against the angle of incidence. When the micropolarity approaches to zero and then the amplitude ratio $|Z_2|$ corresponding to reflected (CD I) wave propagating with speed λ_1^{-1} with the frequency $\bar{\varpi}$=10rad/s. The curve of this amplitude ratio attains its maximum value at angle of incidence between 40^o and 50^o and after that decrease to its minima. Figure (14), shows the variation of modulus of amplitude ratios ($|Z_i|$, i = 4, 6, 7) of refracted waves against the angle of incidence. A longitudinal wave is incident obliquely at the interface when the micropolarity approaches to zero and with the frequency $\bar{\varpi}$=10rad/s. The amplitude ratio $|Z_7|$ corresponding to the set of refracted coupled transverse (CD II) wave propagating with speed $\bar{V}_4$ are quite large in comparison to the other amplitude ratios with the frequency $\bar{\varpi}$=10rad/s and thus amplitude ratios increase at the angle 18^o and then sharply decrease with the angle of incidence. In figure (15), we have plotted the modulus of amplitude ratios $|Z_5|$ of the refracted volume fractional wave against the angle of incidence. The amplitude ratio $|Z_5|$ corresponding to the reflected volume fractional wave propagating with speed $\bar{V}_2$ and with the frequency $\bar{\varpi}$=10rad/s. Amplitude ratio attains its maximum value at the angle 0^o and after that decrease monotonically with the angle of incidence. Figure (16) shows the variation of the modulus of the amplitude ratios ($|Z_1|$ and $|Z_3|$) of reflected LD wave and reflected CD II wave respectively at free surface of micropolar elastic solid (MPES) in medium-I. In figure (16), the amplitude ratio $|Z_1|$ first decrease to their minimum value at the angle 60^o and after that getting maximum value and the amplitude ratio $|Z_3|$, first smoothly increase to their maximum value at the angle 50^o and after that smoothly decrease to its minimum value. In figure (17) the amplitude ratio first rapidly increases to their maximum value at the angle of incidence *10*^{o} and after that rapidly decrease at the angle of incidence near by *13*^{o} and after all gradually approach to its minimum value.

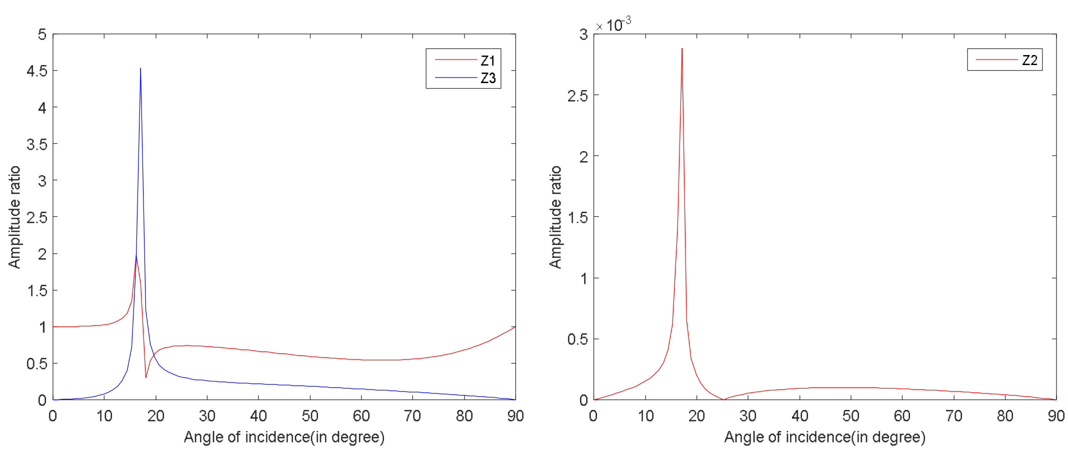

Figure 2-3: (High frequency case) Variation of modulus amplitudes ratios $|Z_i|$, ($i = 1, 2, 3$) with angle of incidence of the incident LD wave

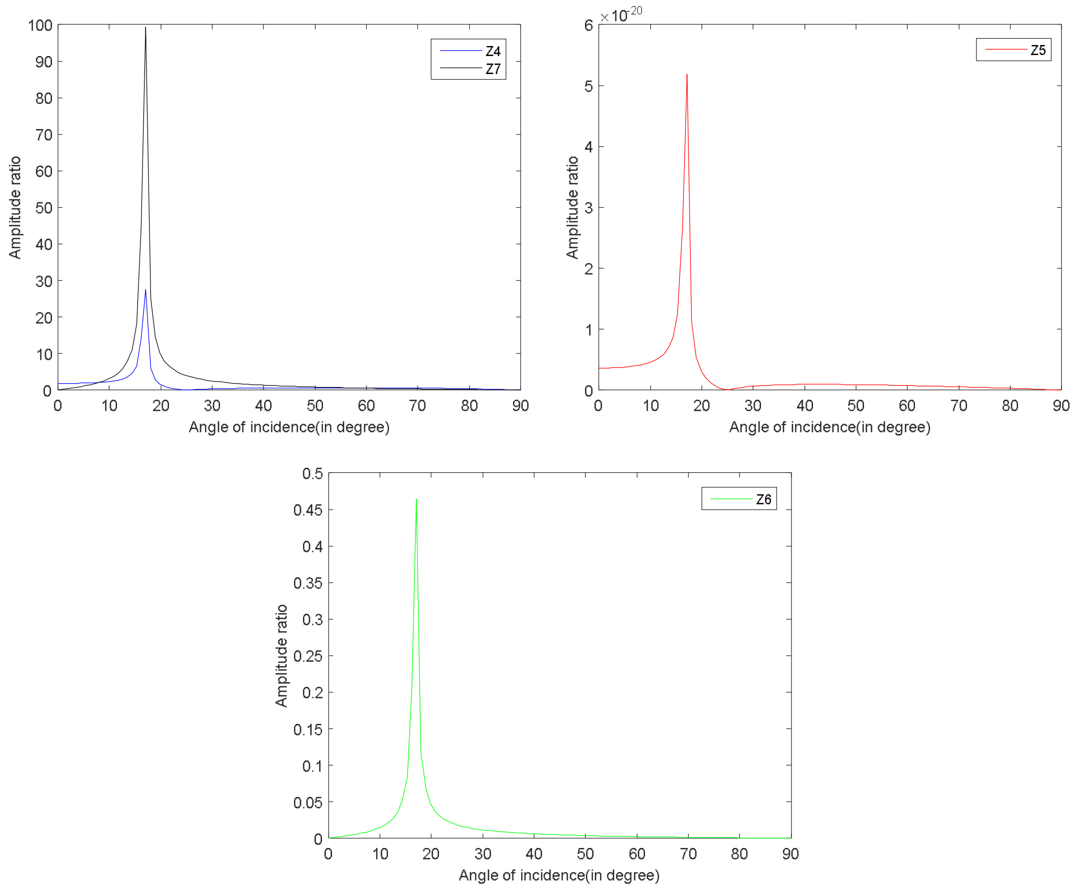

Figure 4-6: (High frequency case) Variation of modulus amplitudes ratios $|Z_i|$, $(i = 4,5,6,7)$ with angle of incidence of the incident LD wave.

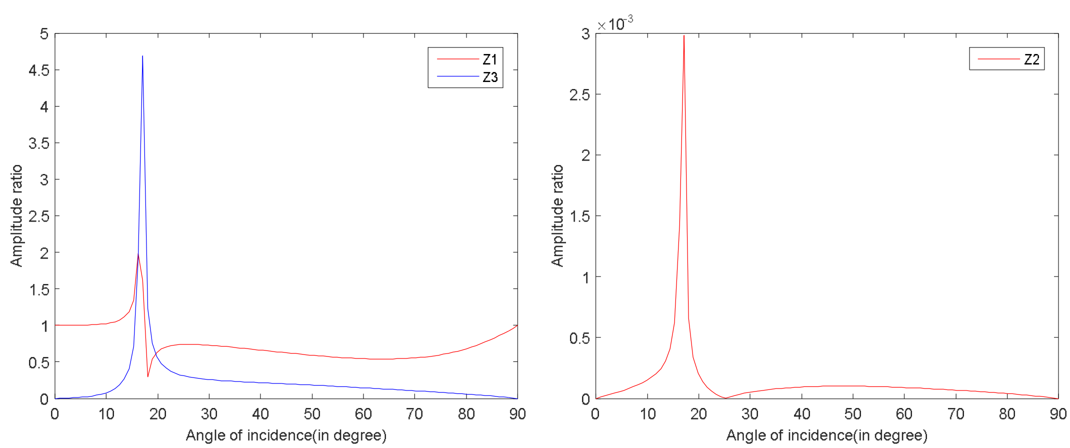

Figure 7-8: (Low frequency case) Variation of modulus amplitudes ratios $|Z_i|$, $(i = 1, 2, 3)$ with angle of incidence of the incident LD wave.

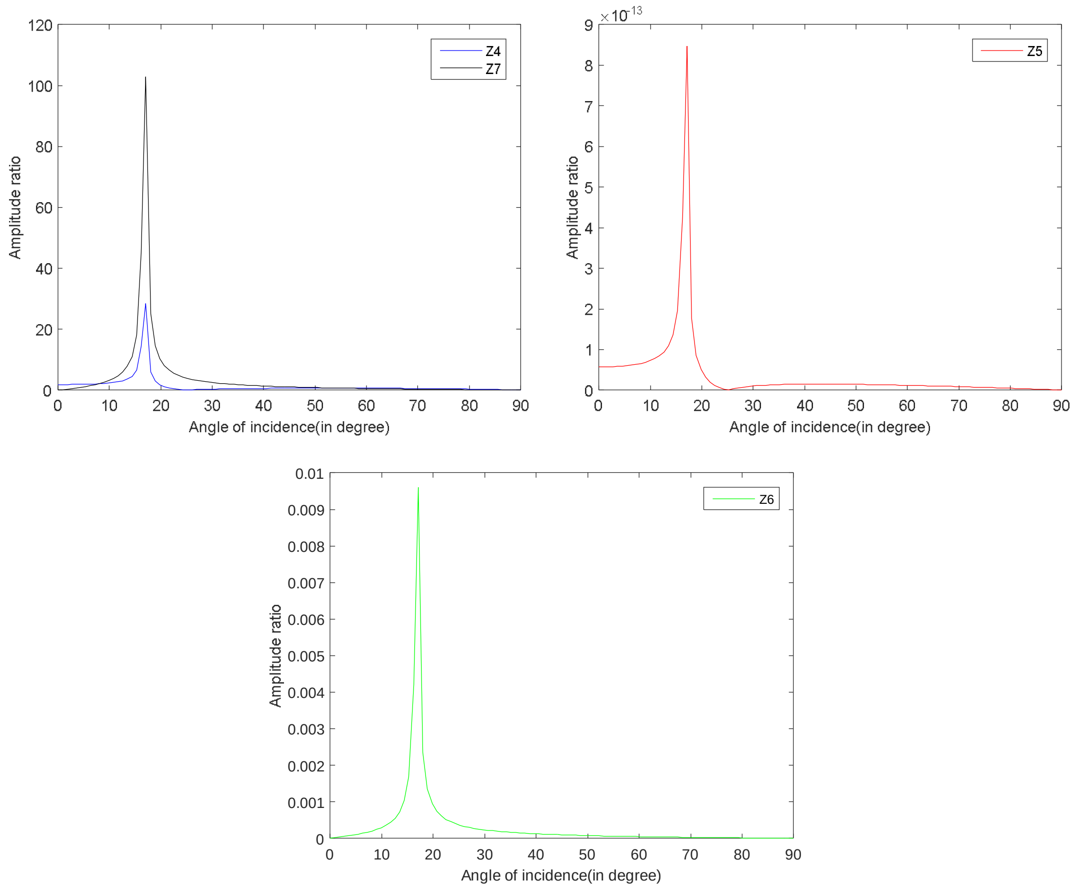

Figure 9-11: (Low frequency case) Variation of modulus amplitudes ratios $|Z_i|$, ($i = 4,5,6,7$) with angle of incidence of the incident LD wave.

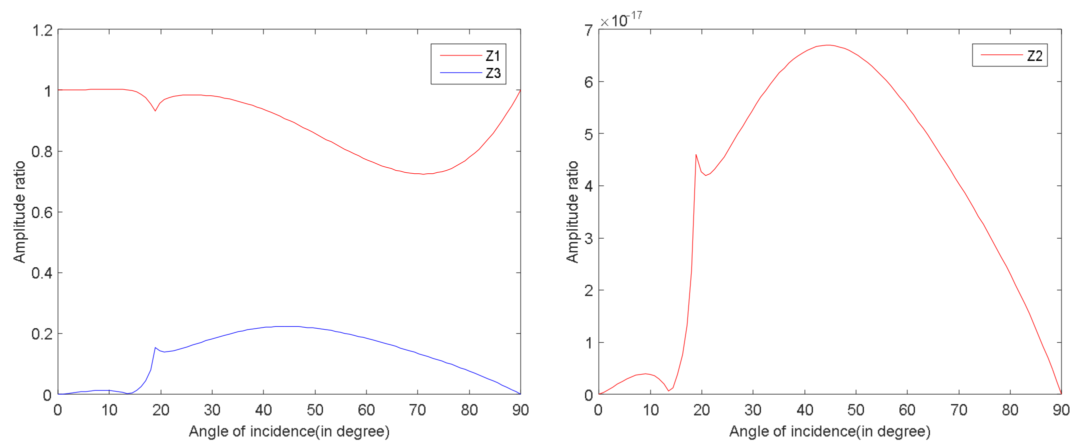

Figure 12-13: (Low frequency case) Variation of modulus amplitudes ratios $|Z_i|$, ($i = 1, 2, 3$) with angle of incidence of the incident LD wave (micropolarity approaches to zero).

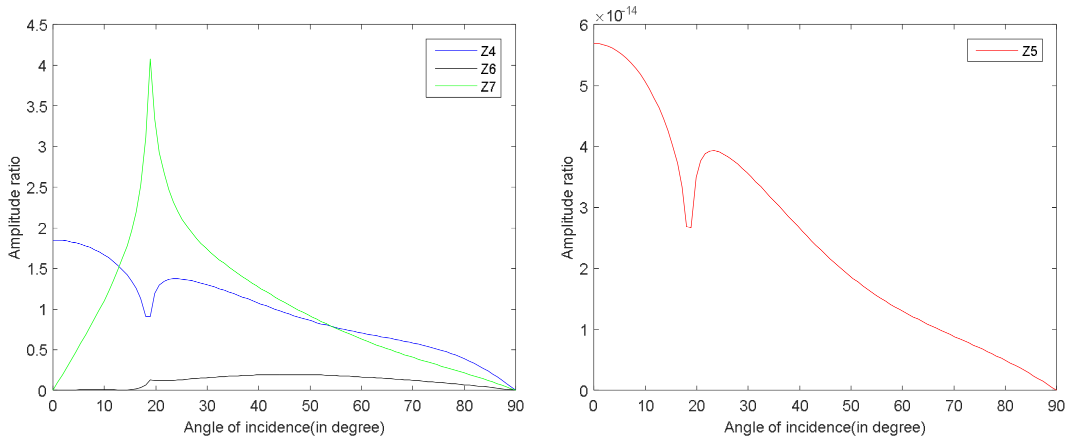

Figure 14-15: (Low frequency case) Variation of modulus amplitudes ratios $|Z_i|$, $(i = 4, 5, 6, 7)$ with angle of incidence of the incident LD wave (micropolarity approaches to zero).

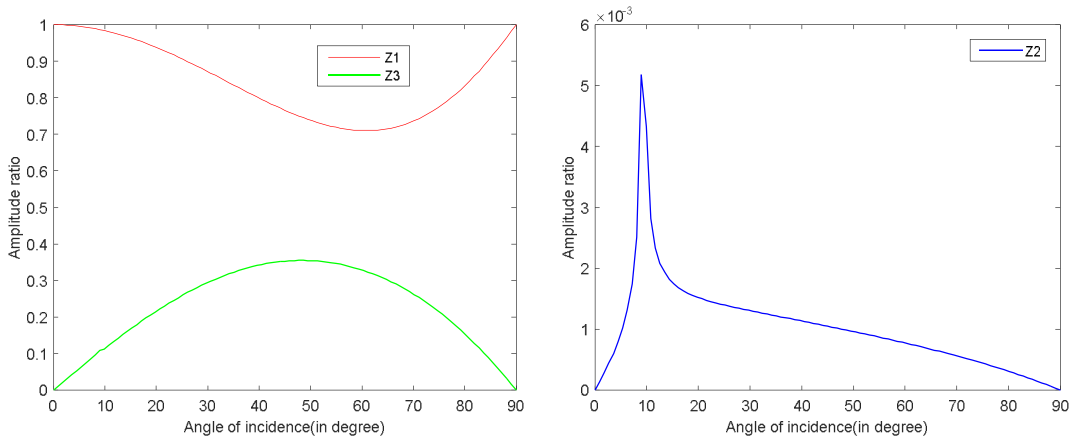

Figure 16-17: Variation of modulus amplitudes ratios $|Z_i|$, $(i = 1, 2, 3)$ with angle of incidence of the incident LD wave (free surface)

CONCLUSION

In this paper, a mathematical study of reflection and refraction of incident longitudinal displacement wave at plane interface between micropolar elastic solid half space and micropolar porous elastic solid half space in perfect contact. Making the use of appropriate set of boundary conditions, the system of simultaneous equations giving the amplitudes of various reflected and refracted waves are obtained.

a. The amplitudes of various reflected and refracted waves are found to be complex valued.
b. The modulus of amplitudes of various reflected and refracted waves depend upon angle of incidence, frequency and elastic properties of materials of the medium.
c. Maximum amount of incident energy is carried along the reflected and refracted longitudinal displacement wave.
d. Reflection phenomena of an incident LD wave at free plane boundary of a micropolar elastic solid half – space have been investigated.
e. For limit high and low frequency cases, the void volume fractional wave is more influenced by the micropolarity of the medium.

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