Comparative Study of Multi-objective Transportation Problems with and without Budgetary Constraints

Fegade M. R.[1], Jadhav V.A.[2], Doke D. M.[3], Irpenwar B. M.[4]

{[1]Research Scholar, [2]Head} Dept. of Statistics, Comp. Sci. and IT, N.E.S. Science College, Nanded, Maharashtra, INDIA.

[3]Head, Department of Mathematics and Statistics, M. L. Dahanukar College, Vile Parle (East), Mumbai, Maharashtra, INDIA.

[4]Head, Department of Statistics, Vivekanand College, Kolhapur, Maharashtra, INDIA.

Corresponding Addresses:

[1]mrfegade@gmail.com, [2]vinayakjadhav2261@gmail.com, [3]d.doke@yahoo.co.in, [4]bhagwanthrao@gmail.com

Research Article

Abstract: In this paper, we consider the situations when transportation problems with and without budgetary constraints where demand and budget are imprecise. We have proposed interval-point method for finding an optimal solution for transportation problems. We compared this method with zero suffix method. For better understanding, the solution procedure is illustrated with a numerical example.

Keywords: *Transportation problem, Optimal solution, Budget, Fuzzy logic.*

1. Introduction

The transportation problem is a special type of linear programming problem, which deals with transport commodities from sources to destinations. In many real life situations, the commodity does vary in some characteristics according to its source and the final commodity mixture reaching at destinations, may then be required to have known specifications. In order to reduce information costs and also to construct a real model, the use of interval and fuzzy transportation problems are more appropriate. In an interval transportation problem, information about the range of variation of some (or all) of the parameters is available, which allows to specify a model with intervals. The transportation problem (TP) is a special class of linear programming problem, which deals with commodities from sources to destinations. In literature, a good amount of research has available to obtain an optimal solution for balanced transportation problems. But in real life situations, the decision maker faces an unbalanced transportation problem in which total supply is less than the total demand. Kishore and Jayswal [2] introduced a method, called fuzzy approach, to solve unbalanced transportation problem with budgetary constraints. Peerayuth Charnsethikul and Saeree Sverasreni [4] discussed a method for solving the constrained bottleneck transportation problem under budgetary condition. Pandian and Natarajan [3] introduced the zero point method for finding an optimal solution to a classical transportation problem. Lin and Cheng [8] gave a genetic algorithm for solving a transportation network under a budget Constraint. Senapati and Tapan Kumar [5] investigated fuzzy multi-index transportation problem with budgetary restriction. Khanna et al. [6] introduced an algorithm for solving transportation flow under budgetary constraints. Tiwari et al. [7] investigated how the preemptive priority structure can be used in fuzzy goal programming problems. Weighted goal programming for unbalanced single objective transportation problem with budgetary constraint has been discussed by Kishore and Jayswal [1].

In this paper, we have proposed interval-point method for finding an optimal solution for transportation problems with and without budgetary constraints. We compared this method with zero suffix method. For better understanding, the solution procedure is illustrated with a numerical example.

2. Preliminaries

We need the following definitions of the basic arithmetic operators and partial ordering on closed bounded intervals which can be found in [4].

Let D = { $[a,b]$, a $\leq$ b and a and b are in R} denote the set of all closed bounded intervals on the real line R.

Definition 1: Let $A=[a,b]$ and $B=[c,d]$ be in D. Then,

$$A \oplus B = [a+c, b+d];$$

$$A \otimes B = [p,q],$$ where

$p = \min\{ac, ad, bc, bd\}$ and

$$q = \max\{ac, ad, bc, bd\}$$

Consider the following transportation problem with budgetary constraints:

(A) Find the values of x_{ij}, i = 1,2,…m; j = =1,2,…n such that the following conditions are satisfied:

$$\sum_{j=1}^{n} x_{ij} \leq a_i \quad i = 1,2,....m \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij} \in [l_j, u_j], \quad j = 1,2,....n \quad (2)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \in [z_1, z_2] \quad (3)$$

$x_{ij} \geq 0$, i=1,2,…m and j=1,2,…n (4)

Where c_{ij} is the cost of shipping one unit from supply point i to the demand point j;

a_i is the supply at supply point i; $[l_j, u_j]$ is the imprecise demand at demand point j;

x_{ij} is the number of units shipped from supply point i to demand point j and $[z_1,z_2]$ is the imprecise budget.

The transportation problem related to the problem (p) is given below:

(IP) Minimize

$$[z_1, z_2] = \left[\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}\right]$$

Subject to

$$\left[\sum_{j=1}^{n} x_{ij}, \sum_{j=1}^{n} x_{ij}\right] = [a_i, a_i], \; i = 1,2,....m$$

$$\left[\sum_{i=1}^{m} x_{ij}, \sum_{i=1}^{m} x_{ij}\right] = [l_j, u_j], \; j = 1,2,....n$$

$x_{ij} \geq 0$, i = 1,2,…m and j = 1,2,…n and integers.

Now, we need the following condition which finds a relation between optimal solutions of an interval transportation problem and a pair of induced transportation problems.

If the set { $y^0{}_{ij}$ for all *i* and *j*} is an optimal solution of the upper bound transportation problem (UP) of (IP) where

(UP) Minimize $z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

subject to $\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1,2,....m$

$\sum_{i=1}^{m} x_{ij} = u_j, \; j = 1,2,....n$

$x_{ij} \geq 0$, i= 1,2,…m and j = 1,2,…n and are integers

And the set { $x^0{}_{ij}$ for all *i* and *j*} is an optimal solution of the lower bound transportation problem (LP) of the problem (IP) where

(LP) Minimize $z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad i = 1,2,....m$$

$$\sum_{i=1}^{m} x_{ij} = l_j, \; j = 1,2,....n$$

$x_{ij} \geq 0$, i=1,2,…m and j=1,2,…n and are integers,

Then the set of intervals { $[x^0_{ij}, y^0_{ij}]$, for all i and j} is an optimal solution of the problem (IP) provided $x^0_{ij} \leq y^0_{ij}$, for all i and j.

3. Interval-Point Method

We, now propose a new method namely, interval-point method for finding an optimal solution to the problem (P). The interval-point method proceeds as follows:

Step 1: Construct an interval transportation problem (IP).

Step 2: Check whether it is balanced or unbalanced. If it is balanced go to step 4 otherwise go to next step.

Step 3: Convert the unbalanced transportation problem in to balanced transportation problem.

Step 4: Construct the upper bound transportation problem (UP) of the problem (IP) and solve the problem (UP) by the zero point method [6]. Let {y, for all i and j} be an optimal solution to the problem (UP).

Step 5: Construct the lower bound transportation problem (LP) of the given the problem (IP) and solve the problem (LP) with the upper bound constraints, $x_{ij} \leq y^{\circ}{}_{ij}$ for all i and j by the zero point method [6].

Let { $x^0{}_{ij}$ for all *i* and *j*} be the optimal solution to the problem (LP) with $x^0_{ij} \leq y^0_{ij}$, for all i and j.

Step 6: The optimal solution to the problem (IP) is {[$x^0_{ij} \leq y^0_{ij}$], for all i and j} by the optimal objective values of the problem (IP) is $(Z_L\text{-}Z_U)$

Step 7: Let $Z \in [Z_1, Z_2]$ be the given budget cost for transportation cost of the given UTPBC. Now, we are write Z as in the form, $Z = Z_L + (Z_U - Z_L)\mu$ for some μ, $0 \leq \mu \leq 1$. This implies $\mu = \frac{Z - Z_L}{(Z_U - Z_L)}$

Here, we considered two cases,

Case 1: Values of Z is known i.e. fixed or given.

Case 2: Value of Z is not known.

Step 8: Compute the values of decision variables

$x_{ij} = [x^0_{ij}, y^0_{ij}] = x^0_{ij} + (y^0_{ij} - x^0_{ij})\mu$,

where μ is as in step 7(including Case I and II).

Step 9: The optimal solution to the given UTPBC is

$$x_{ij} = [x^0_{ij}, y^0_{ij}] = x^0_{ij} + (y^0_{ij} - x^0_{ij})\frac{Z - Z_L}{(Z_U - Z_L)}$$

for the given budget is Z.

4. Numerical Example

The interval-point method for solving transportation problem with budgetary constraints is illustrated by the

following example:
There are four warehouses i.e. source from where the food grains are supplied to three different destinations i.e. demand stations. C_{ij}'s are the cost coefficients expressed in rupee per metric ton and a_i's, l_j's and u_j's are expressed in lakhs of metric ton. The transportation matrix is given in the following table:

	D_1	D_2	D_3	S..
O_1	5	8	3	≤ 10
O_2	7	4	5	≤4
O_3	2	6	9	≤4
O_4	4	6	6	≤12
D..	∈ [6,12]	∈ [7,14]	∈ [7,14]	

Determine an optimal distribution plan to transport the items from the source points to the destination points for the budget Rs.100. Now, the interval transportation problem (IP) to the given problem is given below:

	D_1	D_2	D_3	Supply
O_1	[5,5]	[8,8]	[3,3]	[10,10]
O_2	[7,7]	[4,4]	[5,5]	[4,4]
O_3	[2,2]	[6,6]	[9,9]	[4,4]
O_4	[4,4]	[6,6]	[6,6]	[12,12]
D	[6,12]	[7,14]	[7,14]	

Now, the upper bound problem of (IP), (UP) of the problem (IP) is given below:

	D_1	D_2	D_3	Supply
O_1	5	8	3	10
O_2	7	4	5	4
O_3	2	6	9	4
O_4	4	6	6	12
Demand	12	14	14	

Now, using the zero point method, the optimal solution to the problem (UB) is: $y_{13}^0 = 10$, $y_{22}^0 = 4$, $y_{31}^0 = 4$, $y_{41}^0 = 8$, and $y_{42}^0 = 4$.

Now, the lower bound problem of interval transportation problem with the upper bounded (LB) constraints is:

	D_1	D_2	D_3	Supply
O_1	5	8	3	10
O_2	7	4	5	4
O_3	2	6	9	4
O_4	4	6	6	12
D..	6	7	7	

With $x_{ij} \le y_{ij}^0$, i= 1, 2,…,m and j = 1,2,…n.

Now, using the zero point method, the optimal solution to the problem (LB) is
$x_{13}^0 = 7$, $x_{22}^0 = 4$, $x_{31}^0 = 4$, $x_{41}^0 = 2$, and $x_{42}^0 = 3$.

Now, as in the step 3., we consider the optimal solution to the problem (IP) is $[x^\circ_{13}, y^\circ_{13}] = [7,10]$, $[x^\circ_{22}, y^\circ_{22}] = [4,4]$, $[x^\circ_{31}, y^\circ_{31}] = [4,4]$, $[x^\circ_{41}, y^\circ_{41}] = [2,8]$ and $[x^\circ_{42}, y^\circ_{42}] = [3,4]$ and also, the minimum interval transportation cost is [71,110].

Now, as in step 4, we have the total transportation cost as Z = 71 + 39 μ where Z is given budget. This implies that $\mu = \frac{Z-71}{39}$

Now, as in step 5 and the step 6, we have

$$x_{13} = 7 + 3\left(\frac{Z-71}{39}\right); \quad x_{22}=4; \quad x_{31}=4;$$

$$x_{41} = 2 + 6\left(\frac{Z-71}{39}\right); \quad x_{42} = 3 + \left(\frac{Z-71}{39}\right);$$

Case 1:
Given Z = 100. Then, the optimal solution to the given TPBC for Z = 100 are:
$x_{13} = 9.23$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 6.46$ and $x_{42} = 3.74$.
The total number of units transported = 27.43 tons.

Case 2:
We get, different values of Z for μ ($0 \le \mu \le 1$) is given as,

μ	Z	Decision variables	Transported Unit
0	71	$x_{13} = 7$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 2$ and $x_{42} = 3$	20
0.1	74.9	$x_{13} = 7.3$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 2.6$ and $x_{42} = 3.1$	21
0.2	78.9	$x_{13} = 7.6$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 3.2$ and $x_{42} = 3.2$	22
0.3	82.7	$x_{13} = 7.9$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 3.8$ and $x_{42} = 3.3$	23
0.4	86.6	$x_{13} = 8.2$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 4.4$ and $x_{42} = 3.4$	24
0.5	90.5	$x_{13} = 8.5$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 5$ and $x_{42} = 3.5$	25
0.6	94.4	$x_{13} = 8.8$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 5.6$ and $x_{42} = 3.6$	26
0.7	98.3	$x_{13} = 9.1$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 6.2$ and $x_{42} = 3.7$	27
0.8	102.2	$x_{13} = 9.4$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 6.8$ and $x_{42} = 3.8$	28
0.9	106.1	$x_{13} = 9.7$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 7.4$ and $x_{42} = 3.9$	29
1	110	$x_{13} = 10$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 8$ and $x_{42} = 4$	30

Now we compared the above method with zero suffix method [2], the optimal solution to the problem (UB) is: y_{13}^0 = 10, y_{22}^0=4, y_{31}^0 = 4, y_{41}^0 = 8 , and y_{42}^0= 4. and the optimal solution to the problem (LB) is: x_{13}^0 = 7, x_{22}^0=4, x_{31}^0 = 4, x_{41}^0 = 2, and x_{42}^0= 3.
Remaining process is same as the above.

5. Conclusion

In this paper, we have considered the transportation problem with and without budgetary constraints cases. The proposed without budgetary constraints method is useful for finding an optimal solution for decreasing or reducing the cost and finding a compromise solution. It is very easy to understand and easy to compute. Also, enables the decision makers to optimize the economical activities and make the correct managerial decisions depending on their financial position. The proposed method enables the decision makers to evaluate the economical activities and make self satisfied managerial decisions when they are handling a variety of transportation problems with budgetary constraints. We compared this method with zero suffix method. We found that, by both these methods we got the same result. When transportation problem with and without budgetary constraints situation occurred then our proposed method may be more useful.

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