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Efficiency of Neighbour Balanced Block Designs for Correlated Observations

R. Senthil kumar[1*] and C. Santharam[2†]

[1,2] Department of Statistics, Loyola College, Chennai – 600034 (TN) INDIA.

Corresponding Addresses:

[*]Senthilkumar0185@gmail.com, [†]santharamsdeep@yahoo.com

Research Article

Abstract: Neighbour Balanced Block Designs for observations correlated within a block have been investigated. The performance of a series of Complete Neighbour Balanced Design for Autoregressive(AR) and Nearest Neighbour (NN) error correlation structure is studied when generalized least squares estimation is used by Tomar & Seema Jaggi (2007). In this paper, we have investigated the efficiency of Neighbour Balanced Block Designs for Moving Average with First Order (MA(1)) and Nearest Neighbour (NN) correlation structures. We have observed that the efficiency is high, in the case of Complete Block Designs for NN correlation structure. However, the efficiency of left and right neighbour effects is more as compared to direct effects of treatments under both the structures.

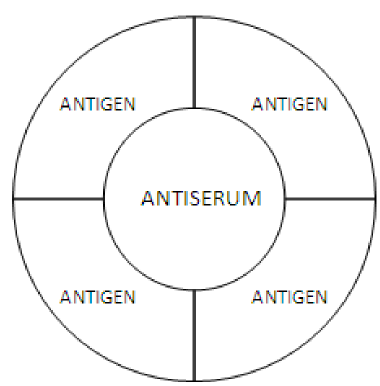
Key Words: Neighbour balanced design; Correlated observations; Generalized least squares; Moving average; Nearest neighbour; Efficiency.

1. Introduction

Experiments conducted in agriculture often show neighbour effects i.e., the response on a given plot is affected by the treatments on the neighbouring plots as well as by the treatment applied to that particular plot. When treatments are varieties, neighbour effects may be caused by differences in height, root vigor, or germination date especially on small plots, which are used in plant breeding experiments. Treatments such as fertilizer, irrigation, or pesticide may spread to adjacent plots causing neighbour effects. Such experiments exhibit neighbour effects, because the effect of having no treatment as a neighbour is different from the neighbour effects of any treatment. Competition or interference between neighbouring units in field experiments can contribute to variability in experimental results and lead to substantial losses in efficiency. In case of block design setup, if each block is a single line of plots and blocks are well separated, extra parameters are needed for the effect of left and right neighbours. An alternative is to have border plots on both ends of every block. Each border plot receives an experimental treatment, but it is not used for measuring the response variable. These border plots do not add too much to the cost of one-dimensional experiments.

Serology is a branch of Biometrics which is concerned with the study of virus and viral preparations. Many studies concerned with viral preparation require the arrangement of antigens in a place so that each antigen has two other antigens as its neighbours. In analysis of such experiment, the classical design may not perform efficiently. Therefore Rees, D. H. (1967) introduced neighbour structure. The following is the experiment considered by Rees, D. H. (1967) for the use of Nearest Neighbour Balanced Block Design. As seen in the figure below, the observations available from the neighbouring plats are correlated, therefore the usual assumption like independence of observation in the analysis of classical comparative experiments may not be valid. Therefore there is necessity for the use of Nearest Neighbour Balanced Block Design. In biometrical science, we can cite many areas where this kind of correlated structure exists. Now, consider the viral preparation given above. Let there be t kinds of antigens to be arranged on b plates, each containing k antigens. Each antigen appear r times (but not necessarily on r different plates) and is a neighbour of every other antigen exactly λ times.

Rees used circular neighbouring block designs, where as in the present paper we are dealing with one dimensional block designs. Rees, D. H. (1967) used incomplete neighbour design $(k < t)$ in his experiment.



The estimates of treatment differences may therefore deviate because of *interference* from neighbouring units. Neighbour balanced block designs, where in the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbours, are used for modeling and controlling interference effects between neighbouring plots. Azais *et al*. (1993) obtained a series of efficient neighbour designs with border plots that are balanced in $v-1$ blocks of size v and v blocks of size $v-1$, where v is the number of treatments. Druilhet (1999) studied optimality of circular neighbour balanced block designs obtained by Azais *et al*. (1993). Bailey (2003) has given some designs for studying one-sided neighbour effects. These neighbour balanced block designs have been developed under the assumption that the observations within a block are uncorrelated. In situations where the correlation structure among the observations within a block is known, may be from the data of past similar experiments, it may be advantageous to use this information in designing an experiment and analyzing the data so as to make more precise inference about treatment effects (Gill and Shukla, 1985). Kunert *et al*. (2003) considered two related models for interference and have shown that optimal designs for one model can be obtained from optimal designs for the other model. Martin and Eccelston (2004) have given variance balanced designs under interference and dependent observations. Tomar and Seema Jaggi (2007) observed that efficiency is quite high, in case of complete block designs for both AR(1) and NN correlation structures. Ruban, Santharam and Ramesh (2012) observed that the MV-optimality Nearest Neighbour Balanced Block Designs (NNBD) using AR(1), MA(1), ARMA(1,1), AR(2) and MA(2) models.

In this article, neighbour balanced block designs for observations correlated within a block have been investigated for the estimation of direct as well as left and right neighbour effects of treatments. The performance of these designs for moving average with first order and nearest neighbour error correlation structure is studied when generalized least squares estimation is used.

2. Model and Definition

The designs considered here are assumed to be in linear blocks, with neighbour effects only in the direction of the blocks (say left-neighbour or right-neighbour or both). Because the effect of having no treatment differs from the neighbor effects of any treatment, designs with border plots have been considered, which is, designs with one point added at each end of each block. The border plots receive treatments but are not used for measuring the response variables. The plots, which are not on the borders, are inner plots. The length of a block is the number of its inner plots. It is further assumed that all the designs are **circular**, that is the treatment on border plots is same as the treatment on the inner plot at the other end of the block.

Let Δ be a class of binary neighbour balanced block designs with $n = bk$ units that form b blocks each containing k units. Y_{ij} be the response from the i^{th} plot in the j^{th} block $(i = 1,2,\ldots,k; j = 1,2,\ldots,b)$. The layout includes border plots at both ends of every block, i.e. at 0^{th} and $(k+1)^{th}$ position and observations for these units are not modeled. The following fixed effects additive model is considered for analyzing a neighbour balanced block design under correlated observations:

$$Y_{ij} = \mu + \tau_{(i,j)} + l_{(i-1,j)} + \gamma_{(i+1,j)} + \beta_j + e_{ij} \qquad (2.1)$$

where μ is the general mean, $\tau_{(i,j)}$ is the direct effect of the treatment in the i^{th} plot of j^{th} block, β_j is the effect of the j^{th} block. $l_{(i-1,j)}$ is the left neighbour effect due to the treatment in the $(i-1)^{th}$ plot of j^{th} block. $\gamma_{(i+1,j)}$ is the right neighbour effect due to the treatment in the $(i+1)^{th}$ plot in j^{th} block. e_{ij} are error terms distributed with mean zero and a variance-covariance structure $V = I_b \otimes \Lambda$ (I_b is an identity matrix of order b and $\otimes$ denotes the kronecker product). Assuming no correlation among the observations between the blocks and correlation structure between plots within a block to be the same in each block, Λ is the correlation matrix of k observations within a block.

If the errors within a block follow a First Order Moving average MA(1) structure, then Λ is a matrix with diagonal entries as 1 and $(i,i')^{th}$ entry $(i,i' = 1,2,\ldots,k)$ as ρ, when $|i-i'| = 1$, otherwise zero (Gill and Shukla, 1985). The Nearest Neighbour

(NN) correlation structure, the Λ is a matrix with diagonal entries as 1 and off-diagonal entries as ρ.

Model (2.1) can be rewritten in the matrix notation as follows:

$$Y = \mu 1 + \Delta' \tau + \Delta'_1 l + \Delta'_2 \gamma + D' \beta + e \quad (2.2)$$

Where Y is $n \times 1$ vector of observations, 1 is $n \times 1$ vector of ones, Δ' is an $n \times v$ incidence matrix of observations versus direct treatments, τ is $v \times 1$ vector of direct treatment effects, Δ'_1 is a $n \times v$ matrix of observations versus left neighbour treatment, Δ'_2 is a $n \times v$ matrix of observations versus right neighbour treatment, l is $v \times 1$ vector of left neighbour effects, γ is $v \times 1$ vector of right neighbour effects, D' is an $n \times b$ incidence matrix of observations versus blocks, β is $b \times 1$ vector of block effects and e is $n \times 1$ vector of errors.

The joint information matrix for estimating the direct and neighbour (left and right) effects under correlated observations estimated by generalized least squares is obtained as follows:

$$C = \begin{bmatrix} \Delta(I_b \otimes \wedge^*)\Delta' & \Delta(I_b \otimes \wedge^*)\Delta'_1 & \Delta(I_b \otimes \wedge^*)\Delta'_2 \\ \Delta_1(I_b \otimes \wedge^*)\Delta' & \Delta_1(I_b \otimes \wedge^*)\Delta'_1 & \Delta_1(I_b \otimes \wedge^*)\Delta'_2 \\ \Delta_2(I_b \otimes \wedge^*)\Delta' & \Delta_2(I_b \otimes \wedge^*)\Delta'_1 & \Delta_2(I_b \otimes \wedge^*)\Delta'_2 \end{bmatrix} \quad (2.3)$$

with

$$\wedge^* = \wedge^{-1} - \left(1'_k \wedge^{-1} 1_k\right)^{-1} \wedge^{-1} 1_k 1'_k \wedge^{-1}$$

The above $3v \times 3v$ information matrix (C) for estimating the direct effects and neighbour effects of treatments in a block design setting is symmetric, non-negative definite with row and column sums equal to zero.

The information matrix for estimating the direct effects of treatments from (2.3) is as follows:

$$C_\tau = C_{11} - C_{12} C_{22}^{-1} C_{21} \quad (2.4)$$

where $C_{11} = \Delta(I_b \otimes \wedge^*)\Delta'$

$C_{12} = \left[\Delta(I_b \otimes \wedge^*)\Delta'_1 \quad \Delta(I_b \otimes \wedge^*)\Delta'_2\right]$ and

$$C_{22} = \begin{bmatrix} \Delta_1(I_b \otimes \wedge^*)\Delta'_1 & \Delta_1(I_b \otimes \wedge^*)\Delta'_2 \\ \Delta_2(I_b \otimes \wedge^*)\Delta'_1 & \Delta_2(I_b \otimes \wedge^*)\Delta'_2 \end{bmatrix}$$

Similarly, the information matrix for estimating the left neighbour effect of treatments (C_l) and right neighbour effect of treatments (C_γ) can be obtained. We have given some definitions associated with the neighbour balanced block designs under correlated observations.

Definition 2.1. A block design is *neighbour balanced* if every treatment has every treatment appearing as a neighbour (left and right) constant number of times (say, λ).

Definition 2.2. A neighbour balanced block design is called *pair-wise uniform* on the plots if each treatment $s(=1,2,\ldots,v)$ occurs equally often in each plot position $i(=1,2,\ldots,k)$ and each pair of treatments s and s', $s \neq s'(=1,2,\ldots,v)$ occurs equally often (α times) within the same block in each unordered pair of plot positions i and i', $i \neq i'(=1,2,\ldots,k)$.

Definition 2.3. A neighbour balanced block design with correlated observations permitting the estimation of direct and neighbour (left and right) effects, is called *variance balanced* if the variance of any estimated elementary contrast among the direct effects is constant, say V_1, the variance of any estimated elementary contrast among the left neighbour effect is constant, say V_2, and the variance of any estimated elementary contrast among the right neighbour effects is constant, say V_3. The constant V_1, V_2 and V_3 may not be equal. A block design is *totally balanced* if $V_1 = V_2 = V_3$.

3. Design

Tomer *et al.* (2005) has constructed neighbour balanced block design with parameters v (prime or prime power), $b = v(v-1)$, $r = (v-1)(v-m)$, $k = (v-m)$, $m = 1,2,\ldots,v-4$ and $\lambda = (v-m)$ using Mutually Orthogonal Latin Squares (MOLS) of order v. This series of design bas been investigated under the correlated error structure. It is seen that the design turns out to be pair-wise uniform with $\alpha = 1$ and also variance balanced for estimating direct (V_1) and neighbour effects $(V_2 = V_3)$.

Example 3.1.

Let $v = 5$ and $m = 0$. The following is a neighbour balanced pair-wise uniform complete block design with parameters $v = 5$, $b = 20$, $r = 20$, $k = 5$, $\lambda = 5$ and $\alpha = 1$:

5	1	2	3	4	5	1
1	2	3	4	5	1	2
2	3	4	5	1	2	3
3	4	5	1	2	3	4
4	5	1	2	3	4	5
1	2	3	4	5	1	2
2	3	4	5	1	2	3
3	4	5	1	2	3	4
4	5	1	2	3	4	5
5	1	2	3	4	5	1
2	3	4	5	1	2	3
3	4	5	1	2	3	4
4	5	1	2	3	4	5
5	1	2	3	4	5	1
1	2	3	4	5	1	2
3	4	5	1	2	3	4
4	5	1	2	3	4	5
5	1	2	3	4	5	1
1	2	3	4	5	1	2
2	3	4	5	1	2	3
4	5	1	2	3	4	5
5	1	2	3	4	5	1
1	2	3	4	5	1	2
2	3	4	5	1	2	3
3	4	5	1	2	3	4

The information matrices for estimating the direct and neighbour effects (left and right) of treatments for MA(1) structure with $\rho = 0.1$ is obtained as given below:

$$C_\tau = 14.420\left[I - \frac{J}{5}\right]$$ and

$$C_l = C_\gamma = 15.320\left[I - \frac{J}{5}\right]$$

Similarly for NN structure,

$$C_\tau = 16.572\left[I - \frac{J}{5}\right]$$

$$C_l = C_\gamma = 17.391\left[I - \frac{J}{5}\right]$$

These matrices have been worked out using **R** package.

Example 3.2.

Let $v = 6$ and $m = 0$. The following is a neighbour balanced pair-wise uniform complete block design with parameters $v = 6$, $b = 30$, $r = 30$, $k = 6$, $\lambda = 6$ and $\alpha = 1$:

1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6

The information matrices for estimating the direct and neighbour effects (left and right) of treatments for MA(1) structure with $\rho = -0.1$ is obtained as given below:

$$C_\tau = 21.728\left[I - \frac{J}{5}\right]$$ and

$$C_l = 22.076 = \left[I - \frac{J}{5}\right]$$

Similarly for NN structure,

$$C_\tau = 21.849\left[I - \frac{J}{5}\right]$$

$$C_l = C_\gamma = 22.227\left[I - \frac{J}{5}\right]$$

4. Comparison of Efficiency

The quantitative measure of efficiency of the designs in Section 3 has been made in comparison to the universally optimal neighbour balanced design for v

treatments in $(v-1)$ complete blocks of Azais *et al*. (1993) considering observations to be correlated within the blocks. We compare the average variance of an elementary treatment contrast $\hat{\tau}_s - \hat{\tau}_{s'}$ in both cases. The average variance of an elementary treatment contrast (Kempthorne, 1956) for direct effects of the neighbour balanced design of Azais *et al*. (1993) estimated by generalized least squares methods, is given by

$$V_A = \frac{2\sigma^2}{v-1}\sum_{s=1}^{v-1}\theta_s^{-1}$$

Where θ_s's are the $(v-1)$ non-zero eigen values of C_τ for Azais *et al*. (1993), σ^2 is the variance of an observation. The efficiency factor (E_τ) for direct effects of the neighbour balanced pair-wise uniform block design is thus given as:

$$E_\tau = \frac{(v-1)\sum_{s=1}^{v-1}\theta_s^{-1}}{(v-m)\sum_{s=1}^{v-1}\delta_s^{-1}}$$

δ_s's are the $(v-1)$ non-zero eigen values of C_τ of the design given in section 3. Similarly the efficiency (E_l) and (E_γ) for neighbour effects (left and right) of treatments is obtained. The ranges of correlation coefficient (ρ) for different correlation structures investigated are $|\rho| \le 0.50$ for MA(1) and NN correlation structures. For these ranges, the matrix of correlation coefficients among observations within a block is positive definite. For $\rho = 0$, the efficiency is that of totally balanced designs obtained by Tomer *et al*. (2005).

5. Conclusion

From Table 1, we have concluded that, the parameters of neighbour balanced pair-wise uniform complete block design as given in Section 3 for $v = 5$ and 6 $(m = 0)$ along with the efficiency for direct, left and right neighbour effects. The efficiencies have been reported under the MA(1) and NN correlation structures with ρ in the interval -0.5 to 0.5. It is seen that efficiency is high, in case of complete block designs for NN correlation structure. Moreover, as the block size increases, the efficiency increases because the plots in a block become more heterogeneous. However, the efficiency of left and right neighbour effects is more as compared to direct effects of treatments under both the structures. When block sizes are large and neighbouring plots are highly correlated, generalized least squares for estimation of direct and neighbour effects (left and right) can be used.

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Appendix: 1

Table 1: Efficiency of neighbour balanced pair-wise uniform complete block designs

Parameters					Correlation Structure							
					MA(1)				NN			
v	b	m	r	$k=\lambda$	ρ	E_τ	E_l	E_γ	ρ	E_τ	E_l	E_γ
5	20	0	20	5	-0.5	0.64261	0.53886	0.54848	-0.5	0.52074	0.51483	0.52693
					-0.4	0.66631	0.63644	0.66063	-0.4	0.56287	0.53710	0.56352
					-0.3	0.71832	0.69436	0.69760	-0.3	0.62530	0.60242	0.58364
					-0.2	0.76633	0.70641	0.71013	-0.2	0.66174	0.65598	0.66174
					-0.1	0.76103	0.77614	0.74742	-0.1	0.71680	0.71779	0.72109
					0.0	0.80000	0.80000	0.80000	0.0	0.80000	0.80000	0.80000
					0.1	0.81629	0.84870	0.83885	0.1	0.89193	0.94829	0.90732
					0.2	0.86839	0.86158	0.86363	0.2	0.99359	0.99120	1.02672
					0.3	0.90889	0.91996	0.88717	0.3	1.13873	1.17027	1.15689
					0.4	0.93365	0.88157	0.88499	0.4	1.37226	1.36867	1.33303
					0.5	0.89447	0.69189	0.69884	0.5	1.64620	1.66292	1.64921
6	30	0	30	6	-0.5	0.70530	0.58822	0.70963	-0.5	0.54261	0.54877	0.53141
					-0.4	0.72261	0.69922	0.71010	-0.4	0.58753	0.58692	0.58983
					-0.3	0.73073	0.73851	0.75539	-0.3	0.63785	0.64068	0.62985
					-0.2	0.77151	0.78147	0.77456	-0.2	0.68384	0.69532	0.65391
					-0.1	0.79159	0.80102	0.81227	-0.1	0.75234	0.76089	0.77723
					0.0	0.80000	0.80000	0.80000	0.0	0.80000	0.80000	0.80000
					0.1	0.88099	0.85001	0.86476	0.1	0.93761	0.91002	0.93524
					0.2	0.89240	0.89178	0.88415	0.2	1.05008	1.04844	1.04667
					0.3	0.92133	0.91276	0.90509	0.3	1.20284	1.20351	1.20192
					0.4	0.94970	0.87412	0.92125	0.4	1.40943	1.40854	1.40744
					0.5	0.95067	1.02657	1.13100	0.5	1.69035	1.69664	1.69907