MHD Mixed Convective Heat and Mass Transfer in a Visco-Elastic Boundary Layer Slip Flow past a Vertical Permeable Plate with Thermal Radiation and Chemical Reaction

Rita Choudhury[1] and Utpal Jyoti Das[2*]

[1]Department of Mathematics, Gauhati University, Guwahati-781014, Assam, INDIA.

[2]Department of Mathematics, Rajiv Gandhi University, Itanagar-791112, Arunachal Pradesh, INDIA.

[*]Corresponding Address:

utpaljyotidas@yahoo.co.in

Research Article

Abstract: An analytical study for the problem of unsteady mixed convection with thermal radiation and first-order chemical reaction on magneto hydrodynamic boundary layer flow of an electrically conducting visco-elastic fluid past a vertical permeable plate has been investigated. Slip boundary layer condition is applied at the porous interface. The classical model is used for studying the effect of radiation for optically thin media. The perturbation scheme has been used to solve the problem. Analytical expressions for dimensionless velocity, temperature, concentration fields, skin friction co-efficient, rate of heat transfer and Sherwood number have been obtained. The profiles of the velocity and skin friction have been presented graphically for different values of the visco-elastic parameters with the combination of the other flow parameters encountered in the problem under investigation.

2000 Mathematics Subject Classification: 76A05, 76A10.

Keywords: MHD, visco-elastic, thermal radiation, heat transfer, porous medium, boundary layer flow, slip flow, first order chemical reaction.

1. Introduction

Mixed convection flows with simultaneous heat and mass transfer in porous media under the influence of a magnetic field and chemical reaction are frequently encountered in many transport processes in nature. Its application is found in many industries viz. in the chemical industry, power and cooling industry for drying, chemical vapour deposition on surfaces, cooling of nuclear reactors and magnetohyrodynamic power generators. Simultaneous heat transfer and evaporation of crude oil in different stages of refining process are physical examples of heat and mass transfer. Many transport processes exist in nature and in industrial applications in which the simultaneous heat and mass transfer occurs as a result of combined buoyancy effects of diffusion of chemical species. A comprehensive description of the theoretical work for both laminar and turbulent mixed convection boundary layer flows has been given in a review paper by Chen and Armaly [1] and in the book by Pop and Ingham [2]. The problem of mixed convection under the influence of magnetic field has attracted numerous researchers viz. Soundalgekar et al. [3], Elbasheshy [4], Abel et al. [5, 6] in view of its applications in geophysics and astrophysics. In the above mentioned studies the radiation effect is ignored. But the effects of thermal radiation heat transfer cannot be neglected when technological processes take place at high temperature (Siegel and Howell [7], Modest [8]). Recent developments in hypersonic flights, missile re-entry rocket combustion chambers, gas cooled nuclear reactors and power plants for inter planetary flight have focussed the attention of many researchers [9]-[17].

Convection in porous media has gained significant attention in recent years because of its importance in engineering applications. Reviews of the applications related to convective flows in porous media can be found in the book by Nield and Bejan [18]. The fundamental problem of flow through and past porous media has been discussed by Cheng [19] and Rudraiah [20] on thermal radiation as a mode of energy transfer and emphasize the need for inclusion of radiative transfer in these processes. The inadequacy of the no-slip condition is quite evident in polymer melts which often exhibit microscopic wall slip. The boundary conditions to be satisfied at the interface between a porous medium and fluid layer are the matching of velocity and stresses. Several authors [21]-[26] have studied in this line.

The combined effect of heat and mass transfer with chemical reaction in porous medium has important engineering applications e.g. tubular reactors, oxidation of solid materials and synthesis of ceramic materials. Chemical reaction can be codified as either a heterogeneous or homogeneous process. This depends on whether it occurs at an interface or a single-phase volume reaction. In most chemical reactions, the reaction rate depends on the concentration of the species itself. A chemical reaction is said to be of first order, if the rate of reaction is directly proportional to concentration itself. During chemical reaction between two species concentration heat is also generated. Chemical reaction effects on heat and mass transfer laminar boundary layer flow have been discussed by various authors [27, 28, 29, 30, 31] in various situations. For the problem of coupled heat and mass transfer in MHD, the effect of Ohmic heating are not studied in the above investigations. However, it is more realistic to include Ohmic effect in order to explore the impact of the magnetic field on the thermal transport in the boundary layer. The effect of Ohmic heating on the MHD free convective heat transfer has been examined by Hossain [32]. Chaudhary et al. [33] have analyzed the effect of radiation on heat transfer in MHD mixed convection flow with simultaneous thermal and mass diffusion from an infinite vertical plate with viscous dissipation. Pal and Mondal [34] analyzed the effect of variable viscosity on MHD non-Darcy mixed convective heat transfer over a stretching sheet embedded in a porous medium with non-uniform heat source/sink and Ohmic dissipation. Recently, Pal and Talukdar [35] studied the combined effect of MHD and Ohmic heating in unsteady two-dimensional boundary layer slip flow, heat and mass transfer of a viscous incompressible fluid past a vertical permeable plate with the diffusion of species in the presence of thermal radiation incorporating the first-order chemical reaction. They used the classical model introduced by Cogley et al. [36] for the radiation effect. In the present study, an attempt has been made to extend the problem studied by Pal and Talukdar to the case of visco-elastic fluid characterised by second-order fluid.

The constitutive equation for the second-order fluid is of the form

$$S = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \quad (1)$$

where S is the stress tensor, p is hydrostatic pressure, I is unit tensor, $A_n\ (n = 1,2)$ are the kinematic Rivlin –Ericksen tensors, μ_1, μ_2, μ_3 are the material co-efficients describing the viscosity, visco-elasticity and cross-viscosity respectively. The material coefficients μ_1, μ_2, μ_3 are taken constants with μ_1 and μ_3 as positive and μ_2 as negative (Coleman and Markovitz [37]). The equation (1) was derived by Coleman and Noll [38] from that of simple fluids by assuming that the stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

2. Formulation of the problem

We consider unsteady two-dimensional visco-elastic flow of an incompressible, electrically conducting and heat-absorbing fluid past a semi-infinite vertical permeable plate embedded in a uniform porous medium which is subject to slip boundary condition at the interface of porous and fluid layers. A uniform transverse magnetic field of magnitude B_0 is applied in the presence of radiation and concentration buoyancy effects in the direction of y^*- axis. The transversely applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field and the Hall effect are negligible. It is assumed that there is no applied voltage which implies the absence of an electric field. Since the motion is two dimensional and length of the plate is large enough so all the physical variables are independent of x^*. The wall is maintained at constant temperature T_w and concentration C_w, higher than the ambient temperature T_∞ and concentration C_∞, respectively. Also, it is assumed that there exists a homogeneous first-order chemical reaction with rate constant R between the diffusing species and the fluid. It is assumed that the porous medium is homogeneous and present everywhere in local thermodynamic equilibrium. Rest of properties of the fluid and the porous medium are assumed to be constant. The governing equations for this investigation are based on the balances of mass, linear momentum, energy and concentration species. Taking into consideration these assumptions, the equations that describe the physical situation can be written as follows:

Continuity Equation:

$$\frac{\partial v^*}{\partial y^*} = 0 \quad (2)$$

Momentum Equation:

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \upsilon_1 \frac{\partial^2 u^*}{\partial y^*} + \upsilon_2 \left(v^* \frac{\partial^3 u^*}{\partial y^{*3}} + \frac{\partial u^*}{\partial y^*}\frac{\partial^2 v^*}{\partial y^{*2}} \right) - \frac{\sigma_1 B_0{}^2 u^*}{\rho} + g\beta_T \left(T^* - T_\infty\right) + g\beta_C \left(C^* - C_\infty\right) \quad (3)$$

Energy Equation:

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*} - \frac{Q_0}{\rho C_p}\left(T^* - T_\infty\right) + Q_1^*\left(C^* - C_\infty\right) \quad (4)$$

Mass Diffusion Equation:

$$\frac{\partial C^*}{\partial t^*}+v^*\frac{\partial C^*}{\partial y^*}=D\frac{\partial^2 C^*}{\partial y^{*2}}-R\left(C^*-C_\infty\right) \quad (5)$$

where x^*, y^* are the dimensional distances along and perpendicular to the plate, respectively. u^* and v^* are the components of dimensional velocities along x^* and y^* directions, respectively. g is the gravitational acceleration, T^* is the dimensional temperature of the fluid near the plate, T_∞ is the free stream dimensional temperature, C^* is the dimensional concentration, C_∞ is the free stream dimensional concentration. β_T and β_C are the thermal and concentration expansion coefficients, respectively. p^* is the pressure, C_p is the specific heat of constant pressure, B_0 is the magnetic field coefficient, q_r^* is the radiative heat flux, ρ is the density, κ is the thermal conductivity, σ_1 is the magnetic permeability of the fluid, D is the molecular diffusivity, Q_0 is the dimensional heat absorption coefficient, Q_1^* is the coefficient of proportionality of the absorption of the radiation, R is the chemical reaction parameter and $\upsilon_i=\frac{\mu_i}{\rho},(i=1,2)$. The fifth and the sixth terms on R.H.S. of the momentum equation (3) denote the thermal and the concentration buoyancy effects, respectively. The second and the third term on the R.H.S. of equation (4) denote the inclusion of the effect of the thermal radiation and heat absorption effects, respectively.

The radiative heat flux is given by (Cogley *et al.* [36]) as

$$\frac{\partial q_r^*}{\partial y^*}=4\left(T^*-T_\infty\right)I' \quad (6)$$

where $I'=\int_0^\infty \kappa_{\lambda w}\frac{\partial e_{b\lambda}}{\partial T}d\lambda$, $\kappa_{\lambda w}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is Planck's function.

Under these assumptions, the appropriate boundary conditions for velocity involving slip flow, temperature and concentration fields are defined as

$$u^*=u^*_{slip}=\frac{\sqrt{\kappa_1}}{\alpha_1}\frac{\partial u^*}{\partial y^*},\quad T^*=T_w+\varepsilon\left(T_w-T_\infty\right)e^{n^*t^*} \quad \text{at} \quad y^*=0 \quad (7)$$

$$C^*=C_w+\varepsilon\left(C_w-C_\infty\right)e^{n^*t^*} \quad \text{at} \quad y^*=0 \quad (8)$$

$$u^*\to U^*_\infty=U_0\left(1+\varepsilon e^{n^*t^*}\right),\ T^*\to T_\infty,$$

$$C^*\to C_\infty \quad \text{as} \quad y^*\to\infty \quad (9)$$

where C_w and T_w are the wall dimensional concentration and temperature, respectively. κ_1 is the permeability of the porous medium, n^* is the frequency and α_1 is the porous parameter. From equation (3), it is clear that the suction velocity at the plate surface is a function of time only. Hence the suction velocity normal to the plate is assumed in the form

$$v^*=-V_0\left(1+\varepsilon Ae^{n^*t^*}\right) \quad (10)$$

where A is a real positive constant, ε and εA are small less than unity, and V_0 is a scale of suction velocity which is non-zero positive constant.

Outside the boundary layer, equation (3) gives

$$-\frac{1}{\rho}\frac{dp^*}{dx^*}=\frac{dU^*_\infty}{dt^*}+\frac{\sigma}{\rho}B_0^2u^*_\infty \quad (11)$$

In order to write the governing equations and boundary conditions in dimensionless form, the following non-dimensional quantities are introduced

$$u=\frac{u^*}{U_0},\ \upsilon_1=\frac{\upsilon_1^*}{V_0},\ \eta=\frac{V_0y^*}{\upsilon_1},\ U_\infty=\frac{U^*_\infty}{U_0},\ t=\frac{V_0^2t^*}{\upsilon_1},$$

$$\theta=\frac{T^*-T_\infty}{T_w-T_\infty},\ C=\frac{C^*-C_\infty}{C_w-C_\infty},\ \eta=\frac{\eta^*\upsilon_1}{V_0^2} \quad (12)$$

In view of the above non-dimensional variables, the basic field of equations (3)-(5) can be expressed in non-dimensional form as

$$\frac{\partial u}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial u}{\partial\eta}=\frac{dU_\infty}{dt}+M\left(U_\infty-u\right)+\frac{\partial^2u}{\partial\eta^2}$$

$$-\alpha\left(1+\varepsilon Ae^{nt}\right)\frac{\partial^3u}{\partial\eta^3}+Gr\,\theta+Gm\,C \quad (13)$$

$$\frac{\partial\theta}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial\theta}{\partial\eta}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\eta^2}-\phi\theta+Q_1C-F\theta \quad (14)$$

$$\frac{\partial C}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial C}{\partial\eta}=\frac{1}{Sc}\frac{\partial^2C}{\partial\eta^2}-\gamma C \quad (15)$$

where Gr is the thermal Grashof number, Gm is the solutal Grashof number, $\Pr$ is the Prandtl number, M is the magnetic field parameter, F is the radiation parameter, Sc the Schmidt number, ϕ is the heat source parameter, α is the visco-elastic parameter and γ is the chemical reaction parameter and Q_1 is the absorption of radiation parameter which are defined as follows:

$$Gr=\frac{\rho g\beta_T\upsilon_1\left(T_w-T_\infty\right)}{U_0V_0^2},\ Gm=\frac{\rho g\beta_C\upsilon_1\left(C_w-C_\infty\right)}{U_0V_0^2},$$

$$\Pr = \frac{\mu_1 C_p}{\kappa}, M = \frac{\sigma B_0^2 \upsilon_1}{\rho V_0^2}, F = \frac{4\upsilon_1 I'}{\rho C_p V_0^2}, Sc = \frac{\upsilon_1}{D},$$

$$\phi = \frac{Q_0 \upsilon_1}{\rho C_p V_0^2}, \alpha = \frac{\upsilon_2}{\upsilon_1^2} V_0^2, \gamma = \frac{R\upsilon_1}{V_0^2},$$

$$Q_1 = \frac{Q_1^* \upsilon_1 (C_w - C_\infty)}{V_0^2 (T_w - T_\infty)}. \quad (16)$$

The corresponding boundary conditions are

$$u = u_{slip} = \phi_1 \frac{\partial u}{\partial \eta}, \theta = 1 + \varepsilon e^{nt}, C = 1 + \varepsilon e^{nt} \text{ at } \eta = 0 \quad (17)$$

$$u \to U_\infty = \left(1 + \varepsilon e^{nt}\right), \ \theta \to 0, \ C \to 0 \text{ as } \eta \to \infty \quad (18)$$

where $\phi_1 = \frac{\sqrt{\kappa_1}}{\alpha_1} \frac{U_0 V_0}{\upsilon_1}$ is the porous permeability parameter.

3. Method of solution

The set of partial differential equations (13)-(15) cannot be solved in closed-form. However, it can be solved analytically after these equations are reduced to a set of ordinary differential equations in dimensionless form which can be done by representing the velocity u, temperature θ and concentration C as

$$u = f_0(\eta) + \varepsilon e^{nt} f_1(\eta) + o(\varepsilon^2) \quad (19)$$

$$\theta = g_0(\eta) + \varepsilon e^{nt} g_1(t) + o(\varepsilon^2) \quad (20)$$

$$C = h_0(\eta) + \varepsilon e^{nt} h_1(\eta) + o(\varepsilon^2) \quad (21)$$

Substituting (19)-(21) into equations (13)-(15) and equating the harmonic and non-harrnonic terms, and neglecting the higher order of $o(\varepsilon^2)$, we obtain

$$f_0'' + f_0' - Mf_0 = -M - Gr\, g_0 - Gm\, h_0 + \alpha\, f_0''' \quad (22)$$

$$f_1'' + f_1' - (M+n) f_1 = -(M+n) - Af_0' - Gr\, g_1 - Gm\, h_1 - \alpha\left(f_1''' + Af_0'''\right) \quad (23)$$

$$g_0'' + \Pr g_0' - \Pr(F + \phi) g_0 = -\Pr Q_1 h_0 \quad (24)$$

$$g_1'' + \Pr g_1' - \Pr(F + \phi + n) g_1 = -A \Pr g_0' - \Pr Q_1 h_1 \quad (25)$$

$$h_0'' + Sc\, h_0' - Sc\,\gamma h_0 = 0 \quad (26)$$

$$h_1'' + Sc\, h_1' - Sc(\gamma + n) h_1 = -A\, Sc\, h_0' \quad (27)$$

where the prime denotes ordinary differentiation with respect to η. The corresponding boundary conditions are

$$f_0 = \phi_1 f_0', \ f_1 = \phi_1 f_1', \ g_0 = 1, \ g_1 = 1, \ h_0 = 1, \ h_1 = 1 \quad \text{at} \quad \eta = 0 \quad (28)$$

$$f_0 = 1, \ f_1 = 1, \ g_0 \to 0, \ g_1 \to 0, \ h_0 \to 0, \ h_1 \to 0 \quad \text{as} \quad \eta \to \infty \quad (29)$$

The solutions of the equations (24)-(27) consistent with the boundary conditions (28) and (29) are obtained but not presented here for the sake of brevity.

For small shear rate we note that $|\alpha| \langle 1$. Substituting

$$f_0(\eta) = f_{00}(\eta) + \alpha\, f_{01}(\eta) + o(\alpha^2)$$

$$f_1(\eta) = f_{10}(\eta) + \alpha\, f_{11}(\eta) + o(\alpha^2) \quad (30)$$

into equations (22), (23) and boundary conditions (28), (29) up to the first order of α and comparing the coefficients of like powers of α, we obtain

$$f_{00}'' + f_{00}' - Mf_{00} = -M - Gr\, g_0 - Gm\, h_0 \quad (31)$$

$$f_{01}'' + f_{01}' - Mf_{01} = f_{00}''' \quad (32)$$

$$f_{10}'' + f_{10}' - (M+n) f_{10} = -(M+n) - Af_0' - Gr\, g_1 - Gm\, h_1 \quad (33)$$

$$f_{11}'' + f_{11}' - (M+n) f_{11} = f_{10}''' - Af_0''' \quad (34)$$

The corresponding boundary conditions are

$$f_{00} = \phi_1 f_{00}', \ f_{01} = \phi_1 f_{01}', \ f_{10} = \phi_1 f_{10}', \ f_{11} = \phi_1 f_{11}' \quad \text{at} \quad \eta = 0 \quad (35)$$

$$f_{00} = 1, \ f_{01} = 0, \ f_{10} = 1, \ f_{11} = 0 \quad \text{as} \quad \eta \to \infty \quad (36)$$

Solutions of the equations (31)-(34) consistent with the boundary conditions (35) and (36) are obtained but not presented here for the sake of brevity.

The non-dimensional skin friction coefficient at the plate is given by

$$\sigma = \frac{\tau_w}{\rho U_0 V_0} = \left(\frac{\partial u}{\partial \eta}\right)_{y=0} + \alpha\left[\frac{\partial^2 u}{\partial \eta \partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial^2 u}{\partial \eta^2}\right]_{y=0} \quad (37)$$

The rate of heat transfer coefficient, which in non-dimensional form, in terms of Nusselt number, is given by

$$Nu_x = \frac{x\left(\frac{\partial T}{\partial y^*}\right)_{y=0}}{T_w - T_\infty} \Rightarrow Nu_x / \mathrm{Re}_x = \left(\frac{\partial \theta}{\partial \eta}\right)_{y=0}$$

Sherwood number is given by

$$Sh_x = \frac{x\left(\frac{\partial C}{\partial y^*}\right)_{y^*=0}}{C_w - C_\infty} \Rightarrow Sh_x / \mathrm{Re}_x = \left(\frac{\partial C}{\partial \eta}\right)_{y=0}$$

where $\mathrm{Re}_x = \frac{V_0 x}{\upsilon_1}$ is the local Reynolds number.

4. Results and Discussion

In the present investigation, we have analyzed the heat and mass transfer on mixed convection flow of a visco-elastic incompressible, electrically conducting fluid over an infinite vertical porous plate in the presence of magnetic field and thermal radiation using the classical model for the radiating heat flux. We have obtained the results for varieties of physical parameters, which are illustrated by means of graphs. The purpose of this study is to bring out the effects of the visco-elastic parameter α on the governing flow with the combination of the other flow parameters.

All the corresponding results for Newtonian fluid are obtained by setting $\alpha = 0$.

Figures 1-10 depict the velocity profile u in the boundary layer against η to observe the visco-elastic effects for various sets of values (Table.1) of the thermal Grashof number Gr, solutal Grashof number Gm, Schmidt number Sc, absorption radiation parameter Q_1, chemical reaction parameter γ, radiation parameter F, heat source parameter ϕ, magnetic field parameter M, Porous permeability parameter ϕ_1 with fixed values of $\Pr = 3, A = 0.3, n = 0.3, \varepsilon = 0.1,$ and $t = 2$. It is evident from the figures that the velocity u in the boundary layer increases with the increasing values of the visco-elastic parameter $|\alpha|, (\alpha = 0, -0.02, -0.04)$ in comparison with the Newtonian cases when (i) Gr increases (Figs. 1 and 2), (ii) Gm increases (Figs. 2 and 3), (iii) Sc increases (Figs. 3 and 4), (iv) Q_1 increases (Figs. 4 and 5), (v) γ increases (Figs. 5 and 6), (vi) F increases (Figs. 6 and 7), (vii) ϕ increases (Figs. 7 and 8), (viii) M increases (Figs 8 and 9), (ix) ϕ_1 increases (Figs. 9 and 10) ; by keeping other flow parameters fixed.

Also, we note from the figures that the velocity u in the boundary layer increases for Newtonian as well as non-Newtonian cases when Gr (Figs. 1 and 2), Gm (Figs 2 and 3), ϕ_1 (Figs. 9 and 10) increase whereas reverse trend is seen for Sc (Figs. 3 and 4), and ϕ (Figs. 7 and 8).

Table1: Description of various cases:

Cases	Gr	Gm	Sc	Q_1	γ	F	ϕ	M	ϕ_1
I	2	0	2	0.3	0.3	2	2	0.4	0.3
II	4	0	2	0.3	0.3	2	2	0.4	0.3
III	4	2	2	0.3	0.3	2	2	0.4	0.3
IV	4	2	4	0.3	0.3	2	2	0.4	0.3
V	4	2	4	5	0.3	2	2	0.4	0.3
VI	4	2	4	5	0.5	2	2	0.4	0.3
VII	4	2	4	5	0.5	4	2	0.4	0.3
VIII	4	2	4	5	0.5	4	4	0.4	0.3
IX	4	2	4	5	0.5	4	4	1.4	0.3
X	4	2	4	5	0.5	4	4	1.4	0.5

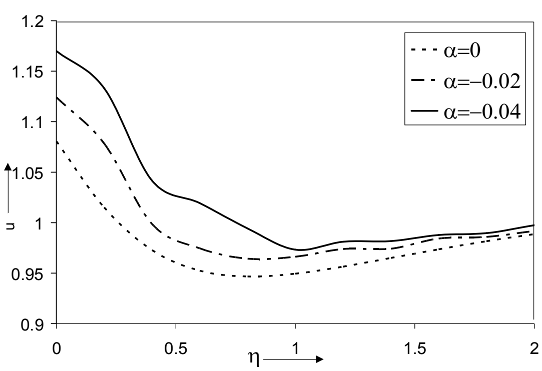

***Fig. 1:** Variation of u against η for case I.*

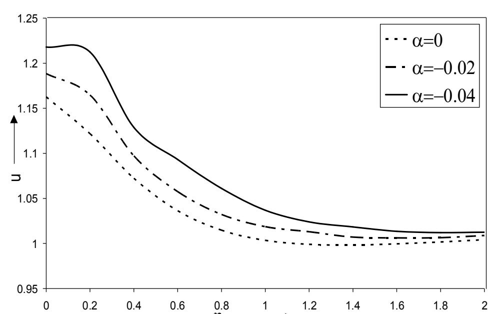

***Fig. 2:** Variation of u against η for case II.*

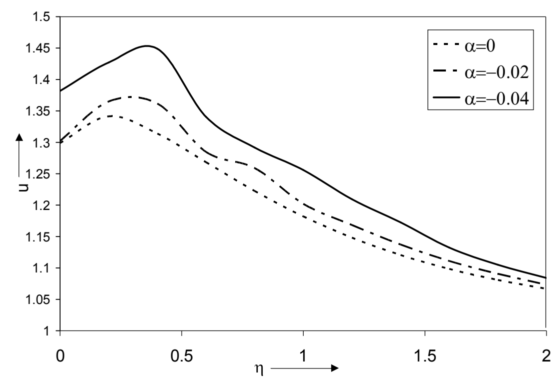

***Fig. 3:** Variation of u against η for case III.*

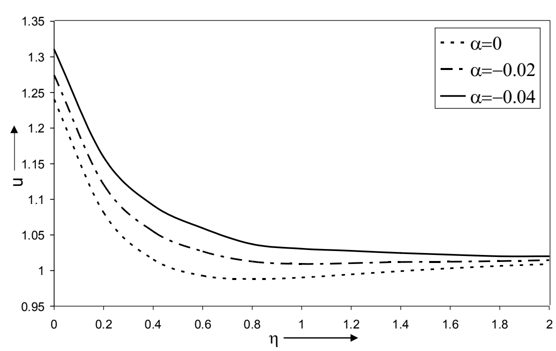

***Fig. 4:** Variation of u against η for case IV.*

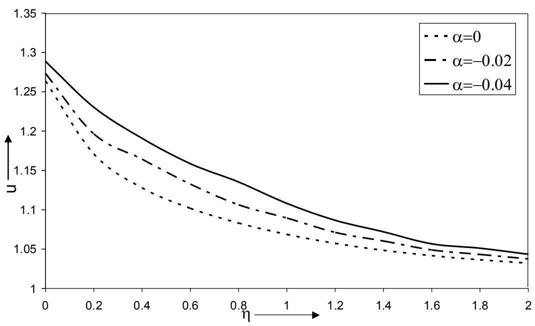

Fig. 5: *Variation of u against* η *for case V.*

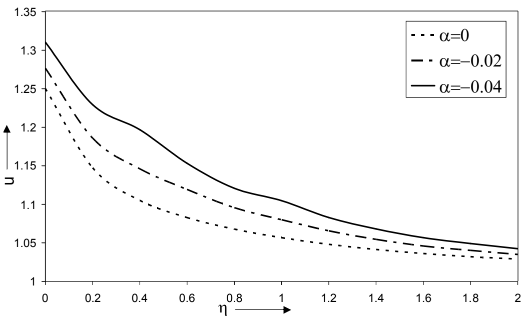

Fig. 6: *Variation of u against* η *for case VI.*

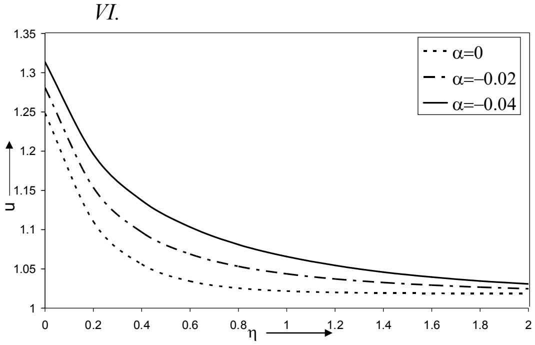

Fig. 7: *Variation of u against* η *for case VII.*

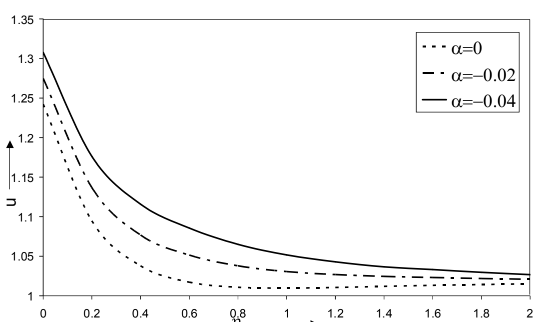

Fig. 8: *Variation of u against* η *for case VIII.*

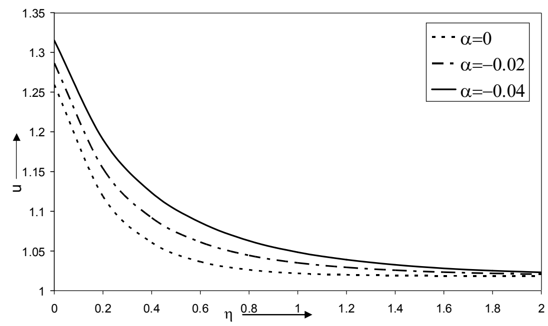

Fig. 9: *Variation of u against* η *for case IX.*

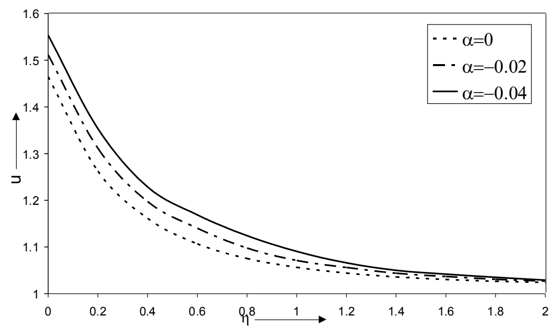

Fig. 10: *Variation of u against* η *for case X.*

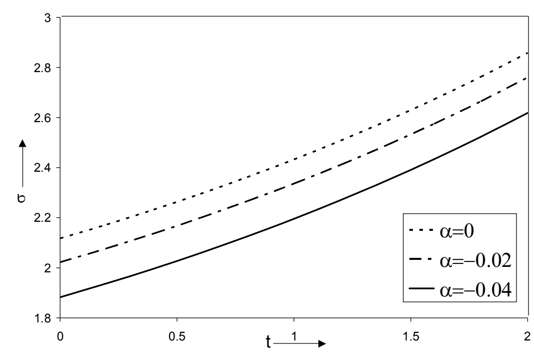

Fig. 11: *Variation of* σ *against t for case I.*

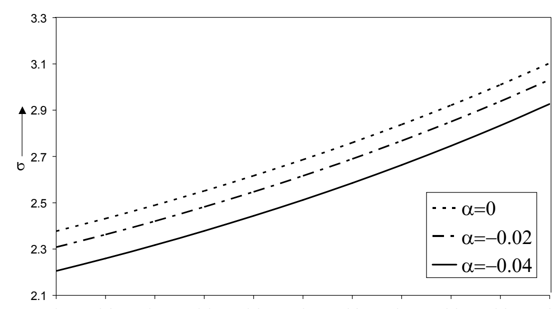

Fig. 12: *Variation of* σ *against t for case II.*

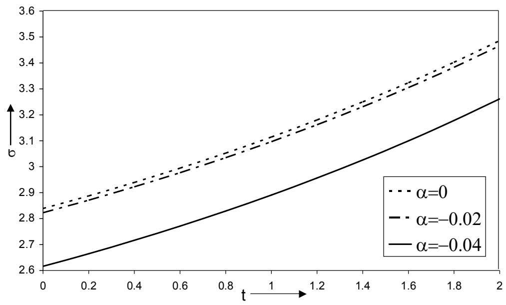

Fig. 13: *Variation of σ against t for case III.*

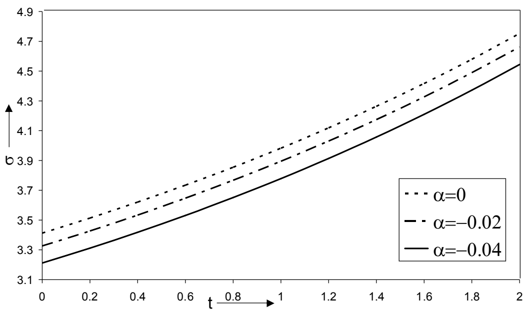

Fig. 14: *Variation of σ against t for case IV.*

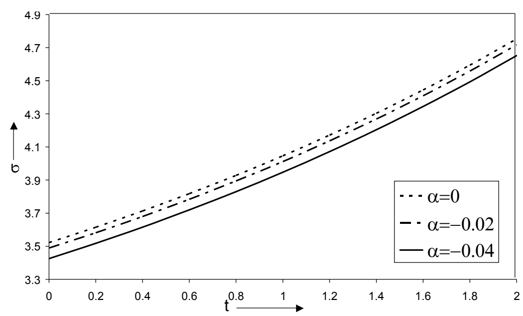

Fig. 15: *Variation of σ against t for case V.*

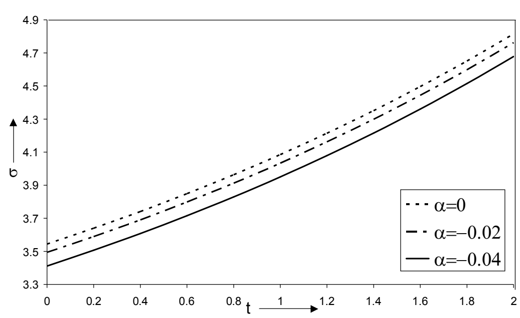

Fig. 16: *Variation of σ against t for case VI.*

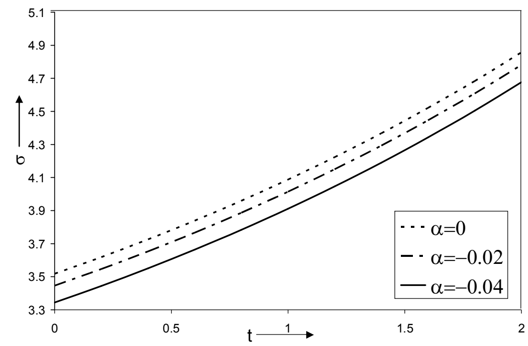

Fig. 17: *Variation of σ against t for case VII.*

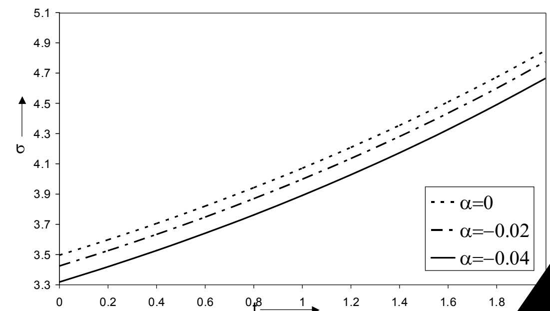

Fig. 18: *Variation of σ against t for case VIII.*

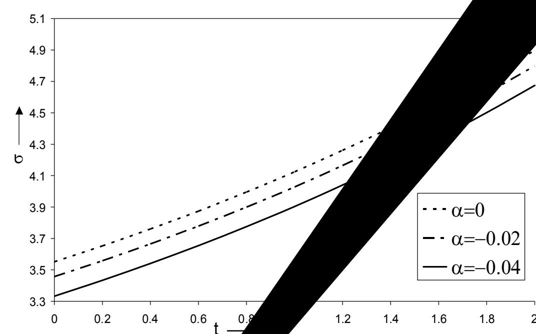

Fig. 19: *Variati... f σ against t for case IX.*

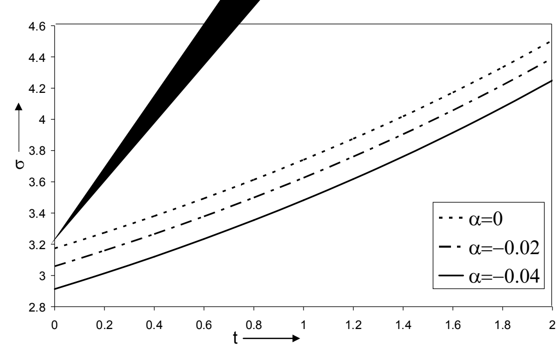

Fig. 20: *Variation of σ against t for case X.*

Figures 11-20 represent the non-dimensional shearing stress σ against t to observe the visco-elastic effects for various sets of values given in table 1. Figures 11-20 reveal that the skin friction coefficient σ decrease with the increasing values of the visco-elastic parameter $|\ |$

13 and 14), (iv) Q_1 increases (Figs. 14 and 15), (v) γ increases (Figs. 15 and 16), (vi) F increases (Figs. 16 and 17), (vii) ϕ increases (Figs. 17 and 18), (viii) M increases (Figs. 18 and 19), and (ix) ϕ_1 increases (Figs. 19 and 20) keeping other flow parameters constant.

It is seen from the figures 11-20 that the skin friction coefficient increase with increasing time t for Newtonian as well as non-Newtonian fluid in all the cases of table 1. Further it is found that skin friction coefficient σ increase for both Newtonian and non-Newtonian cases with the increasing values of Gr (Fig. 11 and 12), Gm (Figs. 12 and 13), Sc (Figs.13 and 14), Q_1 (Figs. 14 and 15), M (Figs. 18 and 19), whereas reverse trend is seen for ϕ (Figs. 17 and 18), ϕ_1 (Figs. 19 and 20).

It has also been observed that the temperature field, concentration field, Nusselt number and Sherwood number are not significantly affected by the visco-elastic parameter.

5. Conclusion

The above study brings out the flowing results of physical interest:

(1) The velocity u in the boundary layer increase with the increasing values of the visco-elastic parameter $|\alpha|$ in comparison with the Newtonian cases. Also, velocity u increase as any one values of the Grashof number for heat transfer, Solutal Grashof number, and porous permeability parameter (ϕ_1) increases but it decrease as any of the values of the Schmidt number and heat source parameter (ϕ) increases.

(2) Skin friction increase with the increasing time for both Newtonian and non-Newtonian fluid. Further, it is found that skin friction co-efficient increase for both Newtonian and non-Newtonian cases with the increasing values of Grashof number, Schimdt number, absorption radiation parameter, or Hartmann number whereas skin friction decrease for heat source parameter or porous permeability parameter increasing.

References

[1] T.S. Chen and B.F. Armaly, Mixed convection in external flow, In: S KaKac, R. K. Shah, W. Aunng, editors, Handbook of single-phase convective heat transfer, Wiley, New York, 1987.

[2] I. Pop and D.B. Ingham, Convective heat transfer: mathematical and computational modelling of viscous fluids and porous media, Pergamon Press, Oxford, 2001.

[3] V.M. Soundalgekar, S.K. Gupta and N.S. Birajdar, Effects of mass transfer and free effects on MHD Stokes problem for a vertical plate, Nuclear Engineering Research 53; 309-346, 1979.

[4] E.M.A. Elbasheshy, Heat and mass transfer along a vertical plate surface tension and concentration in the presence of magnetic field, International journal of Sciences 34(5); 515-522, 1997.

[5] M.S. Abel, P.G. Siddheshwar and N. Mahesha, Effects of thermal buoyancy and variable thermal conductivityon the MHD flow and heat transfer in a power law fluid past a vertical stretching sheet in the presence of a non-uniform heat source, International Journal of Non- Linear Mechanics, 44; 1-12, 2009.

[6] M.S. Abel, P.S. Datti and N. Mahesha, Flow and heat transfer in a power-law fluid over a stretching sheet with variable thermal conductivity and non-uniform heat source. International Journal of Heat Mass Transfer , 52; 2902-2913, 2009.

[7] R. Siegel and J.R. Howell, Thermal radiation transfer, 3rd edition, Hemisphere, New York,, 1992.

[8] M.F. Modest, Radiative heat transfer, 2nd edition, Academic Press: New York, 2003.

[9] M.A. Hossain and H.S. Takhar, Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat Mass Transfer 31; 243-248, 1996.

[10] Emad M. Aboeldahab, Radiation effect on heat transfer in electrically conducting fluid at a stretching surface with uniform free stream, Journal of Physics D Applied Physics, 33; 3180-3185, 2000.

[11] R. Muthukumarswamy and G. Kumar Senthil, Heat and mass transfer effects on moving vertical plate in the presence of thermal radiation, Theoretical Applied Mechanics, 3; 35-46, 2004.

[12] E.M. Abo-Eldahab and M.S. Elgendy, Radiation effect on convective heat transfer in an electrically conducting fluid at a stretching surface with variable viscosity and uniform free stream, Physica Scripta, 62; 321-325, 2000.

[13] M.A. Seddeek, Effects of radiation and variable viscosity on a MHD free convection past a semi-infinite flat plate with an aligned magnetic field in the case of unsteady flow, International Journal of Heat and Mass Transfer, 45; 931-935, 2002.

[14] M.D. Abdus Satter, M.D. Hamid Kalim, Unsteady free-convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate, Journal of Mathematical and Physical Sciences, 30; 25-37, 1996.

[15] R. Cortell, Effects of viscous dissipation and radiation on the thermal boundary layer over a non-linearly stretching sheet, Physics Letters, A 372(5); 631-636, 2008.

[16] M.S. Abel and N. Mahesha, Heat transfer in MHD visco-elastic fluid flow over a stretching sheet with variable thermal conductivity, non-union heat source and radiation, Applied Mathematical Modelling, 32; 1965-1983, 2008.

[17] Y.J. Kim, Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, International Journal of Engineering Sciences, 38; 833-845, 2000.

[18] D. Nield and A. Bejan, Convection in porous media, Springer: New York, 1999.

[19] P. Cheng, Heat transfer in geothermal system, Advances in Heat Transfer, 14; 1-105, 1978.

[20] N. Rudraiah, Flow through and past porous media, Encyclopedia of fluid mechanics, Gulf Publications, 5; 567-647, 1986.

[21] G.S. Beavers and D.D. Joseph, Boundary conditions at a naturally permeable wall, Journal of Fluid Mechanics, 30; 197-207, 1967.

[22] P.G. Saffman, On the boundary conditions at the surface of a porous medium, Studies in Applied Mathematics, 1; 93-101, 1971.

[23] D.A. Nield, Onset of convection in a fluid layer overlying a layer of porous medium, Journal of Fluid Mechanics, 81; 513-522, 1967.

[24] D.A. Nield, Boundary correction for the Rayleigh-Darcy problem: limitations of the Brinkman equation, Journal of Fluid Mechanics, 128; 37-46, 1983.

[25] D. Poulikakos, A. Bejan, B. Salomos and K.R. Blake, High Rayleigh number convection in a fluid overlying a porous bed, International Journal of Heat and Fluid Flow, 7; 109-114, 1986.

[26] N. Rudraiah, D. Pal and P.N.Shivakumar, Effect of slip and magnetic filed on composite systems, Fluid Dynamics Research, 4; 255-270, 1988.

[27] U.N. Das, R.K. Deka and V.M. Soundalgekar, Effect of mass transfer on flow past an impulsivel started vertical plate with constant heat flux and chemical reaction, Forschung im Ingenieurwesen, 60(10); 284-287, 1994.

[28] R. Muthucumarswamy and P. Ganesan, Diffusion and first-order chemical reaction on impulsively started infinte vertical plate with variable temperature, International Journal of Thermal Sciences, 41(5); 475-479, 2002.

[29] K.V. Prasad, S. Abel and P.S. Datti, Diffusion of chemically reactive species of a non-Newtonian fluid immersed in a porous medium over a stretching sheet, International Journal of Non-Linear Mechanics, 38; 651-657, 2003.

[30] A.Y. Ghaly and M.A. Seddeek, Chebyshev finite difference method for the effect of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate with temperature dependent viscosity, Choas Solitons Fractals, 19; 61-70, 2004.

[31] F. T. Akyildiz, H. Bellout and K. Vajravelu, Diffusion of chemically reactive species in a porous medium over a stretching sheet, Journal of Mathematical Analysis and Application, 320; 322-339, 2006.

[32] M.A. Hossain, Viscous and Joule heating effects on MHD free convection flow with variable plate temperature, International Journal of Heat and Mass Transfer, 35; 3485-3487, 1992.

[33] R.C. Chaudhary, B.K. Sharma and A.K. Jha, Radiation effect with simultaneous thermal and mass diffusion in MHD mixed convection flow, Romanian Journal of Physics, 51(7-8); 715-727, 2006.

[34] D. Pal and H. Mondal, Effect of variable viscosity on MHD non-Darcy mixed convective heat transfer over a stretching sheet embedded in a porous medium with non-uniform heat source/sink, Communication in Nonlinear Science and Numerical Simulation 15(6); 1553-1564, 2010.

[35] D. Pal and B. Talukdar, Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction, Communication in Nonlinear Science and Numerical Simulation, 15 (7); 1813-1830, 2010.

[36] A.C. Cogley, W.G. Vincent and S.E. Giles, Differential approximation to radiative heat transfer in a non-grey gas near equilibrium, American Institute of Aeronautics and Astronautics Journal, 6; 551-553, 1968.

[37] B.D. Coleman and H. Markovitz, Incompressible second order fluids, Advances in Applied Mechanics, 8; 69-101, 1964.

[38] B.D. Coleman and W. Noll, An approximation theorem for functionals with applications in continuum mechanics, Archive for Rational Mechanics and Analysis, 6(1); 355-370, 1960.