Comparative Study of Inventory Model for Duopolistic Market under Trade Credit

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Research Article

Abstract: This paper did some researches on stochastic inventory model in duopolistic market when permissible delay in payment was applied and the model was compared when the privilege is not offered. From the perspective of supplier's and businessman mathematical model has been established. Spectral theory is used to derive explicit expression for the transition probabilities of a four state continuous time markov chain representing the status of the systems. These probabilities are used to compute the exact form of the average cost expression. We use concepts from renewal reward processes to develop average cost objective function. Optimal solution is obtained using Newton Rapson method in R programming. Finally sensitivity analysis for varying parameter values has been carried out.

Keywords: Future supply uncertainty, Permissible delay in payment, Duopolistic market, Two suppliers.

1. Introduction

This paper represent practical life situation by assuming that the supplier's market is not monopolistic as competitive spirit in the business is increased especially after induction of multinational companies. In other words, it is assumed that the inventory manager may place his order with any one of two suppliers who are randomly available. Here we assume that the decision maker deals with two suppliers who may be ON or OFF. Here there are three states that correspond to the availability of at least one supplier that is states 0, 1 and 2 where as state 3 denotes the non availability of either of them. State 0 indicates that supplier 1 and supplier 2 both are available. Here it is assumed that one may place order to either one of the two suppliers or partly to both. State 1 represents that supplier 1 is available but supplier 2 is not available. State 2 represents that supplier 1 is not available but supplier 2 is available.

Silver [11] appears to be the first author to discuss the need for models that deal with supplier uncertainty. Articles by Parlar and Berkin [9] consider the supply uncertainty problem for a class of EOQ model with a single supplier where the availability and unavailability periods constitute an alternating Poisson process. Parlar and Berkin [9] assume that at any time the decision maker is aware of the availability status of the product although he does not know when the ON (available) and OFF (unavailable) periods will start and end. When the inventory level reaches the reorder point of zero and the status is ON, the order is received; otherwise the decision maker must wait until the product becomes available. Parlar and Perry [8] developed inventory model for non deteriorating items with future supply uncertainty considering demand rate d=1 for two suppliers. Kandpal and Gujarathi [4,5] has extended the model of Parlar & Perry [8] by considering demand rate greater than one and for deteriorating items for single supplier. Kandpal and Tinani [6] developed inventory model for deteriorating items with future supply uncertainty under inflation and permissible delay in payment for single supplier.

The traditional economic order quantity (EOQ) model is widely used as a decision tool for the control of inventory. The EOQ assumes that the retailer's capitals are unrestricting and must be paid for the items as soon as the items are received. However, in practice, the supplier will offer the retailer a delay period which is the trade credit period, in paying for the amount of purchasing cost. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the trade credit period. Permissible delay in payments is generally considered to be the equivalent form of discount contract. Goyal [3] has studied an EOQ system with deterministic demand and delay in payments is permissible which was reinvestigated by Chand and Ward [2]. Shah [10] developed model for deteriorating items when delay in payments is permissible by assuming deterministic demand. Aggarwal and Jaggi [1] developed a model to determine the optimum order quantity for deteriorating items under a permissible delay in payment.Kandpal and Tinani [7] developed perishable-inventory model under inflation and delay in payment allowing partial payment for single supplier.

Case 1: When permissible delay in payment is allowed.

2. Notations, Assumptions and Model

The inventory model here is developed on the basis of following assumptions.

(a) Demand rate d is deterministic and it is d>1.

(b) We define *Xi* and *Yi* be the random variables corresponding to the length of ON and OFF period respectively for i^{th} supplier where *i*=1, 2. We specifically assume that $X_i \sim exp\ (\lambda_i)$ and $Y_i \sim exp\ (\mu_i)$. Further *Xi* and *Yi* are independently distributed.

(c) Ordering cost is Rs. *k*/order.
(d) Holding cost is Rs. *h*/unit/unit time.
(e) Shortage cost is Rs. π/unit.
(f) Time dependent part of the backorder cost is Rs. $\hat{\pi}$ /unit/time.
(g) q_i= order upto level i=0, 1, 2
(h) r=reorder upto level; qi and r are decision variables.
(i) Purchase cost is Rs. c/unit.
(j) T_{1i} is a credit period allowed by i^{th} supplier where i=1, 2 which is a known constant.
(k) T_{00} is cycle period.
(l) ie_i=Interest rate earned when purchase made from i^{th} supplier where i=1, 2
ic_i=Interest rate charged by i^{th} supplier where i=1, 2
(m) α_i= Indicator variable for i^{th} supplier i=1, 2
$\alpha_1 = 0$ if account is settled completely at T_{11}
= 1 otherwise
$\alpha_2 = 0$ if account is settled completely at T_{12}
= 1 otherwise
(n) $Ie(1i)$=Interest earned over period (0 to T_{1i}) = $dcT_{00}T_{1i}ie_i$
(o) $Ie(2i)$=Interest earned over period (T_{1i} to T_{00}) upon interest earned $(Ie(1i))$ previously.

$$Ie(2i) = (dcT_{00} + Ie(1i))(T_{00} - T_{1i})ie_i$$

(p) Interest charged by the i^{th} supplier clearly (ic_i>ie_i) i=1, 2

$$Ic_i = \alpha_i dcic_i (T_{00} - T_{1i})$$

In this paper, we assume that Supplier allows a fixed period 'T_{1i}' to settle the account. During this fixed period no interest is charged by the i^{th} supplier but beyond this period, interest is charged by the i^{th} supplier under the terms and conditions agreed upon. Interest charged is usually higher than interest earned. The account is settled completely either at the end of the credit period or at the end of the cycle.During the fixed credit period T_{1i}, revenue from sales is deposited in an interest bearing account.

The policy we have chosen is denoted by (q_0, q_1, q_2, r). An order is placed for q $_i$ units i=0, 1, 2, whenever inventory drops to the reorder point r and the state found is i=0, 1, 2.When both suppliers are available, q_0 is the total ordered from either one or both suppliers. If the process is found in state 3 that is both the suppliers are not available nothing can be ordered in which case the buffer stock of r units is reduced. If the process stays in state 3 for longer time then the shortages start accumulating at rate of d units/time. When the process leaves state 3 and supplier becomes available, enough units are ordered to increase the inventory to q_i +r units where i=0, 1, 2.

$A(q_i, r)$=cost of ordering+ cost of holding inventory during a single interval that starts with an inventory of q_i+r units and ends with r units.

$$A(q_i, r) = k + \frac{1}{2}\frac{hq_i^2}{d} + \frac{hrq_i}{d} \qquad i=0, 1, 2$$

$P_{ij}(t)$=P (Being in state j at time t/starting in state i at time 0) i, j=0, 1, 2, 3

p_i =long run probabilities i=0, 1, 2, 3

3. Optimal Policy Decision for the Model

For calculation of average cost objective function, we need to identify the cycles. Below given figure gives us the idea about cycles and their identification.

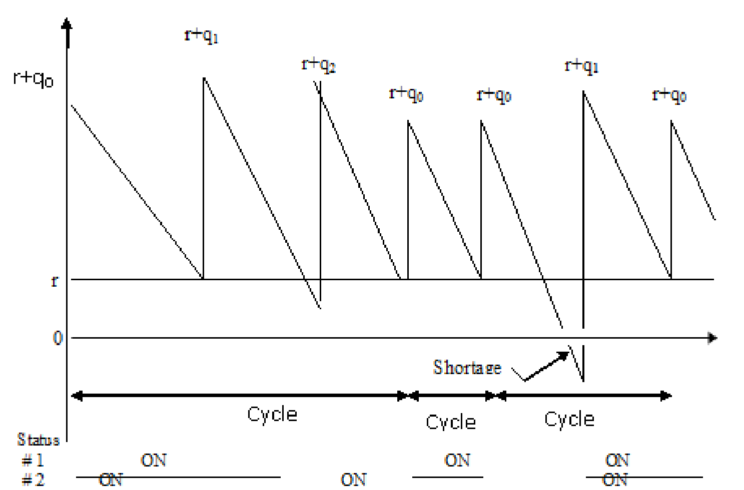

Fig 3.1: *Inventory level and status process with two suppliers*

Referring to Figure 3.1, we see that the cycles of this process start when the inventory goes up to a level of q_0+r units. Once the cycle is identified, we construct the average cost objective function as a ratio of the expected cost per cycle to the expected cycle length.

i.e. Ac (q_0, q_1, q_2, r) = $\frac{C_{00}}{T_{00}}$ where C_{00}=E (cost per cycle) and T_{00}=E (length of a cycle)

Analysis of the average cost function requires the exact determination of the transition probabilities $P_{ij}(t)$, i, j=0, 1, 2, 3 for the four state CTMC. The solution is provided in the following lemma.

Lemma 3.1: Let $P(t)=[P_{ij}(t)]$ t≥0, i, j=0, 1, 2, 3 be 4×4 matrix of transition functions for the continuous time markov chain(CTMC). The exact transient solution is given as $P(t)=UD(t)U^{-1}$ .where,

$$U=\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -\mu_2/\lambda_2 & -\mu_2/\lambda_2 \\ 1 & -\mu_1/\lambda_1 & 1 & -\mu_1/\lambda_1 \\ 1 & -\mu_1/\lambda_1 & -\mu_2/\lambda_2 & \mu_1\mu_2/\lambda_1\lambda_2 \end{bmatrix}$$

$$D(t)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-(\lambda_1+\mu_1)t} & 0 & 0 \\ 0 & 0 & e^{-(\lambda_2+\mu_2)t} & 0 \\ 0 & 0 & 0 & e^{-(\lambda_1+\mu_1+\lambda_2+\mu_2)t} \end{bmatrix}$$

$$U^{-1}=\frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}\begin{bmatrix} \mu_1\mu_2 & \lambda_2\mu_1 & \lambda_1\mu_2 & \lambda_1\lambda_2 \\ \lambda_1\mu_2 & \lambda_1\lambda_2 & -\lambda_1\mu_2 & -\lambda_1\lambda_2 \\ \lambda_2\mu_1 & -\lambda_2\mu_1 & \lambda_1\lambda_2 & -\lambda_1\lambda_2 \\ \lambda_1\lambda_2 & -\lambda_1\lambda_2 & -\lambda_1\lambda_2 & \lambda_1\lambda_2 \end{bmatrix}$$

Proof: For proof refer Parlar and Perry[1996]

Corollary 3.2: The long run probabilities $P_j=\lim_{t\to\infty}P_{ij}(t)$ are

$$[p_0,p_1,p_2,p_3]=\frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}[\mu_1\mu_2,\lambda_2\mu_1,\lambda_1\mu_2,\lambda_1\lambda_2]$$

Proof: For proof refer Parlar and Perry[1996].

Define C_{i0}=E (cost incurred to the beginning of the next cycle from the time when inventory drops to r at state i=0, 1, 2, 3 and q_i units are ordered if i=0, 1 or 2)

Lemma 3.3: C_{i0} is given by

$$C_{i0}=P_{i0}\left(\frac{q_i}{d}\right)A(q_i,r)+\sum_{j=1}^{3}P_{ij}\left(\frac{q_i}{d}\right)\left[A(q_i,r)+C_{j0}\right] \quad \text{i=0, 1, 2} \qquad (3.3.1)$$

$$C_{30}=\overline{C}+\sum_{i=1}^{2}\rho_iC_{i0} \qquad (3.3.2)$$

Where $\rho_i=\frac{\mu_i}{\delta}$ with $\delta=\mu_1+\mu_2$ and

$$\overline{C}=\frac{e^{\frac{-\delta r}{d}}}{\delta^2}\left[he^{\frac{\delta r}{d}}(\delta r-d)+\pi\delta d+hd+\hat{\pi}\right]$$

Proof: First consider i=0. Conditioning on the state of the supplier availability process when inventory drops to r, we obtain

$$C_{00} = P_{00}\left(\frac{q_0}{d}\right)A(q_0, r) + \sum_{j=1}^{3} P_{0j}\left(\frac{q_0}{d}\right)\left[A(q_0, r) + C_{j0}\right]$$

The equation follows because $q_0 + r$ being the initial inventory, when q_0 units are used up we either observe state 0, 1, 2 or 3 with probabilities

$P_{00}\left(\frac{q_0}{d}\right)$, $P_{01}\left(\frac{q_0}{d}\right)$, $P_{02}\left(\frac{q_0}{d}\right)$ and $P_{03}\left(\frac{q_0}{d}\right)$ respectively. If we are in state 0 when r is reached, we must have incurred a cost of $A(q_0, r)$. On the other hand, if state j=1, 2, 3 is observed when inventory drops to r, then the expected cost will be $A(q_0, r) + C_{j0}$ with probability $P_{0j}\left(\frac{q_0}{d}\right)$. The equation relating C_{10} and C_{20} are very similar but C_{30} is obtained as

$$C_{30} = \left[\overline{C} + C_{10}\right]\frac{\mu_1}{\mu_1 + \mu_2} + \left[\overline{C} + C_{20}\right]\frac{\mu_2}{\mu_1 + \mu_2} \quad (3.3.1)$$

Here, $\overline{C}$ is defined as the expected cost from the time inventory drops to r until either of the suppliers becomes available and it is computed as follows:
Now referring to fig 3.1., note that the cost incurred from the time when inventory drops to r and the state is OFF to the beginning of next cycle is equal to

$$\frac{1}{2}hy^2 d + hy(r - yd) \quad y < \frac{r}{d}$$

$$\frac{1}{2}\frac{hr^2}{d} + \pi\left(y - \frac{r}{d}\right)d + \frac{\hat{\pi}}{2}\left(y - \frac{r}{d}\right)^2 \quad y \geq \frac{r}{d}$$

Hence,

$$\overline{C} = \int_0^{r/d}\left\{\frac{1}{2}hy^2 d + hy(r - yd)\right\}\delta e^{-\delta y}dy + \int_{r/d}^{\infty}\left\{\frac{hr^2}{d} + \pi\left[y - \frac{r}{d}\right]d + \frac{\hat{\pi}}{2}\left[y - \frac{r}{d}\right]^2\right\}\delta e^{-\delta y}dy$$

$$\overline{C} = \frac{e^{\frac{-\delta r}{d}}}{\delta^2}\left[he^{\frac{\delta r}{d}}(\delta r - d) + (\pi\delta d + hd + \hat{\pi})\right]$$

with $\delta = \mu_1 + \mu_2$ as the rate of departure from state 3. This follows because if supplier availability process is in state 3 (OFF for both suppliers) when inventory drops to r, then the expected holding and backorder costs are equal to $\overline{C}$. If the process makes a transition to state 1, the total expected cost would then be $\overline{C} + C_{10}$. The probability of a transition from state 3 to state 1 is

$$P(Y_1 < Y_2) = \int_0^{\infty} P(Y_1 < Y_2 / Y_2 = t)\mu_2 e^{-\mu_2 t}dt = \frac{\mu_1}{\mu_1 + \mu_2}$$

Multiplying this probability with the expected cost term above gives the first term of (3.3.1). The second term is obtained in a similar manner. Combining the results proves the lemma.

The following lemma provides a simpler means of expressing C_{00} in an exact manner. To simplify the notation, we let $A_i = A(q_i, r)$, i=0, 1, 2 and $P_{ij} = P_{ij}\left(\frac{q_i}{d}\right)$ i, j=0, 1, 2, 3.

Lemma 3.4: The exact expression for C_{00} is

$$C_{00} = A_0 + P_{01}C_{10} + P_{02}C_{20} + P_{03}\left(\overline{C} + \rho_1 C_{10} + \rho_2 C_{20}\right) \quad (3.4.1)$$

where the pair $[C_{10}, C_{20}]$ solves the system

$$\begin{bmatrix} 1-P_{11}-P_{13}\rho_1 & -(P_{12}+P_{13}\rho_2) \\ -(P_{21}+P_{23}\rho_1) & 1-P_{22}-P_{23}\rho_2 \end{bmatrix}\begin{bmatrix} C_{10} \\ C_{20} \end{bmatrix}=\begin{bmatrix} A_1+P_{13}\overline{C} \\ A_2+P_{23}\overline{C} \end{bmatrix} \quad (3.4.2)$$

Proof: Rearranging the linear system of four equations in lemma(3.3) in matrix form gives

$$\begin{bmatrix} 1 & -P_{01} & -P_{02} & -P_{03} \\ 0 & 1-P_{11} & -P_{12} & -P_{13} \\ 0 & -P_{21} & 1-P_{22} & -P_{23} \\ 0 & -\rho_1 & -\rho_2 & 1 \end{bmatrix}\begin{bmatrix} C_{00} \\ C_{10} \\ C_{20} \\ C_{30} \end{bmatrix}=\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ \overline{C} \end{bmatrix} \quad (3.4.3)$$

We have $C_{30}=\overline{C}+\rho_1 C_{10}+\rho_2 C_{20}$ from the last row of the system. Substituting this result in rows two and three and rearranging gives the system in (3.4.2). From the first row of (3.4.3) we obtain $C_{00}=A_0+\sum_{j=1}^{3}P_{0j}C_{j0}$.

Hence above lemma is proved.

Define, T_{i0}=E [Time to the beginning of the next cycle from the time when inventory drops to r at state i=0, 1, 2, 3 and q_i units are ordered if i=0, 1, 2]

Lemma 3.5: Expected cycle length is given by

$$T_{i0}=P_{i0}\left(\frac{q_i}{d}\right)\frac{q_i}{d}+\sum_{j=1}^{3}P_{ij}\left(\frac{q_i}{d}\right)\left[\frac{q_i}{d}+T_{j0}\right] \qquad i=0,\,1\,,2$$

$$T_{30}=\overline{T}+\sum_{j=1}^{2}\rho_i T_{i0}$$

where $\overline{T}=\frac{1}{\mu_1+\mu_2}$ is the expected time from the time inventory drops to r until either supplier 1 or 2 becomes available.

Lemma 3.6: The exact expression for T_{00} is

$$T_{00}=\frac{q_0}{d}+P_{01}T_{10}+P_{02}T_{20}+P_{03}(\overline{T}+\rho_1 T_{10}+\rho_2 T_{20})$$

where the pair [T_{10}, T_{20}] solves the system.

$$\begin{bmatrix} 1-P_{11}-P_{13}\rho_1 & -(P_{12}+P_{13}\rho_2) \\ -(P_{21}+P_{23}\rho_1) & (1-P_{22}-P_{23}\rho_2) \end{bmatrix}\begin{bmatrix} T_{10} \\ T_{20} \end{bmatrix}=\begin{bmatrix} q_1+P_{13}\overline{T} \\ q_2+P_{23}\overline{T} \end{bmatrix}$$

The proof of the above two lemmas i.e. (3.5) and (3.6) are very similar to lemma (3.3) and (3.4).

Theorem 3.7: The Average cost objective function for two suppliers when delay in payment is considered is given by

$$\text{Ac}=\frac{C_{00}}{T_{00}}=\frac{A(q_0,r)+P_{01}(C_{10}-(Ie(11)+Ie(21)+Ic_1)+P_{02}(C_{20}-(Ie(12)+Ie(22)+Ic_2)+P_{03}(\overline{C}+\rho_1(C_{10}-(Ie(11)+Ie(21)+Ic_1)+\rho_2(C_{20}-(Ie(12)+Ie(22))+Ic_2))}{\frac{q_0}{d}+P_{01}T_{10}+P_{02}T_{20}+P_{03}(\overline{T}+\rho_1 T_{10}+\rho_2 T_{20})}$$

Proof: Proof follows using Renewal reward theorem(RRT). The optimal solution for q_0, q_1, q_2 and r is obtained by using Newton Rapson method in R programming.

4. Numerical Example

In this section we verify the results by a numerical example. We assume that

(i) k =Rs. 5/order, c=Rs.1/unit, d=20/units , h=Rs. 5/unit/time, π=Rs. 350/unit, $\hat{\pi}$=Rs.25/unit/time, α_1=1, α_2=1, ic_1=0.11, ie_1=0.02, ic_2=0.13, ie_2=0.04, T_{11}=0.6, T_{12}=0.8 , λ_1=0.58, λ_2=0.45, μ_1=3.4, μ_2=2.5

The last four parameters indicate that the expected lengths of the ON and OFF periods for first and second supplier are $1/\lambda_1$=1.72413794, $1/\lambda_2$=2.2222, $1/\mu_1$=.2941176 and $1/\mu_2$=.4 respectively. The long run probabilities are obtained as p_0=0.7239588, p_1=0.1303126, p_2 =0.1234989 and p_3=0.02222979. The optimal solution is obtained as

q_0=4.032518, q_1=27.63127, q_2=27.43041, r=1.351202 and Ac= $\frac{C_{00}}{T_{00}}$ =6.499962

(ii) Keeping other parameters as it is, we consider α_1=0 and α_2=0. The optimal solution is obtained as

q_0=5.72531, q_1=30.10025, q_2=30.0786,r=1.418232 and Ac= $\frac{C_{00}}{T_{00}}$ = 6.156267

(iii) Keeping other parameters as it is, we consider α_1=1 and α_2=0. The optimal solution is obtained as

q_0=4.837963, q_1=28.71599, q_2=28.61197,r=1.407225 and Ac= $\frac{C_{00}}{T_{00}}$ = 6.315563

(iv) Keeping other parameters as it is, we consider α_1=0 and α_2=1. The optimal solution is obtained as

q_0=4.68275, q_1=28.45212, q_2=28.32906,r=1.403514 and Ac= $\frac{C_{00}}{T_{00}}$ =6.362211

From the above numerical example, we conclude that the cost is minimum when account is settled at credit time given by the i^{th} supplier.

Case 2: When permissible delay in payment is not allowed

The Average cost objective function for two suppliers when delay in payment is not considered reduces to

$$Ac = \frac{C_{00}}{T_{00}} = \frac{A(q_0, r) + P_{01}C_{10} + P_{02}C_{20} + P_{03}(\overline{C} + \rho_1 C_{10} + \rho_2 C_{20})}{\frac{q_0}{d} + P_{01}T_{10} + P_{02}T_{20} + P_{03}(\overline{T} + \rho_1 T_{10} + \rho_2 T_{20})}$$

Numerical Example:

We assume thatk =Rs. 5/order, c=Rs.1/unit, d=20/units, h=Rs. 5/unit/time, π=Rs. 350/unit, $\hat{\pi}$=Rs.25/unit/time, λ_1=0.58, λ_2=0.45, μ_1=3.4, μ_2=2.5. The optimal solution is obtained as

q_0=2.9845, q_1=25.7849, q_2=25.3637, r=1.2538 and Ac= $\frac{C_{00}}{T_{00}}$ =6.6247

On comparing the average cost of all the situations of case 1 we find that cost is maximum for case 2. This implies that the privilege offered proves to be bliss for entrepreneurs with minimum average cost than without privilege, so they are in business resulting in keen competition in the business.

5. Sensitivity Analysis for Case 1

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the valueλ_1 and keeping other parameter values fixed where α_1=1 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and Ac. The optimal values of q_0, q_1, q_2 and Ac are plotted in Fig.5.1.

Table 5.1: Sensitivity Analysis Table by varying the parameter values of λ_1 α_1=1and α_2=1

λ_1	q_0	q_1	q_2	r	Ac
0.5	3.717054	27.44038	27.2929	0.879031	6.551717
0.54	3.843987	27.48572	27.30287	1.123268	6.5296
0.58	4.032518	27.63127	27.43041	1.351202	6.499962
0.62	4.332144	27.97548	27.78156	1.563168	6.462281
0.66	4.89136	28.82827	28.68549	1.752149	6.414409

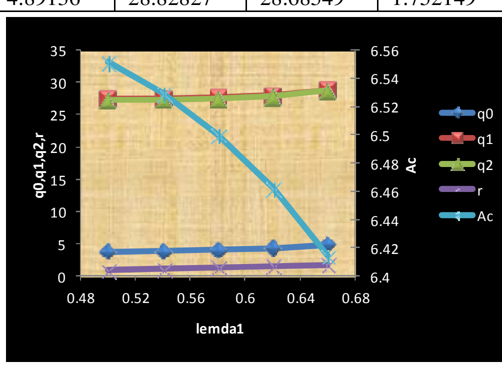

***Fig 5.1:** Sensitivity analysis graph for λ_1(α_1=1and α_2=1)*

We see that as λ_1 increases i.e. expected length of ON period for 1st supplier decreases the value of q_0, q_1, q_2 and r increases which result in decrease in average cost.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 and keeping other parameter values fixed where α_1=0 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and Ac. The optimal values of q_0, q_1, q_2 and Ac are plotted in Fig.5.2.

Table 5.2: Sensitivity Analysis Table by varying the parameter values of λ_1 α_1=0and α_2=0

λ_1	q_0	q_1	q_2	r	Ac
0.5	4.701216	28.5303	28.47871	0.99042	6.267463
0.54	5.102404	29.07288	29.02172	1.223672	6.219188
0.58	5.72531	30.10025	30.0786	1.418232	6.156267
0.62	6.714279	32.06434	32.09975	1.544664	6.073685
0.66	8.130382	35.44868	35.53845	1.56233	5.962785

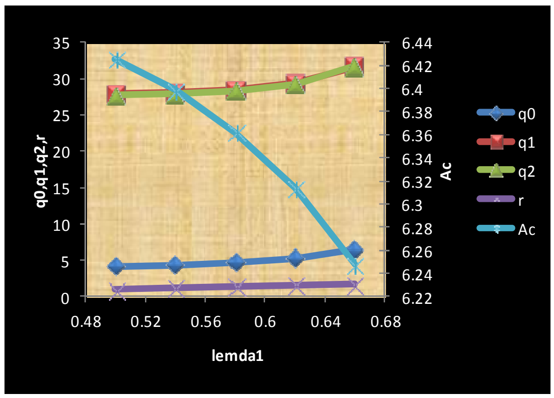

Fig 5.2: *Sensitivity analysis graph for λ_1(α_1=0and α_2=0)*

We see that as λ_1 increases i.e. expected length of ON period for 1st supplier decreases the value of q_0, q_1, q_2 and r increases which result in decrease in average cost.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 and keeping other parameter values fixed where α_1=1 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and Ac. The optimal values of q_0, q_1, q_2 and Ac are plotted in Fig.5.3.

Table 5.3: Sensitivity Analysis Table by varying the parameter values of λ_1 α_1=1and α_2=0

λ_1	q_0	q_1	q_2	r	Ac
0.5	4.17051	27.91604	27.81633	0.940845	6.406776
0.54	4.433138	28.17991	28.06476	1.185896	6.366733
0.58	4.837963	28.71599	28.61197	1.407225	6.315563
0.62	5.524972	29.85845	29.80357	1.59063	6.250326
.66	6.787964	32.46545	32.50053	1.683555	6.16309

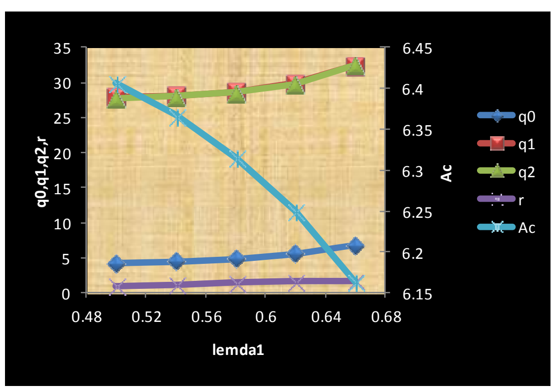

Fig 5.3: Sensitivity analysis graph for λ_1(α_1=1and α_2=0)

We see that as λ_1 increases i.e. expected length of ON period for 1st supplier decreases the value of q_0, q_1, q_2 and r increases which result in decrease in average cost.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 and keeping other parameter values fixed where α_1=0 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and Ac. The optimal values of q_0, q_1, q_2 and Ac are plotted in Fig.5.4.

Table 5.4: Sensitivity Analysis Table by varying the parameter values of λ_1 α_1=0and α_2=1

λ_1	q_0	q_1	q_2	r	Ac
0.5	4.13484	27.83933	27.73441	0.939671	6.425488
0.54	4.349138	28.03646	27.91042	1.181905	6.398667
0.58	4.68275	28.45212	28.32906	1.403514	6.362211
0.62	5.261809	29.36611	29.28288	1.595496	6.314107
0.66	6.438238	31.68256	31.69321	1.710866	6.247621

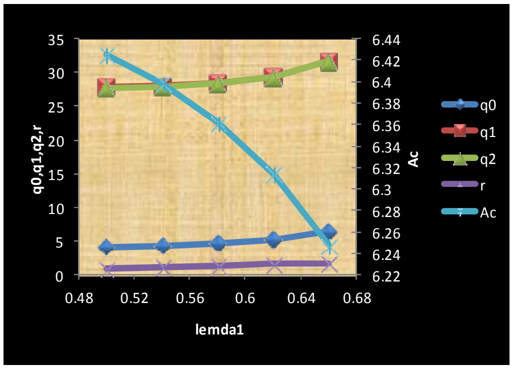

Fig 5.4: *Sensitivity analysis graph for $\lambda_1(\alpha_1=0$ and $\alpha_2=1)$*

We see that as λ_1 increases i.e. expected length of ON period for 1st supplier decreases the value of q_0, q_1, q_2 and r increases which result in decrease in average cost.

Conclusion

From the above sensitivity analysis, we conclude that the cost is minimum when account is settled at credit time given by the i[th] supplier and is maximum when account is not settled at credit time given by the i[th] supplier. So it is advisable for the business man to settle the account at the credit time period given by the supplier, which means that we encourage the small business man by offering the privilege but simultaneously we discourage them for not clearing the account after the given credit time period.

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