An Inventory Model for Weibull Deteriorating Items with Linear Demand and Permissible Delay in Payments under Inflation

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Research Article

Abstract: This paper proposes an optimal replenishment policy for Weibull deteriorating items with a permissible delay in payment under inflation over a finite planning horizon and the demand rate is a linear function of time.

Keywords: Inventory, Inflation, Weibull-distribution, Permissible delay in payments.

Introduction

Many Mathematical models have been developed for controlling inventory and in the earlier models many researchers consider the constant demand rate which is a feature of static environment while the dynamic environment nothing is fixed or constant. So in most of the cases the demand for items increases with time. But most of the organizations are working towards increasing demand of their items with time. Basically the items that are stored for future use always loose part of their value with passage of time. In inventory this phenomenon is known as deterioration of items. The rate of deterioration is very small in some items like hardware, glassware, toys and steel. The items such as medicine, vegetables, gasoline alcohol, radioactive chemicals and food grains deteriorate rapidly overtime so the effect of deterioration of physical goods cannot be disregarded in many inventory systems. The deterioration of goods is a realistic phenomenon in systems. The controlling of manv inventorv deteriorating items becomes a measure problem in any inventory system. The researchers have continuously modified the deteriorating inventory models so as to become more practicable and realistic. Initially Chare & Schrader [6] consider an inventory model with a constant rate of decay. Covert and Philip [5] extended Ghare and Schrader's model for variable rate of deterioration by assuming Weibull's two parameter distribution function. Later these models are modified by Philip [14], Mishra [11], Chakrabarty, Giri and Choudhary [4]. Hence the realistic models treat with a time varying deterioration rate. In the models mentioned above the inflation and time value of money have no

significant role with inventory policy. It is realized that most of the countries are suffering from high inflation that leads decline of time value of money. Hence in many inventory models ignorance of inflation and time value of money is disregarded. Buzacott [2] developed an inventory model with inflation, several researchers developed their model by considering various inflation rate are time value of money. Bierman and thomas [1], Chandra and Bahner [3], Gor and Shah [7] Kuolung Huo [9], Mishra [12], Moon and Giri [13] developed the value able models by considering various inflation rate.

In the traditional EOQ models the purchaser pays for his items as soon as he received the items but in business organizations the supplier may allow a credit period to encourage the customers delay in payment by the purchaser to the supplier in an alternate way of price discount. The valuable models in this regards are given by Goyal [8], Lio [10]. It is empirically observed that the failure and life expectancy of many items can be expressed in terms of Weibull distribution function.

In the present paper we consider a Weibull distribution function of two parameters for deterioration. The demand rate is considered as a linear function of time and solves the model analytically by numerical examples.

Notations and Assumptions:- We consider the following assumptions

| Н | = | Length of finite planning |
|------|---|---|
| | | Horizon. |
| R | = | (a+bt), Demand Rate |
| р | = | Selling price per unit at time t |
| | | = o |
| P(t) | = | pe ^{rT} , Instantaneous selling |
| | | price per unit. |
| C(t) | = | ce ^{rT} , Instantaneous ordering |
| | | cost per order. |
| Ie | = | Interest earned per annum by |
| | | the Retailer. |

| Ic | = | Interest charged in stock per annum by the supplier. | Ic | = | Interest charged for unsold items at initial time. |
|-------------|---|--|-------------|---------------|---|
| T_1 | = | Allowable delay period during settlement of the account. | I_E | = | Interest earned from sales during the permission |
| Q | = | Optimum ordering quantity. | | | delay period [o, T_1]. |
| Т | = | Optimum length of cycle time. | Hence total | l cost TC (T) | |
| $\theta(t)$ | = | $\alpha\beta t^{\beta-1}$, Weibull distribution, | and the gro | ss revenue | $= R(P-C) e^{rT}$ |
| | | $0 < \alpha < 1, \beta > 1.$ | Assumptio | ons:- | |
| r | = | inflation rate. | (i) | Inflation ra | te is constant. |
| В | = | e ^{r∺} − 1 | (ii) | U | are not allowed |
| I (t) | = | Instantaneous inventory level | (iii) | Lead time i | |
| TC (T) | = | Total relevant cost over [O,H] | (iv) | - | or replacement of deteriorated |
| NP (T) | = | Total relevant profit over | | items durin | 6 1 |
| | | finite planning horizon. | (v) | | d of trade credit period the |
| Oc | = | Ordering cost | | 1 | pay off all units ordered and |
| Dc | = | Deterioration cost | | 0 1 0 | ing for the interest charged on |
| Hc | = | Holding cost | | the items in | 1 STOCK. |

Mathematical Formulation

Let H=nT, where n is the number of replenishments occur during the period H, be the planning horizon. Then the differential equation of instantaneous level of inventory I(t) is given by,

$$\frac{dI}{dt} + \theta(t)I = -(a + bt), o < t < T$$

$$\frac{dI}{dt} + \alpha\beta t^{\beta-1}I = -(a + bt), o < t < T$$
.....(1)

With boundary conditions

$$I(0) = Q$$
, and $I(T) = 0$

Then the solution of (1) is

$$I(t) = a(T - t) + \frac{b}{2}(T^{2} - t^{2}) - \frac{a\alpha\beta}{(\beta + 1)}(T^{\beta + 1} - t^{\beta + 1}) - \frac{b\alpha\beta}{2(\beta + 2)}(T^{\beta + 2} - t^{\beta + 2}).....(2)$$

Using the initial condition I (o) = Q, Then the initial ordering of quantity is

$$Q=aT + \frac{bT}{2}^{2} - \frac{a\alpha\beta T^{\beta+1}}{\beta+1} - \frac{b\alpha\beta T^{\beta+2}}{2(\beta+2)} \dots (3)$$

The ordering cost or set up cost is given by
 $O_{c} = A (o) + A (T) + A (2T) + \dots + A \{(n-1)T\}$
Using $A (t) = A e^{rT}$, then
 $O_{c} = A_{r} + A e^{rT} + A^{2rT} + \dots + A e^{(n-1)rT}$
 $= \frac{A[e^{rH} - 1]}{e^{rT} - 1}, \text{ where } H = nT$
 $= ABe^{-rT} \left[\frac{1}{rT} + \frac{rT}{12} + \frac{1}{2} \right], \text{ where } B = e^{rH} - 1$
Thus
 $O_{c} = ABe^{-rT} \left[\frac{1}{rT} + \frac{rT}{12} + \frac{1}{2} \right] - \dots + (4)$
The deterioration cost is given by
 $D_{c} = I (o) - RT$
 $D_{c} = \frac{BC}{(e^{rT} - 1)} \left[\frac{bT^{2}}{2} - \frac{a\alpha\beta T^{\beta+1}}{\beta+1} - \frac{b\alpha\beta T^{\beta+2}}{2(\beta+2)} - btT \right]$

$$D_{C} = BCe^{-rT} \left(\frac{1}{2} + \frac{1}{rT} + \frac{rT}{12} \right) \left(\frac{bT^{2}}{2} - \frac{a \alpha \beta T^{\beta+1}}{\beta+1} - \frac{b \alpha \beta T^{\beta+2}}{2(\beta+2)} - btT \right)^{-\dots-(5)}$$

The inventory holding cost is given by

$$H_{c} = h \sum_{k=0}^{n-1} C(KT) \int_{0}^{T} I(t) dt$$

$$H_{c} = \frac{BCh}{(e^{rT} - 1)} \left[\frac{aT^{2}}{2} + \frac{bT^{3}}{3} - \frac{a\alpha\beta T^{\beta+2}}{\beta+2} - \frac{b\alpha\beta T^{\beta+3}}{2(\beta+3)} \right] \dots \dots (6)$$

Depending upon the cycle time T and customer's choice the following two cases arise

Case I: - Interest charged in [O, H]

Since in [O, H], the cycle length T is greater than the permissible delay time T_1 , Then the interest charged in $[T_1, T]$ is

$$IC_{1} = Ic \sum_{k=0}^{n-1} C(KT) \int_{T_{1}}^{1} I(t) dt$$

$$= \frac{BCIc}{e^{rT} - 1} \left[\frac{a}{2} \left(T^{2} - 2TT_{1} + T_{1}^{2} \right) + \frac{b}{2} \left(\frac{2T^{3}}{3} - T_{1}T^{2} + \frac{T_{1}^{3}}{3} \right) - \frac{a\alpha\beta}{\beta + 1} \left(\frac{(\beta + 1)T^{\beta + 2}}{\beta + 2} - T_{1}T^{\beta + 1} + \frac{T_{1}^{\beta + 2}}{\beta + 2} \right) - \frac{b\alpha\beta}{2(\beta + 2)} \left(\frac{(\beta + 2)T^{\beta + 3}}{\beta + 3} - T_{1}T^{\beta + 2} + \frac{T_{1}^{\beta + 3}}{\beta + 3} \right) \right]^{-(2T)}$$

$$(7)$$

The Interest earned in [0, H] is given by

$$IE_{1} = Ie \sum_{k=o}^{n-1} P(KT) \int_{o}^{T_{1}} (a+bt) t dt$$
$$IE_{1} = \frac{BPIe}{e^{rT} - 1} \left(\frac{aT_{1}^{2}}{2} + \frac{bT_{1}^{3}}{3} \right) \dots \dots (8)$$

The Average cost: - The total average cost per unit in the interval [0, H] is,

$$TC(T) = \frac{1}{T} \left[O_{c} + D_{c} + H_{c} + Ic_{1} - IE_{1} \right] \dots (9)$$

Net Profit: - The net profit in the interval [0, H] is given by

 $NP_{1}(T) = R (P-C)e^{rT} - TC (T)$ $NP_{1}(T) = [(a+bt) (P-C)e^{rT} - TC (T)].....(10)$ For optimality condition of total average cost: - We have

$$\frac{\partial}{\partial T} [TC (T)] = \frac{\partial}{\partial T} \left[\frac{Oc}{T} \right] + \frac{\partial}{\partial T} \left[\frac{Dc}{T} \right] + \frac{\partial}{\partial T} \left[\frac{Hc}{T} \right] + \frac{\partial}{\partial T} \left[\frac{I_{C_1}}{T} \right] - \frac{\partial}{\partial T} \left[\frac{IE_1}{T} \right]$$
Now find separately $\frac{\partial}{\partial T} \left(\frac{O_c}{T} \right)$, $\frac{\partial}{\partial T} \left(\frac{DC}{T} \right)$, etc.
So $\frac{\partial}{\partial T} \left(\frac{O_c}{T} \right) = -ABe^{-rT} \left[\frac{rT}{2} + \frac{3}{2T^2} + \frac{r^2}{12} + \frac{2}{rT^3} \right]$ (11)
 $\frac{\partial^2}{\partial T^2} \left(\frac{O_c}{T} \right) = ABe^{-rT} \left[\frac{r^2}{2T} + \frac{2r}{T^2} + \frac{5}{T^3} + \frac{r^3}{12} \right]$ (12)
Since $\frac{\partial^2}{\partial T^2} \left(\frac{O_c}{T} \right) > O$
 $\frac{\partial(D_c)}{\partial T} = BCe^{-rT} \left[\frac{b(rT-1)}{2} + \frac{bt}{T} - \frac{brT}{4} \left(1 - \frac{rt}{3} \right) - \frac{r^2T^2b}{24} + \frac{a\alpha\beta T^{\beta-1}}{\beta+1} \right]$

$$\begin{split} &+ \frac{ra\beta}{2} T^{\beta} \left(\frac{b}{2(\beta+2)} + \frac{a}{\beta+1} \right) + \frac{ra\beta}{4} T^{\beta+1} \left(\frac{b}{\beta+2} + \frac{a}{3(\beta+1)} \right) + \frac{b\alpha\beta}{24} r^{2} T^{\beta+2} + \frac{b}{(r^{2}+2)} + \frac{b}{4} \left(1 - \frac{rT}{r^{2}} \right) + \frac{brT}{12} - \frac{a\alpha\beta(\beta-1)T^{\beta+2}}{r(\beta+1)} - \frac{a\beta}{2} T^{\beta+2} + \frac{a}{\beta+1} \right) - \frac{\alpha\beta(\beta+1)T^{\beta+2}}{24(\beta+2)} - \frac{\alpha\beta\beta^{2}T^{\beta+1}}{\beta+1} - \frac{\alpha\beta\beta^{2}T^{\beta+1}}{24(\beta+2)} - \frac{br\beta\beta(\beta+2)T^{\beta+1}}{24(\beta+2)} + \frac{a}{\beta+1} \right) \\ &- \frac{\alpha\beta\beta(\beta-1)T^{\beta+2}}{2} + \frac{a}{3(\beta+1)} - \frac{b}{T} + \frac{br^{2}T}{24} - \frac{ar\betaT^{\beta+1}}{\beta+1} - \frac{\alpha\beta\beta^{2}T^{\beta+1}}{\beta+1} - \frac{\alpha\beta\beta^{2}T^{\beta+1}}{24(\beta+2)} + \frac{a}{\beta+1} \right) \\ &- \frac{\alpha\beta\beta(\beta-1)T^{\beta+2}}{2} + \frac{a}{3(\beta+1)} - \frac{b}{T} - \frac{b}{24} \frac{a}{\beta+2} + \frac{a}{\beta+1} \right) + \frac{r\alpha\beta\beta(\beta+1)T^{\beta}}{24} - \frac{ar\betaT^{\beta+1}}{2} - \frac{a\betaT^{\beta+1}}{2(\beta+2)} + \frac{a}{\beta+1} \right) \\ &- \frac{\alpha\beta(\beta-1)T^{\beta-2}}{\beta+1} + \frac{r\alpha\beta^{2}T^{\beta+1}}{2} \left(\frac{b}{2(\beta+2)} + \frac{a}{\beta+1} \right) + \frac{r\alpha\beta(\beta+1)T^{\beta}}{4} \left(\frac{b}{\beta+2} + \frac{ar}{3(\beta+1)} \right) + \frac{b\alpha\beta(\beta^{2}T^{\beta+1})}{24} - \frac{a\betaT^{\beta+2}}{24} + \frac{b}{\beta+2} \right) \\ &- \frac{a\alpha\beta(\beta-1)T^{\beta-2}}{r(\beta+1)} - \frac{a(\beta-1)\beta^{2}T^{\beta-1}}{2} \left(\frac{b}{2(\beta+2)} + \frac{a}{\beta+1} \right) - \frac{a(\beta+1)\beta^{2}T^{\beta+1}}{4} \left(\frac{b}{\beta+2} + \frac{ar}{3(\beta+1)} \right) - \frac{b\alpha\beta(\beta^{2}T^{\beta+1})}{24} - \frac{a\betaT^{\beta+2}}{24} + \frac{b}{\beta+2} \right) \\ &- \frac{a\alpha\beta(\beta-1)T^{\beta-2}}{r(\beta+1)} - \frac{a(\beta-1)\beta^{2}T^{\beta-1}}{2} \left(\frac{b}{2(\beta+2)} + \frac{a}{\beta+1} \right) - \frac{a(\beta+1)\beta^{2}T^{\beta+1}}{4} \left(\frac{b}{\beta+2} + \frac{ar}{3(\beta+1)} \right) - \frac{b\alpha\beta\beta^{2}T^{\beta+1}}{24} + \frac{b}{24} \right) \\ &- \frac{a\alpha\beta(\beta-1)(\beta-2)r^{\beta-3}}{r(\beta+2)} - \frac{a(\beta-1)\beta^{2}T^{\beta-1}}{2} \left(\frac{b}{2(\beta+2)} + \frac{a}{\beta+1} \right) - \frac{b\alpha\beta\beta^{2}T^{\beta+1}}{4} \left(\frac{b}{\beta+2} + \frac{ar}{3(\beta+1)} \right) - \frac{b\alpha\beta\beta}{24} \left(\frac{b}{\beta+2} + \frac{ar}{3(\beta+2)} \right) \\ &- \frac{a\beta\beta}{24} \left(\frac{b}{\beta+2} + \frac{ar}{3(\beta+2)} \right) \frac{a\beta}{24} \left(\frac{b}{2} + \frac{a}{2} + \frac{b}{2} \right) \\ &- \frac{a\beta\beta}{24} \left(\frac{b}{\beta+2} + \frac{ar}{3(\beta+2)} \right) - \frac{b\alpha\beta\beta^{2}T^{\beta+1}}{24} \left(\frac{a\beta\beta}{\beta+2} + \frac{a\beta\beta}{3(\beta+2)} \right) \\ &- \frac{a\beta\beta}{2} \left(\frac{b}{2} + \frac{ar}{3(\beta+2)} \right) - \frac{b\alpha\beta}{2} \left(\frac{c}{\beta+1} + \frac{ar}{3(\beta+2)} \right) \\ &- \frac{a\beta\beta}{2} \left(\frac{b}{\beta+2} + \frac{ar}{3(\beta+2)} \right) - \frac{b\alpha\beta}{2} \left(\frac{c}{\beta+2} + \frac{a\beta\beta}{3(\beta+2)} \right) \\ &- \frac{a\beta\beta}{2} \left(\frac{b}{\beta+2} + \frac{a\beta\beta}{3(\beta+2)} \right) - \frac{b\alpha\beta}{2} \left(\frac{c}{\beta+2} + \frac{a\beta\beta}{3(\beta+2)} \right) \\ &- \frac{a\beta\beta}{2} \left(\frac{b}{\beta+2} + \frac{a\beta\beta}{3(\beta+2)} \right) - \frac{b\alpha\beta}{2} \left(\frac$$

Now
$$\frac{\partial}{\partial T} [NP_1(T)] = (a + b t)(P - C)re^{rT} - \frac{\partial}{\partial T} [TC(T)]$$

And, $\frac{\partial}{\partial T} [NP_1(T)] = 0$ gives the optimal condition for net profit
So, $\frac{\partial^2}{\partial T^2} [NP(T)] < 0$ gives the sufficient condition for net profit at $T = \alpha$ where $T = \alpha$ is the solution of $\frac{\partial}{\partial T} [NP_1(T)] = 0$

Case II –Interest earned:-If the optimal cycle length T is less than the permissible delay time T_1 then the interest earned during [0, H] is given by $T < T_1$ so

$$2 \quad rT \quad r$$

So $\frac{\partial^2}{\partial T^2} \left(\frac{IE_2}{T}\right) < 0$

Numerical problem

Let A = 100, a = 10, b = 5, C = 20, P = 30, r = 0.01 α = 0.05, β = 2.0, H = 1 year, h = 0.03, I_e = 0.10, I_c = 0.15, T₁ = $\frac{30}{365}$, T = $\frac{30 \times 24}{365}$, At, t = 0, Then the total cost is TC (T) = 53317.035

Numerical problem:- Let A = 100, a = 10, b = 5, C = 20, P = 30, r = 0.0125 $\alpha = 0.0625, \beta = 2.5, H = 1 \text{ year, } h = 0.03, I_e = 0.10, I_c = 0.15,$ $T_1 = \frac{30}{365}, T = \frac{30 \times 24}{365},$ And t = 0, Then the total cost is TC (T) = 256841.0801

Conclusion

Under Constant Weibull deterioration, keeping the parameters, optimum cycle time T and allowable credit period T_1 fixed and increase in parameter r (inflation rate) increases the total cost. Hence the model becomes more profitable and very useful in business organizations dealing with perishable products, domestic goods and other products.

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