MHD Boundary Layer Flow and Heat Transfer over a Continuously Moving Flat Plate

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Research Article

Abstract: The steady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid past a continuously moving surface is considered in the presence of uniform transverse magnetic field. Taking suitable similarity variables, the governing boundary layer equations are transformed to ordinary differential equations and then solved numerically by standard techniques. The effects of the various parameters involved are discussed graphically on velocity and temperature distributions, whereas the values of f''(0) and –θ'(0) are tabulated for different parameters.

Key Words: MHD boundary layer, flat plate, moving surface, heat transfer, viscous dissipation.

Introduction

Flow of an incompressible viscous fluid over moving surfaces has an important bearing on several technical applications, such as in metallurgy and chemical processes industries. An example for a moving continuous surface is a polymer sheet or filament extruded continuously from a die, or a long thread travelling between a feed roll and a wind up rolls. The heat treatment of material travelling between a feed roll and a wind up roll, the heat extrusion of steel, the lamination and melt-spinning process in the extrusion of polymers or the cooling of a large metallic plate in a bath, which may be an electrolyte, glass blowing, continuous casting and spinning of fibres also involve the flow due to a moving surface. In all these cases, a study of the flow field and heat transfer can be of significant importance because the quality of the final product depends to a large extent on the skin-friction and the surface heat transfer rate. Stokes [1] was probably the first to study the flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate. Sakiadis [2] considered the problem of forced convection along an isothermal moving plate. Erickson et al. [3] studied the heat and mass transfer on a moving continuous flat plate with suction or injection. Tsou et al. [4] studied flow and heat transfer in the boundary layer on a continuously moving surface. Afzal and Varshney [5] studied the problem of cooling of a low heat resistance stretching sheet moving through a fluid whereas Hussaini et al. [6] obtained the similarity solution of a boundary layer problem with an upstream moving wall. The problem of flow and heat transfer over a moving surface has drawn considerable attention and a very good amount of literature has been generated on this problem (Afzal et al. [7], Howell et al. [8], Rao et al. [9], Fang [10] and Ishak et al. [11]). Kumari and Nath [12] discussed the problem of MHD boundary layer flow of a non-Newtonian fluid over a continuously moving surface with a parallel free stream. Recently, Jat and Chaudhary [13] studied the flow of incompressible viscous conducting fluid past a continuous moving surface in the presence of transverse magnetic field.

The object of the present paper is to study the effects on both momentum and heat transfer problem with viscous dissipation and Joule heat for an electrically conducting fluid past a continuously moving plate in the presence of uniform transverse magnetic field.

Formulation of the problem

Consider the two-dimensional steady flow of an electrically conducting, viscous, incompressible fluid past a continuously moving flat plate with uniform velocity U_w and surface temperature T_w in the presence of uniform transverse magnetic field of strength B_0 (Figure 1.0). The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. The fluid properties are assumed to be isotropic and constant. Therefore, under the usual boundary layer approximations, the governing equations of motion are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad ...(1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_e B_0^2 u}{\rho} \quad ...(2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma_e B_0^2 u^2}{\rho c_p}$$

...(3)

where u and v are velocity components in $x-$ and $y-$ directions respectively, $\nu=\frac{\mu}{\rho}$ is the kinematic viscosity, ρ is the density, μ is the coefficient of viscosity, σ_e is the electrical conductivity, T is the temperature, α is the thermal diffusivity of the fluid and c_p is the specific heat at constant pressure.

The boundary conditions are:

$$y=0:\ u=U_w,\qquad v=0,\quad T=T_w$$

$$y\to\infty:\qquad u\to U_\infty,\qquad T\to T_\infty \quad ...(4)$$

where U_∞ and U_w are constants and represent the free stream and sheet velocities, respectively.

Analysis

The equation of continuity (1) is identically satisfied, if we take the stream function $\psi(x,y)$ such that

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \quad ..(5)$$

The momentum and energy equations (2) and (3) can be transformed into the corresponding ordinary differential equations by introducing the following similarity transformations:

$$\psi(x,y)=\sqrt{2\nu Ux}\,f(\eta) \quad ...(6)$$

$$\frac{T-T_\infty}{T_w-T_\infty}=\theta(\eta) \quad ...(7)$$

where $\eta=y\sqrt{\frac{U}{2\nu x}}$ ...(8)

and $U=U_w+U_\infty$ is composite reference velocity.

The momentum and energy equations (2) and (3) after some simplifications, reduce to the following forms:

$$f'''+ff''-Mf'=0 \quad ...(9)$$

$$\theta''+\Pr f\theta'+\Pr Ecf''^2+M\Pr Ecf'^2=0$$

...(10)

The corresponding boundary conditions are:

$$\eta=0: f=0, f'=1-r, \theta=1$$

$$\eta\to\infty: f'\to r, \theta\to 0 \quad ...(11)$$

where $r=\frac{U_\infty}{U}=\frac{U_\infty}{U_w+U_\infty}$ is a moving parameter $(0\le r\le 1)$ and the prime $(')$ denotes differentiation with respect to η .

$M=\frac{2\sigma_e x B_0^2}{\rho U}$ is the magnetic parameter, $\Pr=\frac{\nu}{\alpha}$ is the Prandtl number and $Ec=\frac{U^2}{c_p(T_w-T_\infty)}$ is the Eckert number.

It is noted that the equation (9) for $r=1$ i.e. $U_w=0$ is the Rossow [14] and Bansal [15] flat-plate flow problems and for $r=0$ i.e. $U_\infty=0$ is the equation for continuous moving surface proposed by Jat and Chaudhary [13]. For $0<r<1$ both the wall and the free stream are moving in positive $x-$ direction have been presented here.

The physical quantities of interest of the problem are the skin-friction coefficient C_f and the Nusselt number Nu, can be expressed, respectively as

$$C_f=\frac{\tau_w}{\frac{\rho U^2}{2}}$$

$$Nu=\frac{xq_w}{\kappa(T_w-T_\infty)} \quad ...(12)$$

where the wall-shear stress τ_w and the heat flux q_w are given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad \text{and} \quad q_w=-\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Thus by using similarity variables, we obtain in non-dimensional form as

$$C_f=\sqrt{\frac{2}{\mathrm{Re}}}f''(0)$$

$$Nu=-\sqrt{\frac{\mathrm{Re}}{2}}\theta'(0)$$

...(13)

where $\mathrm{Re}=\frac{U\ x}{\nu}$ is the local Reynolds number.

Results and Discussions

The equations (9) and (10) with the boundary conditions (11) are the non-linear boundary value problems prescribed at two boundaries, the analytical solution of which is not feasible, therefore these equations have been solved numerically on computer using Runge-Kutta fourth order scheme with a systematic guessing of f''(0) and θ'(0) by the shooting technique until the boundary conditions at infinity are satisfied. The step size Δη=0.001 is used while obtaining the numerical solution and accuracy upto the seventh decimal place i.e. $1\text{x}10^{-7}$, which is very sufficient for convergence. The computations were done by a programme which uses a symbolic and computational computer language Matlab. The numerical values of f''(0) for various values of the moving parameter r and magnetic parameter M are tabulated in table 1.0. it may be observed from the table that f''(0) increases with the increasing value of r for a fixed M where as opposite phenomenon occurs for M for a fixed r. Again the numerical values of θ'(0) for various values of the parameters involved are shown in table 2.0 to table 6.0. It may be observed from the tables that θ'(0) decreases for increasing values of M for a particular values of other parameters, whereas θ'(0) increases with other parameter for a fixed value of M. The corresponding velocity and temperature profiles are shown in figure 2.0 to 4.0 and figure 5.0 to 7.0 respectively against η for various values of parameters. It may be noted from the figures that the thickness of the velocity boundary layer decreases for increasing values of M for a fixed value of r and also increasing value of r for a fixed value of M, whereas the thickness of thermal boundary layer increases for increasing values of any one parameter, where other parameters have values.

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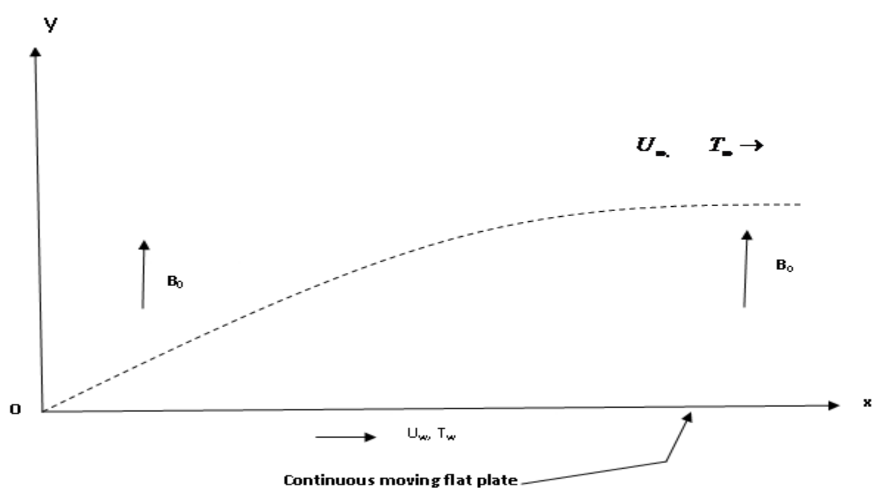

Fig. 1.0. Physical model and coordinate system

Table 1.0. The velocity gradient at the wall f''(0) for $0 \leq r \leq 1$

M	r = 0	r = 0.1	r = 0.2	r = 0.3	r=0.5	r=0.75	r=0.9	r=1.0
0	-0.6357	-0.4987	-0.3663	-0.2387	-0.11561	0.261813	0.394832	0.471153
0.1	-0.7018	-0.5684	-0.4396	-0.3157	-0.08438	0.1682	0.295714	0.368246
0.2	-0.7639	-0.6328	-0.5062	-0.3846	-0.15781	0.089676	0.214558	0.285696
0.3	-0.8225	-0.6925	-0.5671	-0.4466	-0.22167	0.024511	0.149658	0.22174
0.4	-0.8781	-0.7484	-0.6231	-0.5027	-0.27742	-0.02931	0.098453	0.173315
0.5	-0.931	-0.8007	-0.6749	-0.5538	-0.32643	-0.07392	0.058142	0.136966

Table 2.0. -θ'(0) for various values of r when Pr = 0.7, Ec = 0.2

M	r = 0	r = 0.1	r = 0.25
0	0.468941	0.47961	0.486811
0.1	0.450512	0.460087	0.465991
0.25	0.425115	0.433741	0.438522
0.5	0.387778	0.396274	0.401064

Table 3.0. -θ'(0) for various values of Pr when r = 0.1, Ec = 0.2

M	Pr = 0.044	Pr = 0.7	Pr = 1.0	Pr = 7.0
0	0.26325	0.47961	0.575528	1.610184
0.1	0.261969	0.460087	0.549564	1.520087
0.25	0.260266	0.433741	0.514205	1.393814
0.5	0.257887	0.396274	0.463279	1.203547

Table 4.0. -θ'(0) for various values of Pr when r = 0.25, Ec = 0.2

M	Pr = 0.044	Pr = 0.7	Pr = 1.0	Pr = 7.0
0	0.264272	0.486811	0.581348	1.603521
0.1	0.262859	0.465997	0.55416	1.520293
0.25	0.261033	0.438522	0.517838	1.405106
0.5	0.258609	0.401064	0.467483	1.236087

Table 5.0. -θ'(0) for various values of Pr when r = 0.5, Ec = 0.2

M	Pr = 0.044	Pr = 0.7	Pr = 1.0	Pr = 7.0
0	0.264519	0.476359	0.560585	1.449105
0.1	0.26292	0.454252	0.532484	1.381345
0.25	0.26092	0.425657	0.495501	1.286507
0.5	0.258445	0.388806	0.446682	1.149201

Table 6.0. -θ'(0) for various values of Pr when r = 0.75, Ec = 0.2

M	Pr = 0.044	Pr = 0.7	Pr = 1.0	Pr = 7.0
0	0.262961	0.439888	0.505056	1.126966
0.1	0.261215	0.417349	0.477149	1.078594
0.25	0.259081	0.388437	0.440436	1.004917
0.5	0.256604	0.352972	0.393886	0.892373

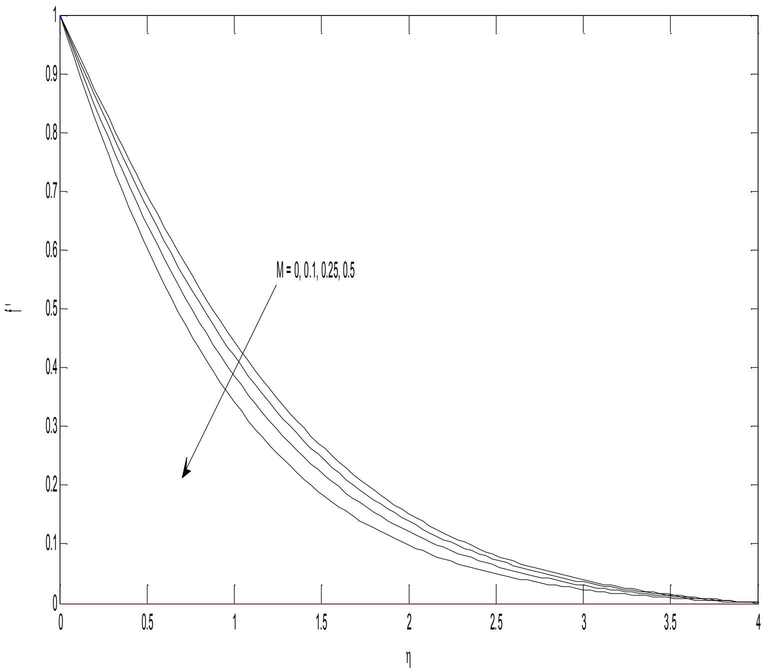

Fig. 2.0 Velocity profile against η for various values of M when r = 0

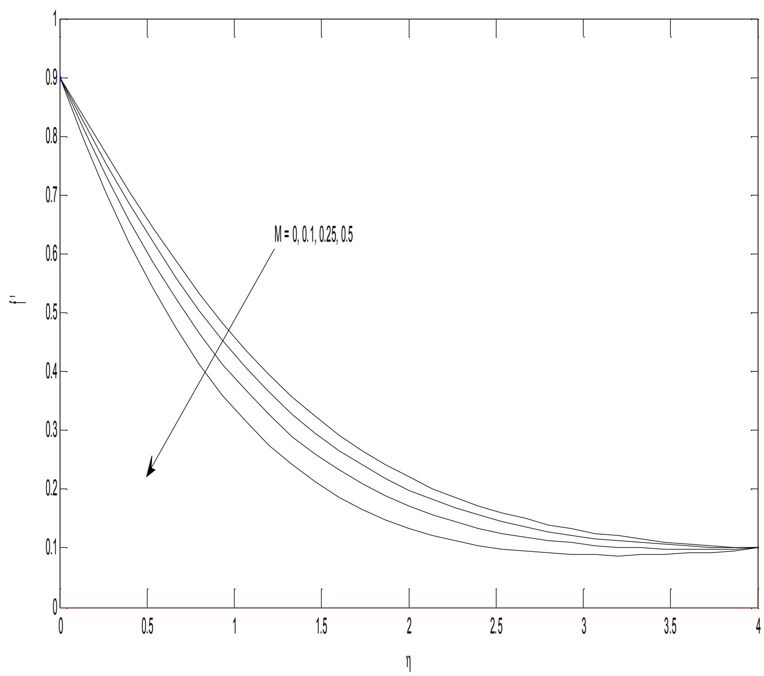

Fig. 3.0 Velocity profile against η for various values of M when r = 0.1

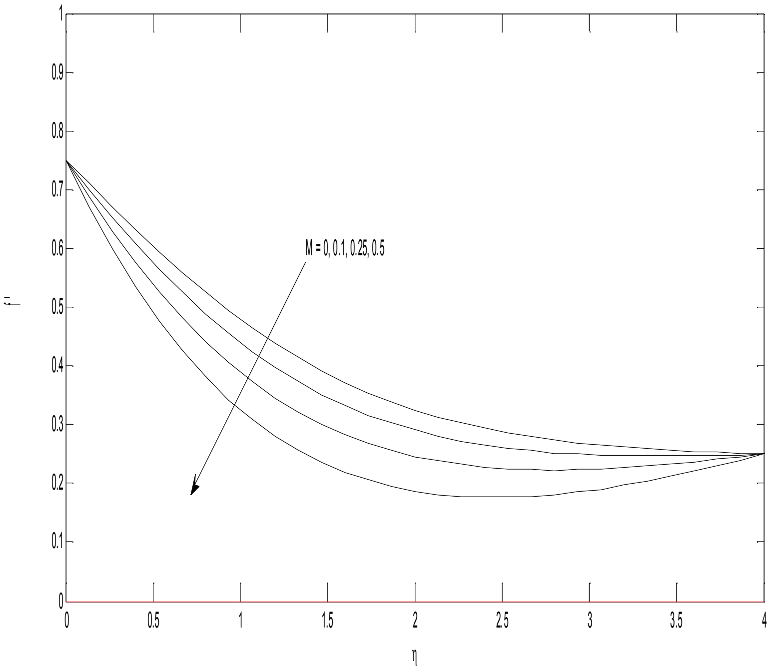

Fig. 4.0 Velocity profile against η for various values of M when r = 0.25

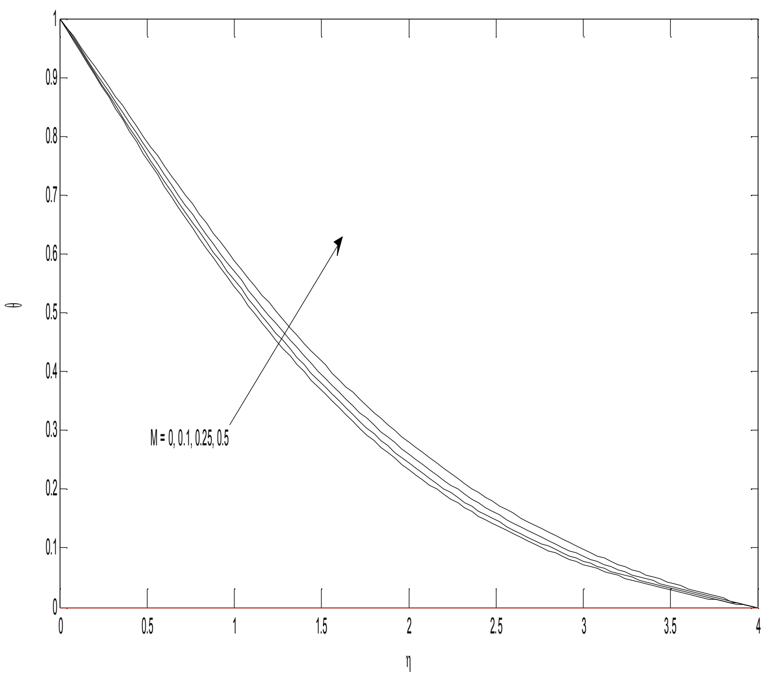

Fig. 5.0 Temperature profile against η for various values of M when r = 0.1, Pr = 0.7 and Ec = 0.2

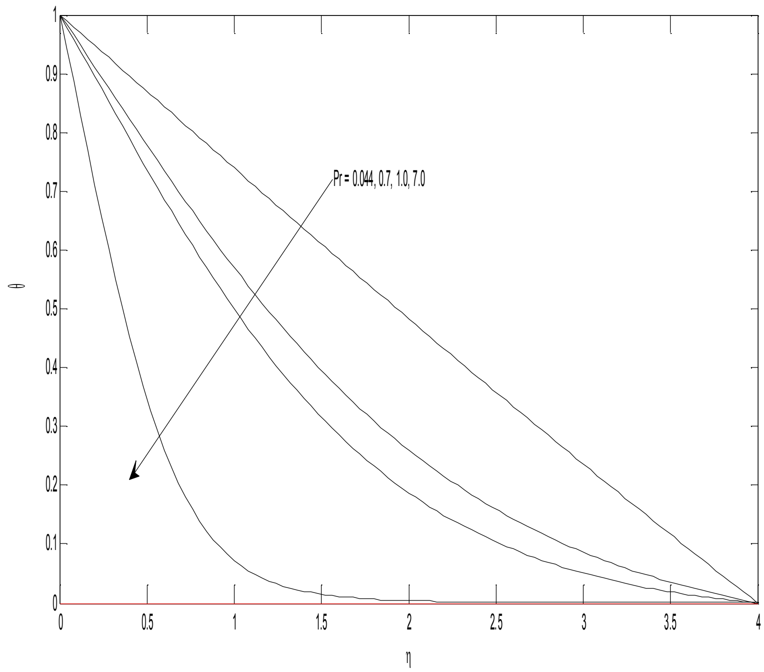

Fig. 6.0 Temperature profile against η for various values of Pr when r = 0.1, M = 0.25 and Ec = 0.2

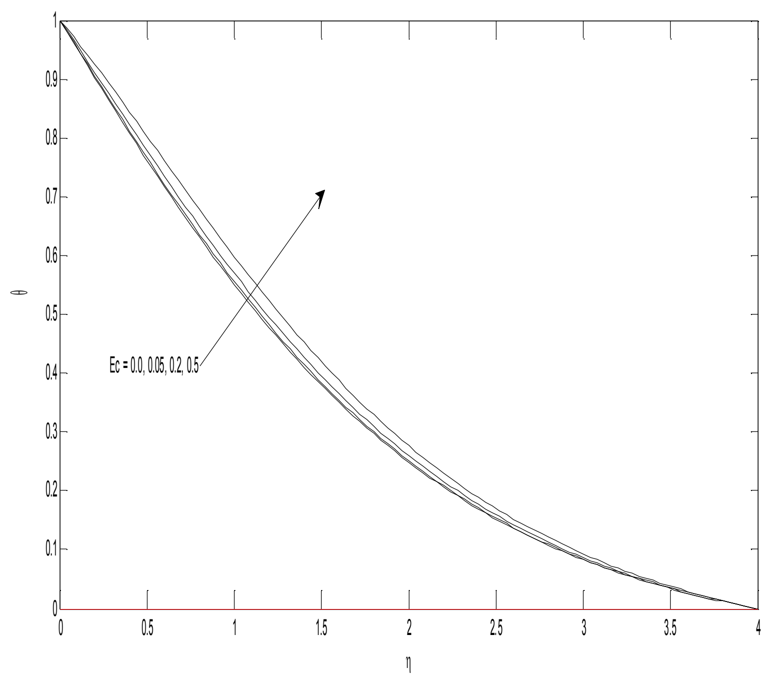

Fig. 7.0 Temperature profile against η for various values of Ec when r = 0.1, M = 0.25 and Pr = 0.7