Hydromagnetic Oscillatory Flow through a Porous Channel in the Presence of Hall Current with Variable Suction and Permeability

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Research Article

Abstract: An exact solution of hydromagnetic, oscillatory flow of an incompressible, electrically conducting and viscous fluid in horizontal porous channel embedded with porous medium of time dependent permeability is obtained in the presence of hall current. A magnetic field of uniform strength is applied along an axis perpendicular to the plane of the plate about which the entire system rotates. The effects of various parameters on the exact solution so obtained are discussed with the help of graphs and tables in detail.

Mathematical Subject Classification: 76A05

Keywords*:* Hydromagnetic, oscillatory, rotating, porous medium, hall current.

Introduction:

Channels are frequently used in various applications in designing ventilating and heating of buildings, cooling electronic components, drying several types of agriculture products grain and food, and packed bed thermal storage. Convective flows in channels driven by temperature differences of boundary walls have been studied and reported, extensively in literature. On the other hand flows of fluid through porous medium are very important particularly in the fields of agriculture engineering for irrigation processes; in petroleum technology to study petroleum transport; in chemical engineering for filtration and purification processes. Abdussattar [1] and Ahmed [3] have studied free convection and mass transfer flow of a viscous fluid through porous medium. In most of these studied the permeability of the porous medium is assumed to be constant while the porosity of medium may not necessarily be constant because porous material containing the fluid is a non homogenous medium and there can be numerous in homogeneities present in a porous medium. Acharya *et al* [2] have analyzed free convection and mass transfer in steady flow through porous medium with constant suction in the presences of magnetic field. A number of investigation have been made by Raptis *et al* [9-11] into steady two – dimensional flow past a vertical walls of constant – permeability of porous medium. Notable among them are Attia and Kotb [4], Crammer and Pai[5], Ferraro and Plumpton [6], Shercliff [12] investigated a two-dimensional MHD flow between two porous parallel, insulated horizontal plates and the heat transfer through when the lower plate is kept stationary and upper plate is moving with uniform velocity. Singh *et al* [14] investigated a three –dimensional fluctuating flow through a porous medium when the permeability various both in time and space. Singh *et al* [13] have discussed hydromagnetic free convective and mass transfer flow of a viscous stratified fluid considering variation in permeability with direction. Further the flow of electrically conducting fluids in channel and pipes under the effect of transverse magnetic field occur in magnetohydrodynamic (MHD) generators, accelerators, pump and flow meters. In view of these and many others important applications of these types of flows a number of scholars have shown their interest.

There are other industrial applications of flows of electrically conducting fluids in the fields of geothermal system, nuclear reactor, filtration etc, where the conducting fluid flows through porous medium which rotates about an axis. In view of the importance of rotating flows a number of studies appeared in literatures. Mazumder [8] studied an oscillatory Ekman boundary layer flow bounded by horizontal plates one of which oscillate and other is at rest. Singh and Kumar [15] studied an oscillatory MHD flow through a porous medium bounded by rotating porous channel in the presences of hall current by taking into consideration the constant suction and permeability.

In this paper we present an investigation of the effect of magnetic field, hall current and rotation on a viscous incompressible and electrically conducting fluid in a porous horizontal channel filled with porous material by considering periodic suction and permeability.

Mathematical Analysis:

Consider an unsteady flow of an electrically conducting, viscous, incompressible fluid through a porous medium bounded between two insulated infinite parallel plates' distances d apart. Choose the origin at the lower plate lying in $x^* - y^*$plane and x^*-axis parallel to the direction of motion of the upper plate. A strong magnetic field of uniform strength H_0 is applied along z^*-axis taken perpendicular to the planes of the plates. The entire system rotates with angular velocity Ω^*about z^* -axis. The magnetic Reynolds number is considered to be small so that the induced magnetic field is neglected. All physical quantities depend on $z^*$and t^* for this problem of fully developed laminar flow.

The equation of continuity $\nabla . V = 0$ gives on integration $w^* = -w_0\left(1 + \varepsilon e^{i\omega t}\right)$ at z^* where $V = (u^*, v^*, w^*)$and solenoidal relation for the magnetic field $\nabla . H = 0$ gives $H_z^* = H_0$(constant) everywhere in the flow field. The physical configuration of the problem is shown in Figure 1.

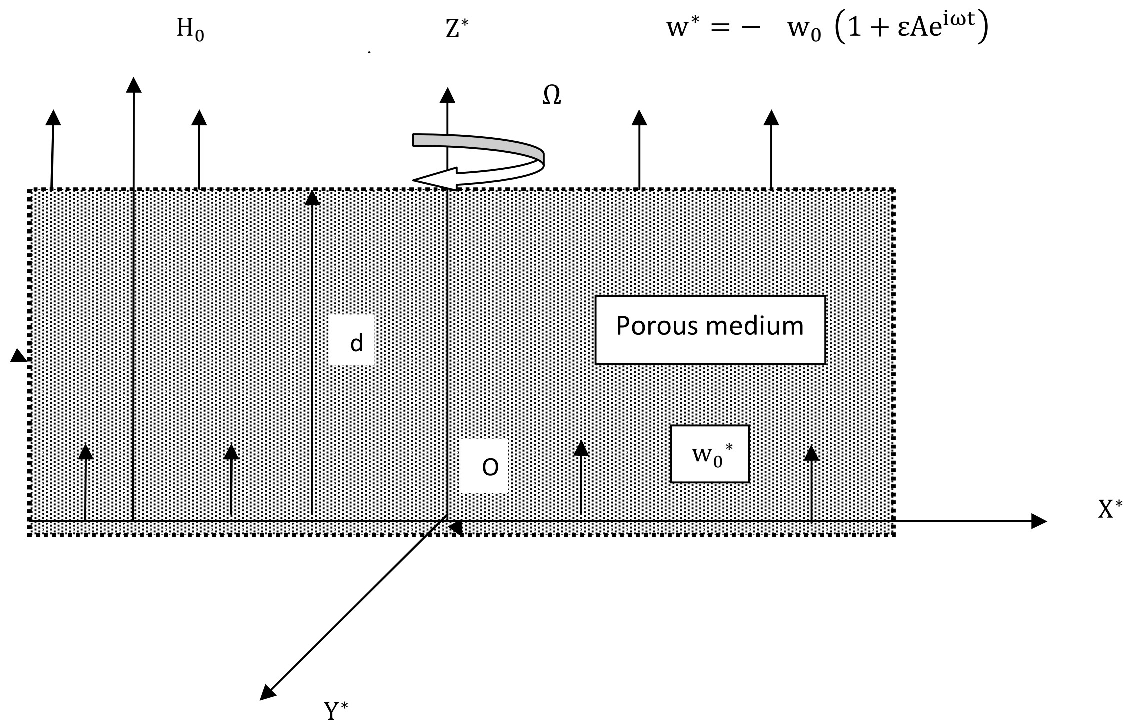


Figure 1: The physical configuration of the problem

Basic Equation:

The equation of conservation of electric charge $\nabla . J = 0$ gives J_z^*= constant. This constant is zero i.e. $J_z^* = 0$ at the plates which are electrically non-conducting. Taking Hall current into account the generalized Ohm's law is of the form

$$J + \frac{\omega_e \tau_e}{H_0} J \times H = \sigma(E + \mu_e V \times H) \quad (1)$$

Where V is the velocity vector ,H is the magnetic field J,is the current density, E is the electric field, σ is the electric conductivity, μ_e is the magnetic permeability, ω_e is the cyclotron frequency and τ_e is the electron collision time.

For very large magnetic field, the x^* and y^* components of Ohm's law (1), which include Hall current, are

$$J_x^* + \omega_e \tau_e J_y^* = \sigma(E_x^* + \mu_e H_0 v^*) \quad (2)$$

$$J_y^* + \omega_e \tau_e J_x^* = \sigma\left(E_y^* - \mu_e H_0 u^*\right) \quad (3)$$

Since the external electric field arising due to polarization of charges is negligible.

Hence$E_x^* = E_y^* = 0$ Therefore, solving for J_x^* and J_y^*, we get

$$J_x^* = \frac{\sigma\mu_e H_0(mu^*+v^*)}{(1+m^2)} \text{ and } J_y^* = \frac{\sigma\mu_e H_0(mv^*-u^*)}{(1+m^2)} \quad (4)$$

Thus within the frame work of these assumptions and making use of equation(4),the flow in the presence of Hall current by the following equations:

$$\frac{\partial u^*}{\partial t^*} + w^*\frac{\partial u^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu\frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega^* v^* + \frac{\sigma H_0^2(mv^*-u^*)}{\rho(1+m^2)} - \frac{\nu u^*}{K^*} \quad (5)$$

$$\frac{\partial v^*}{\partial t^*} + w^*\frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu\frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^* u^* + \frac{\sigma H_0^2(mu^*+v^*)}{\rho(1+m^2)} - \frac{\nu v^*}{K^*} \quad (6)$$

where ρ is the fluid density $t^*$is time K^* is the permeability of the porous medium. $m = \omega_e\tau_e$, is the Hall parameter. The permeability is taken in the form $K^* = K_0(1+\epsilon B e^{i\omega^* t^*})$.

The boundary conditions of the problem are

$$\left.\begin{array}{c} u^* = v^* = 0, \\ w^* = -w_0\left(1+\epsilon A e^{i\omega^* t^*}\right), \quad \text{at } z^* = 0 \\ \text{and} \\ u^* = U^*(t^*) = U_0(1+\varepsilon\cos\omega^* t^*), v^* = 0 \\ w^* = -w_0\left(1+\epsilon A e^{i\omega^* t^*}\right), \quad \text{at } z^* = d \end{array}\right\} \quad (7)$$

The flow in presences of Hall current is governed by the following equations

$$\frac{\partial u^*}{\partial t^*} - w_0\left(1+\varepsilon A e^{i\omega^* t^*}\right)\frac{\partial u^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu\frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega^* v^* \frac{\sigma H_0^2(mv^*-u^*)}{\rho(1+m^2)} \quad \frac{\nu u^*}{K_0(1+\epsilon B e^{i\omega^* t^*})} \quad (8)$$

$$\frac{\partial v^*}{\partial t^*} - w_0\left(1+\varepsilon A e^{i\omega^* t^*}\right)\frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu\frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^* u^* + \frac{\sigma H_0^2(mu^*+v^*)}{\rho(1+m^2)} - \frac{\nu v^*}{K_0(1+\epsilon B e^{i\omega^* t^*})} \quad (9)$$

Introducing the following non dimensional quantities

$\eta = \frac{z^*}{d}$ $\quad t = \omega^* t^*$ $\quad u = \frac{u^*}{U_0}$ $\quad v = \frac{v^*}{U_0}$ $\quad \omega = \frac{\omega^* d^2}{\upsilon}$ is the frequcency of oscillations $\lambda = \frac{w_0 d}{\nu}$ is the injection and suction parameter, $M = H_0 d\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, $\Omega = \frac{\Omega^* d^2}{\nu}$ is the rotation parameter ,$K = \frac{K_0}{d^2}$ is the permeability of the porous medium.

Eliminating the modified pressure gradient under the usual boundary layer approximation, equation (8) and (9) become

$$\frac{\partial u^*}{\partial t^*} - w_0\left(1+\varepsilon A e^{i\omega^* t^*}\right)\frac{\partial u^*}{\partial z^*} = \nu\frac{\partial^2 u^*}{\partial z^{*2}} + \frac{dU^*}{dt^*} + 2\Omega^* v^* + \frac{\sigma H_0^2(mv^*-u^*+U^*)}{\rho(1+m^2)} - \frac{\upsilon(u^*-U^*)}{K_0(1+\epsilon B e^{i\omega^* t^*}} \quad (10)$$

$$\frac{\partial v^*}{\partial t^*} - w_0\left(1+\varepsilon A e^{i\omega^* t^*}\right)\frac{\partial v^*}{\partial z^*} = \nu\frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^*(u^*-U^*) + \frac{\sigma H_0^2(mu^*+v^*-mU^*)}{\rho(1+m^2)} - \frac{\upsilon v^*}{K_0(1+\epsilon B e^{i\omega^* t^*})} \quad (11)$$

Using the non dimensional quantities the equation (10) and (11) become

$$\omega\frac{\partial u}{\partial t} - \lambda\left(1+\epsilon A e^{it}\right)\frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2} + \omega\frac{dU}{dt} + 2\Omega v + \frac{M^2}{1+m^2}(mv-u+U) - \frac{(u-U)}{K(1+\varepsilon B e^{it})} \quad (12)$$

$$\omega\frac{\partial v}{\partial t} - \lambda\left(1+\epsilon A e^{it}\right)\frac{\partial v}{\partial \eta} = \frac{\partial^2 v}{\partial \eta^2} - 2\Omega(u-U) - \frac{M^2}{1+m^2}(mu+v-mU) - \frac{v}{K(1+\varepsilon B e^{it})} \quad (13)$$

The corresponding transformed boundary condition becomes

$$\left.\begin{array}{c} u = v = 0 \quad \eta = 0 \\ \text{and} \\ u = U(t) = 1+\varepsilon e^{it} \quad v = 0 \quad \eta = 1 \end{array}\right\} \quad (14)$$

Equation (12) and (13) can be combined into a single equation by using the complex velocity of the form

$$q = u + iv \quad (15)$$

$$\omega\frac{\partial q}{\partial t} - \lambda\left(1+\epsilon A e^{it}\right)\frac{\partial q}{\partial \eta} = \frac{\partial^2 q}{\partial \eta^2} + \omega\frac{dU}{dt} - \frac{(q-U)}{K(1+\varepsilon B e^{it})} - (q-U)\,S \quad (16)$$

The boundary conditions in complex notation is given by

$$\left.\begin{array}{lcr} q = 0 & & \eta = 0 \\ & \text{at} & \\ q = U(t) = 1+\varepsilon e^{it} & & \eta = 1 \end{array}\right\} \quad (17)$$

Method of Solution:

$q = q_0(\eta, t) + \epsilon e^{it} q_1(\eta, t)$ (18)

Substituting (18) into (16) and comparing harmonic and non harmonic part we have

$$q_0''(\eta) + \lambda q_0'(\eta) - q_0\left(\frac{1}{K} + S\right) = \left(\frac{-1}{K} + S\right) \qquad (19)$$

$$q_1''(\eta) + \lambda q_1'(\eta) - \left(S + \frac{1}{K} + i\omega\right) q_1 = -\left(Bq_0''(\eta) + \lambda Bq_0'(\eta) + \lambda Aq_0'(\eta) - SBq_0(\eta)\right) - (S + 1/K + i\omega + SB)$$

(20)

With corresponding transformed boundary condition

$$\left.\begin{array}{lll} q_0 = q_1 = 0 & & \eta = 0 \\ & \text{at} & \\ q_0 = q_1 = 1 & & \eta = 1 \end{array}\right\} \qquad (21)$$

The solution of the equation (19) and (20) under the boundary condition (21) are obtained as:

$$q_0(\eta) = 1 - \frac{e^{A_2} e^{A_1\eta}}{e^{A_2} - e^{A_1}} + \frac{e^{A_1} e^{A_2\eta}}{e^{A_2} - e^{A_1}} \qquad (22)$$

$$q_1 = \frac{1}{e^{A_4} - e^{A_3}}\Big[A_{13} e^{A_1\eta}\left(e^{A_4} - e^{A_3}\right) + A_{14} e^{A_2\eta}\left(e^{A_4} - e^{A_3}\right) + A_{15}\left(e^{A_4} - e^{A_3}\right) + e^{A_4\eta}\left\{1 - A_{13}\left(e^{A_1} - e^{A_3}\right) - A_{14}\left(e^{A_2} - e^{A_3}\right) - A_{15}\left(1 - e^{A_3}\right)\right\} - e^{A_3\eta}\left\{1 + A_{13}\left(e^{A_4} - e^{A_1}\right) + A_{14}\left(e^{A_4} - e^{A_2}\right) + A_{15}\left(e^{A_4} - 1\right)\right\}\Big] \qquad (23)$$

Results and Discussion:

Now for the resultant velocities and the shear stresses of the steady and unsteady flow, we write

$u_0(\eta) + iv_0(\eta) = q_0(\eta)$ (24)

$u_1(\eta) + iv_1(\eta) = q_1(\eta)e^{it}$ (25)

The solution corresponds to the steady part which u_0 as the primary and v_0 as the secondary velocity components. The amplitude and the phase differences due to these primary and secondary velocities for the steady flow are given by

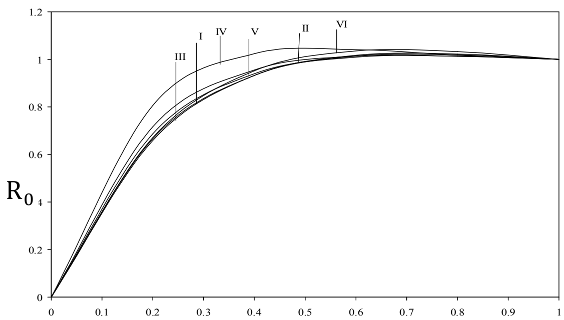
$R_0 = \sqrt{u_0^2 + v_0^2}$ $\quad \phi_0 = \tan^{-1}(\frac{v_0}{u_0})$ (26)

The resultant velocity or amplitude and the phase differences of the unsteady flow are given by

$R_1 = \sqrt{u_1^2 + v_1^2}$ $\quad \phi_1 = \tan^{-1}(\frac{v_1}{u_1})$ (27)

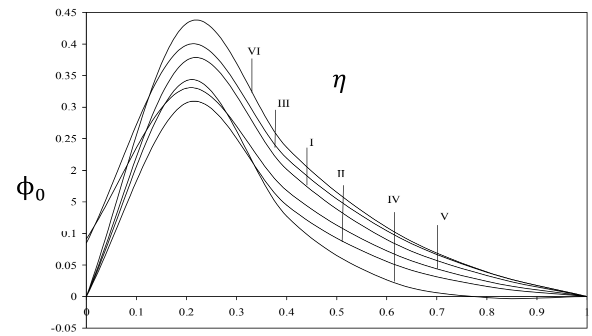
The resultant velocities R_0 and $R_1$and the phase angle ϕ_0, $\phi_1$for the steady and unsteady part of the flow are respectively shown graphically in figures 2 to 5. It is observed from figures 2 and 4 that R_0 and R_1 both increase rapidly from zero near the stationary plate and then approach unity in the form damped oscillations. Figure 2 and 4 for the steady and unsteady resultant velocities R_0 and R_1 respectively shown. The resultant velocities R_0 decrease slightly with the increase value of hall parameter m and it increase with the increase value of suction parameter λ, permeability k and the rotation parameter Ω, the Hartman number M. Unsteady resultant velocities R_1 decrease with the increase value of frequency of oscillation ω and increase with the increase value of hall parameter m, suction parameter λ, permeability K , the rotation parameter Ω and the Hartmann number M.

It is also found from figure 3 and 5 that the phase differences ϕ_0 increase quickly from zero at the stationary plate and then approach zero again at the moving plate in the form of damped oscillations. Particularly for large values of rotations a phase lag is also observed for both steady and unsteady phase angles ϕ_0 and ϕ_1. The phase differences ϕ_0 increase with the increasing value of hall parameter m and permeability K and the phase differences ϕ_0 decreases with the increasing value of Hartmann number M, suction parameter λ and rotation parameter Ω. The phase differences ϕ_1 is increase in the boundary and decrease at the middle of the channel and then increase at the boundary of the channel. The phase differences $\phi_1$increase with the increasing of hall parameter m, rotation parameter Ω, suction parameter λ, frequcency of oscillation ω and permeability K in the beginning and in the middle it decrease and than again a phase lag occur and than at the boundary it again increase and with the decreasing value of A and B the phase differences decrease in the starting and in the middle of the channel it increase and then it again decrease and than a phase lag occur and then it again increase at the boundary of the channel.



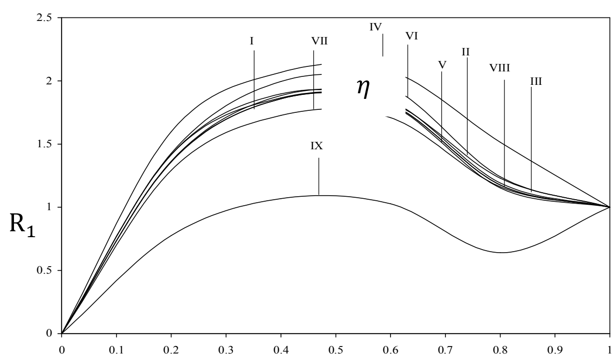
M	m	Ω	λ	K	Curve
2	1	10	0.2	0.2	I
4	1	10	0.2	0.2	II
2	3	10	0.2	0.2	III
2	1	20	0.2	0.2	IV
2	1	10	1	0.2	V
2	1	10	0.2	1	VI

Figure 2: The Resultant velocity R_0 due to u_0 and v_0.



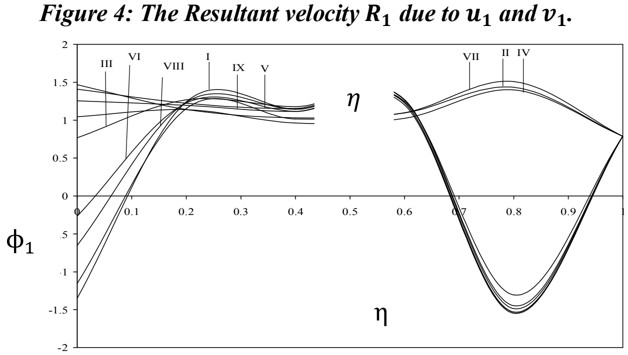
M	m	Ω	λ	K	Curve
2	1	10	0.2	0.2	I
4	1	10	0.2	0.2	II
2	3	10	0.2	0.2	III
2	1	20	0.2	0.2	IV
2	1	10	1	0.2	V
2	1	10	0.2	1	VI

Figure 3: Phase angle ϕ_0 due to u_0 and v_0.



M	m	Ω	λ	K	ω	A	B	Curve
2	1	10	0.2	0.2	5	0.5	0.5	I
4	1	10	0.2	0.2	5	0.5	0.5	II
2	3	10	0.2	0.2	5	0.5	0.5	III
2	1	20	0.2	0.2	5	0.5	0.5	IV
2	1	10	1	0.2	5	0.5	0.5	V
2	1	10	0.2	1	5	0.5	0.5	VI
2	1	10	0.2	0.2	15	0.5	0.5	VII
2	1	10	0.2	0.2	5	0	0.5	VIII
2	1	10	0.2	0.2	5	0.5	0	IX

Figure 4: The Resultant velocity R_1 due to u_1 and v_1.



M	m	Ω	λ	K	ω	A	B	Curve
2	1	10	0.2	0.2	5	0.5	0.5	I
4	1	10	0.2	0.2	5	0.5	0.5	II
2	3	10	0.2	0.2	5	0.5	0.5	III
2	1	20	0.2	0.2	5	0.5	0.5	IV
2	1	10	1	0.2	5	0.5	0.5	V
2	1	10	0.2	1	5	0.5	0.5	VI
2	1	10	0.2	0.2	15	0.5	0.5	VII
2	1	10	0.2	0.2	5	0	0.5	VIII
2	1	10	0.2	0.2	5	0.5	0	IX

Figure 5: Phase angle ϕ_1 due to u_1 and v_1

For the steady flow the amplitude and the phase differences of shear stress at the stationary plate ($\eta = 0$) can be obtained as

$$\tau_{0r} = \sqrt{{\tau_{0x}}^2 + {\tau_{0y}}^2},\ \phi_{0r} = \tan^{-1}\left(\frac{\tau_{0y}}{\tau_{0x}}\right) \qquad (28)$$

$$\tau_{0x} + i\tau_{0y} = \left(\frac{\partial q_0}{\partial \eta}\right)_{\eta=0} = -\frac{A_1 e^{A_2} e^{A_1 \eta}}{e^{A_2} - e^{A_1}} + \frac{A_2 e^{A_1} e^{A_2 \eta}}{e^{A_2} - e^{A_1}} \quad (29)$$

Here τ_{0x} and τ_{0y} are respectively, the shear stress at the stationary plate due to the primary and the secondary velocities components. The numerical values for the resultant shear stress τ_{0r} and the phase angleϕ_{0r} are listed in table-1. This table shows that τ_{0r} and phase angle ϕ_{0r} goes on increasing with the increasing rotations Ω of the channel.

Table1: Amplitude τ_{0r} and the phase angle ϕ_{0r} due to u_0 and v_0

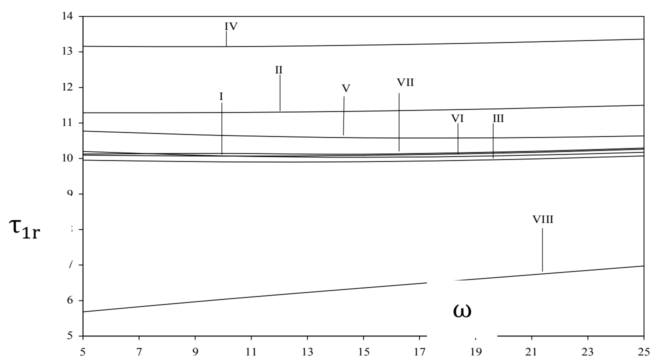
M	m	Ω	λ	K	τ_{0r}	ϕ_{0r}
2	1	10	0.2	0.2	4.8896	0.78540
4	1	10	0.2	0.2	5.6420	0.55842
2	3	10	0.2	0.2	4.7618	0.64822
2	1	20	0.2	0.2	6.6017	0.69284
2	1	10	1	0.2	5.2293	0.57062
2	1	10	0.2	1	4.7958	0.70385

For the unsteady part of the flow the amplitude and the phase differences of the shear stresses at the stationary plate ($\eta = 0$) can be obtained as, for $t = \frac{\pi}{4}$ as

$$\tau_{1x} + i\tau_{1y} = \left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0} + i\left(\frac{\partial v_1}{\partial \eta}\right)_{\eta=0} = \left(\frac{\partial q_1}{\partial \eta}\right)_{\eta=0} = \frac{1}{\sqrt{2}}\left[\frac{1+i}{e^{A_4}-e^{A_3}}\left\{A_{13}\left(e^{A_4}-e^{A_3}\right)A_1 + A_{14}A_2\left(e^{A_4}-e^{A_3}\right) + \left(1 - A_{13}\left(e^{A_1}-e^{A_3}\right) - A_{14}\left(e^{A_2}-e^{A_3}\right) - A_{15}\left(1-e^{A_3}\right)\right) - A_3\left(1 + A_{13}\left(e^{A_4}-e^{A_1}\right) + A_{14}\left(e^{A_4}-e^{A_2}\right) + A_{15}\left(e^{A_4}-1\right)\right)\right\}\right] \quad (30)$$

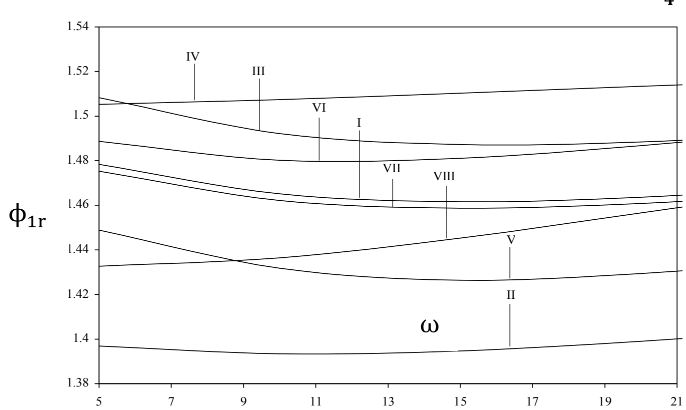
$$\tau_{1r} = \sqrt{\tau_{1x}^2 + \tau_{1y}^2},\ \phi_{1r} = \tan^{-1}\left(\frac{\tau_{1y}}{\tau_{1x}}\right) \quad (31)$$

From figure 6 it is conclude that τ_{1r} decrease slightly with increase value of hall current m and permeability of the porous medium k, A and it increase with increase of Hartman number M ,rotation parameter Ω ,suction parameter λ and B. From figure 7 it is conclude that the phase differences ϕ_{1r} increases with the increasing value of permeability of the porous medium k, hall parameter m and rotation parameter Ω. The phase differences ϕ_{1r} decreases with the increasing value of suction parameter λ and Hartman number M.



M	m	Ω	λ	K	A	B	Curve
2	1	10	0.2	0.2	0.5	0.5	I
4	1	10	0.2	0.2	0.5	0.5	II
2	3	10	0.2	0.2	0.5	0.5	III
2	1	20	0.2	0.2	0.5	0.5	IV
2	1	10	1	0.2	0.5	0.5	V
2	1	10	0.2	1	0.5	0.5	VI
2	1	10	0.2	0.2	0	0.5	VII
2	1	10	0.2	0.2	0.5	0	VIII

Figure 6: The amplitude τ_{1r} of unsteady shear stress for $t = \frac{\pi}{4}$.



M	m	Ω	λ	K	A	B	Curve
2	1	10	0.2	0.2	0.5	0.5	I
4	1	10	0.2	0.2	0.5	0.5	II
2	3	10	0.2	0.2	0.5	0.5	III
2	1	20	0.2	0.2	0.5	0.5	IV
2	1	10	1	0.2	0.5	0.5	V
2	1	10	0.2	1	0.5	0.5	VI
2	1	10	0.2	0.2	0	0.5	VII
2	1	10	0.2	0.2	0.5	0	VIII

Figure 7: Phase angle ϕ_{1r} of unsteady shear stress at $t = \frac{\pi}{4}$.

Concluding Remarks:

The resultant velocities R_0 and R_1 increases with the increasing value of Hartmann number M, permeability parameter k, and rotation parameter Ω. The resultant velocities R_1 decrease with the increasing value of frequency of oscillation. The resultant velocities R_0 and R_1 both increases rapidly from zero near the stationary plate and then approach unity in the form of damped oscillation.

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Appendix

$$A_1 = \frac{-\lambda + \sqrt{\lambda^2 + 4\left(S + \frac{1}{K}\right)}}{2}, \quad A_2 = \frac{-\lambda - \sqrt{\lambda^2 + 4\left(S + \frac{1}{K}\right)}}{2},$$

$$A_3 = \frac{-\lambda + \sqrt{\lambda^2 + 4(S + i\omega + \frac{1}{K})}}{2} \qquad A_4 = \frac{-\lambda - \sqrt{\lambda^2 + 4(S + i\omega + \frac{1}{K})}}{2}, \qquad A_5 = \frac{Be^{A_2} A_1^2}{\left(e^{A_2} - e^{A_1}\right)\left(A_1^2 + \lambda A_1 - (S + i\omega + \frac{1}{K}\right)},$$

$$A_6 = \frac{-Be^{A_1} A_2^2}{\left(e^{A_2} - e^{A_1}\right)\left(A_2^2 + \lambda A_2 - (S + i\omega + \frac{1}{K}\right)}, A_7 = \frac{\lambda(A+B)e^{A_2} A_1}{\left(e^{A_2} - e^{A_1}\right)\left(A_1^2 + \lambda A_1 - (S + i\omega + \frac{1}{K}\right)}$$

$$A_8 = \frac{-\lambda(A+B)e^{A_1} A_2}{\left(e^{A_2} - e^{A_1}\right)\left(A_2^2 + \lambda A_2 - (S + i\omega + \frac{1}{K}\right)}, A_9 = \frac{-BS}{\left(S + i\omega + \frac{1}{K}\right)} \quad A_{10} = \frac{-e^{A_2} BS}{\left(e^{A_2} - e^{A_1}\right)\left(A_1^2 + \lambda A_1 - (S + i\omega + \frac{1}{K}\right)}$$

$$,A_{11} = \frac{BSe^{A_1}}{\left(e^{A_2} - e^{A_1}\right)\left(A_2^2 + \lambda A_2 - \left(S + i\omega + \frac{1}{K}\right)\right)},$$

$$A_{12} = \frac{\left(S + i\omega + \frac{1}{K} + SB\right)}{-\left(S + i\omega + \frac{1}{K}\right)}, S = 2\Omega i + \frac{M^2}{1+m^2}(1 + im), A_{13} = A_5 + A_7 + A_{10}, \quad A_{14} = A_6 + A_8 + A_{11}, A_{15} = A_9 + A_{12}$$