Contra (πp, μ_y)-Continuity on Generalized Topological Spaces

K. Binoy Balan[*], C. Janaki[**]

[*]Assistant Professor, Dept. of Mathematics, Sree Narayana Guru College, Coimbatore- 105, Tamil Nadu INDIA.

[**]Assistant Professor, Dept. of Mathematics, L.R.G. Govt. Arts College for Women, Tirupur-4, Tamil NaduINDIA.

Corresponding Addresses:

[*]binoy_sngc@yahoo.co.in, [**]janakicsekar@yahoo.com

Research Article

Abstract: In this paper we introduce a new notion called contra (πp, μ_y) – continuous function on generalized topological space. The properties and characterizations of such functions are investigated.

Key words: μ-$\pi r\alpha$ space, $T_{\pi p}$ space, μ-$\pi r\alpha$ T_1,μ-$\pi r\alpha$ T_2, μ-$\pi r\alpha$ connected, μ-Urysohn, μ-$\pi r\alpha$ locally indiscrete, contra (πp, μ_y) – continuous, contra (πp, μ_y) – closed, μ-$\pi r\alpha$ closed, μ-$\pi r\alpha$ open.

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1. Introduction

Á.Császár [3]- [13] has introduced the notions of generalized topological space, obtain characterizations for generalized continuous functions and associated interior and closure operators. In [5] he introduced characterizations for generalized continuous functions. Also in [3] he investigated the notions of μ-α-open sets, μ- semi open sets, μ- pre open sets and μ-β open sets in generalized topological space. W.K. Min [15] has introduced and studied the notions of (α, μ_y) – continuous functions, (σ, μ_y) – continuous functions, (π, μ_y) – continuous functions, and (β, μ_y) – continuous in generalized topological spaces. Also D. Jayanthi [14] has introduced some contra continuous functions on generalized topological spaces such as contra (μ_x, μ_y) – continuous functions, contra (α, μ_y) – continuous, contra (σ, μ_y) – continuous functions, and contra (β, μ_y) – continuous functions. In this paper we introduce contra (πp, μ_y) – continuous functions and investigate their characterizations and relationships among these functions.

2. Preliminaries

We recall some basic concepts and results.

Let X be a nonempty set and let exp(X) be the power set of X. $\mu\subset$exp(X) is called a generalized topology [5](briefly, GT) on X, if $\emptyset\in\mu$ and unions of elements of μ belong to μ. The pair (X, μ) is called a generalized topological space (briefly, GTS). The elements of μ are called μ-open [3] subsets of X and the complements are called μ-closed sets. If (X, μ) is a GTS and A$\subseteq$X, then the interior of (denoted by i_μ(A)) is the union of all G$\subseteq$A, G$\in\mu$ and the closure of A (denoted by c_μ(A)) is the intersection of all μ-closed sets containing A.

Note that $c_\mu(A) = X-i_\mu(X-S)$ and $i_\mu(A) = X - c_\mu(X-A)$ [5].

Definition 2.1[5] Let (X, μ_x) be a generalized topological space and A$\subseteq$ X. Then A is said to be

(i) μ- semi open if $A\subseteq c_\mu(i_\mu(A))$.
(ii) μ- pre open if $A\subseteq i_\mu(c_\mu(A))$.
(iii) μ-α-open if $A\subseteq i_\mu(c_\mu(i_\mu(A)))$.
(iv) μ-β-open if $A\subseteq c_\mu(i_\mu(c_\mu(A)))$.
(v) μ-r-open [17] if $A = i_\mu(c_\mu(A))$
(vi) μ-rα-open [2] if there is a μ-r-open set U such that $U\subset A\subset c_\alpha(U)$.

Definition 2.2 [2] Let (X, μ_x) be a generalized topological space and A$\subseteq$ X. Then A is said to be μ-$\pi r\alpha$ closed set if $c_\pi(A) \subseteq U$ whenever $A\subseteq U$ and U is μ-rα-open set. The complement of μ-$\pi r\alpha$ closed set is said to be μ-$\pi r\alpha$ open set.

The complement of μ-semi open (μ-pre open, μ-α-open, μ-β-open, μ-r-open, μ-rα-open) set is called μ-semi closed (μ- pre closed, μ-α- closed, μ-β- closed, μ-r- closed, μ-rα-closed) set.

Let us denote the class of all μ-semi open sets, μ-pre open sets, μ-α-open sets, μ-β-open sets, and μ-$\pi r\alpha$ open sets on X by $\sigma(\mu_x)$ (σ for short), $\pi(\mu_x)$ (π for short), $\alpha(\mu_x)$ (α for short), $\beta(\mu_x)$ (β for short) and $\pi p(\mu_x)$ (πp for short)respectively. Let μ be a generalized topology on a non empty set X and S$\subseteq$X. The μ-α-closure (resp. μ-semi closure, μ-pre closure, μ-β-closure, μ-$\pi r\alpha$-closure) of a subset S of X denoted by $c_\alpha(S)$ (resp. $c_\sigma(S)$, $c_\pi(S)$, $c_\beta(S)$, $c_{\pi p}(S)$) is the intersection of μ-α-closed(resp. μ- semi closed, μ- pre closed, μ-β-closed, μ-$\pi r\alpha$ closed) sets including S. The μ-α-interior (resp. μ-semi interior, μ-pre interior, μ-β-interior, μ-$\pi r\alpha$-interior) of a subset S of X denoted by $i_\alpha(S)$ (resp. $i_\sigma(S)$, $i_\pi(S)$, $i_\beta(S)$, $i_{\pi p}(S)$) is the union of μ-α-open (resp. μ-semi open, μ- pre open, μ-β-open, μ-$\pi r\alpha$ open) sets contained in S.

Definition 2.3[2] A space (X, μ) is called μ-$\pi r\alpha$ $T_{1/2}$ space if every μ-$\pi r\alpha$ closed set is μ- pre closed.

Definition 2.4 [2] Let (X, μ) be a generalized topological space and let x$\in$ X, a subset N of X is said to be μ-$\pi r\alpha$-nbhd of x iff there exists a μ-$\pi r\alpha$- open set G such that $x\in G\subset N$.

Definition 2.5 [2] A function f between the generalized topological spaces (X, μ_x) and (Y, μ_y) is called

(i) (μ_x, μ_y)- πrα – continuous function if $f^{-1}(A) \in \mu$-πrα (X, μ_x) for each $A \in (Y, \mu_y)$.
(ii) (μ_x, μ_y)- πrα – irresolute function if $f^{-1}(A) \in \mu$-πrα (X, μ_x) for each $A \in \mu$-πrα(Y, μ_y).

Definition 2.6 [14] Let (X, μ_x) and (Y, μ_y) be GTS's. Then a function f: X→Y is said to be

(i) contra (μ_x, μ_y) – continuous if for each μ-open set U in Y, $f^{-1}(U)$ is μ- closed in X.
(ii) contra (α, μ_y) – continuous if for each μ- open set U in Y, $f^{-1}(U)$ is μ- α-closed in X.
(iii) contra (σ, μ_y) – continuous if for each μ- open set U in Y, $f^{-1}(U)$ is μ- semi closed in X.
(iv) contra (π, μ_y) – continuous if for each μ- open set U in Y, $f^{-1}(U)$ is μ- pre closed in X.
(v) contra (β, μ_y) – continuous if for each μ- open set U in Y, $f^{-1}(U)$ is μ- β-closed in X.

3. Contra (πp, μ_y) – continuous functions

Definition 3.1 Let (X, μ_x) and (Y, μ_y) be GTS's.
Then a function f: X→Y is said to be contra $(\pi p, \mu_y)$ – continuous, if for each μ-open set U in Y, $f^{-1}(U)$ is μ-πrα closed in X**.**

Theorem 3.2 (i) Every contra (μ_x, μ_y) – continuous function is contra $(\pi p, \mu_y)$ – continuous.
(ii) Every contra (α, μ_y) – continuous function is contra $(\pi p, \mu_y)$ – continuous.
(iii) Every contra (π, μ_y) – continuous function is contra $(\pi p, \mu_y)$ –continuous.
Proof: Straight forward. Converse of the above statement is not true as shown in the following examples.
Remark: contra $(\pi p, \mu_y)$ – continuous and contra (α, μ_y) – continuous, contra (β, μ_y) – continuous are independent concepts.

Example 3.3 Let X = {a, b, c, d}. Consider a generalized topology μ_x= {∅, {a}, {a, b, c}} on X and define f: $(X, \mu_x) \to (X, \mu_x)$ as follows f(a) = f(b) = d and f(c) = f(d)= a. Then $f^{-1}(\{a\})$ = {c, d}, $f^{-1}(\{a, b, c\})$ = {c, d}.
We have f is contra $(\pi p, \mu_x)$ – continuous but not contra (μ_x, μ_x) – continuous and contra (β, μ_x) – continuous.

Example 3.4 Let X=Y= {a, b, c}. Consider two generalized topologies μ_x= {∅, {b}, {a, c}, {b, c}, X} and μ_y= {∅, {c}} on X and Y respectively. Define f: $(X, \mu_x) \to (Y, \mu_y)$ as follows f(a) = b, f(b) = a and f(c) = c. Then $f^{-1}(\{c\})$ = {c}. We have f is contra $(\pi p, \mu_y)$ – continuous but not contra (α, μ_y) – continuous.

Example 3.5 Let X =Y= {a, b, c}. Consider two generalized topologies μ_x= {∅ , {b}, {b, c}, {a, c}, X} and μ_y= { ∅,{c}} on X.
Define f: $(X, \mu_x) \to (Y, \mu_y)$ as follows f(a) = b , f(b) = c and f(c) = c. Then $f^{-1}(\{c\})$ = {b, c}. We have f is contra $(\pi p, \mu_y)$ – continuous but not contra (π, μ_y) – continuous, contra (β, μ_y) – continuous and contra (σ, μ_y) – continuous.

Example 3.6 Let X= {a, b, c, d}. Consider a generalized topology μ_x= {∅ , {a}, {a, b, c}} on X.
Define f: $(X, \mu_x) \to (X, \mu_x)$ as follows f(a) = d, f (b) = a and f (c) = f (d) = d. Then $f^{-1}(\{a\})$ = {b}, $f^{-1}(\{a, b, c\})$ = {b}. We have f is contra (σ, μ_x) – continuous and contra (β, μ_x) – continuous but not a contra $(\pi p, \mu_x)$ – continuous.

Theorem 3.7 Let (X, μ_x) and (Y, μ_y) be GTS's. Then the following are equivalent for a function f: X→Y.
(i) f is contra $(\pi p, \mu_y)$-continuous.
(ii) The inverse image of every μ-closed set of Y is μ-πrα open in X.
Proof: (i) ⇒(ii) Let U be any μ-closed set of Y. Since Y\U is μ-open then by (i) it follows that $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$ is μ-πrα closed in X. This shows that $f^{-1}(U)$ is μ-πrα open in X. (ii) ⇒(i) similarly.

Definition 3.8[16] Let (X, μ) be a GTS. The generalized Kernel of A⊆ X is denoted by μ- ker (A) and defined as
μ- ker (A) = ∩{G∈ μ;A ⊆G}

Lemma 3.9 [16] Let (X, μ) be a GTS and A⊆ X.
Then μ-ker(A) = {x∈X; $c_\mu(\{x\}) \cap A \neq \emptyset$}.

Theorem 3.10 Suppose that μ-πrαO(X) is open under arbitrary union then for a function f: $(X, \mu_x) \to (Y,\mu_y)$ the following properties are equivalent.

(i) f is contra $(\pi p, \mu_y)$- continuous.
(ii) For every μ- closed subset F of Y, $f^{-1}(Y)$ is μ-πrα open in X.
(iii) For each x∈X and each μ-closed subset F of Y containing f(x) there exists a μ-πrα open set U of X containing x such that f(U)⊆F.
(iv) $f(c_{\pi p}(A)) \subseteq \mu$-ker(f(A)), for every subset A of X.
(v) $c_{\pi p}(f^{-1}(B)) \subseteq f^{-1}(\mu$-ker(B)), for every subset B of Y.

Proof: (i) ⇒(ii) is obvious.
(iii) ⇒(iii) Let x∈ X and F be a μ-closed set in Y containing f(x). By hypothesis, $f^{-1}(F)$ is a μ-πrα open in X.
(iv) Let U = $f^{-1}(F)$ then f(U)= $f(f^{-1}(F)) \subseteq F$. Thus f(U)⊆F.

(iii)⇒(ii) Let F be any μ-closed set of Y and $x \in f^{-1}(F)$. Since f(x)∈F, by(iii) there exists a μ-πrα open set U_x of X such that $f(U_x) \subseteq F$. $f^{-1}(F) = \{\cup U_x / x \in f^{-1}(F)$. Hence $f^{-1}(F)$ is μ-πrα open in X.
(ii)⇒(iv) Let A be any subset of X .Suppose that y∉ μ-ker(f(A)),there exist a μ-closed set F containing y such that f(A)∩F =∅. Thus we have $A \cap f^{-1}(F)$ =∅.Therefore we obtain $f(c_{\pi p}(A)) \cap F = \emptyset$ and $y \notin f(c_{\pi p}(A))$. This implies that $f(c_{\pi p}(A)) \subseteq \mu$-ker(f(A)).
(iv)⇒(v) Let B be any subset of Y.
By (iv), we have $f(c_{\pi p}(f^{-1}(B)) \subseteq \mu$-ker$(f(f^{-1}(B)) \subseteq \mu$-ker(B) and thus $c_{\pi p}(f^{-1}(B)) \subseteq f^{-1}(\mu$-ker(B)).
(v)⇒(i) Let V be any μ-open set of Y. Then by theorem3.10, we have $c_{\pi p}(f^{-1}(V)) \subset f^{-1}(\mu$-ker(V)) = $f^{-1}(V)$ and
$c_{\pi p}(f^{-1}(V)) = f^{-1}(V)$. Hence $f^{-1}(V)$ is a μ-πrα closed set in X.

Definition 3.11 A generalized topological space (X, μ_x) is called (i) μ-πrα locally indiscrete if every μ-πrα open set is μ-closed.
(ii) $T_{\pi p}$- space if every μ-πrα closed set is μ-pre closed.
(iii) μ-πrα space if every μ-πrα closed set is μ-closed.

Theorem 3.12 Let (X, μ_x) and (Y, μ_y) be two GTS's.

(i) If a function f: $(X, \mu_x)\to(Y,\mu_y)$ be a (μ_x,μ_y)-πrα continuous and (X, μ_x) is μ-πrα locally indiscrete then f is contra $(\pi p, \mu_y)$- continuous.
(ii) If a function f: $(X, \mu_x)\longrightarrow(Y,\mu_y)$ is a contra $(\pi p,\mu_y)$- continuous and (X, μ_x) is μ-πrα $T_{1/2}$ space then f is contra(π, μ_y)- continuous.
(iii) If a function f: $(X, \mu_x)\to(Y, \mu_y)$ is contra $(\pi p, \mu_y)$- continuous and (X, μ_x) is μ-πrα space then f is contra (μ_x, μ_y)- continuous.
(iv) If a function f: $(X, \mu_x)\to(Y,\mu_y)$ is contra$(\pi p, \mu_y)$- continuous and (X, μ_x) is $T_{\pi p}$- space then f is contra(β, μ_y)- continuous.

Proof: (i) Let V be an μ-open set in Y. By assumption $f^{-1}(V)$ is μ-πrα open in X. Since X is μ-πrα locally indiscrete, $f^{-1}(V)$ is μ-closed in X. Hence f is contra (μ_x, μ_y) – continuous.
(ii) Let V be an μ- open set in Y. By assumption $f^{-1}(V)$ is μ-πrα closed in X. Since X is μ-πrα-$T_{1/2}$ space then $f^{-1}(V)$ is μ-pre closed set in X. Hence f is contra (π, μ_y) – continuous.
(iii) Let V be an μ- open set in Y. By assumption $f^{-1}(V)$ is μ-πrα closed set in X. Since X is μ-πrα space then $f^{-1}(V)$ is μ-closed in X. Hence f is contra (μ_x , μ_y)- continuous.
(iv) Let V be an μ- open in Y. By assumption $f^{-1}(V)$ is μ-πrα closed in X. Since X is $T_{\pi p}$ space then $f^{-1}(V)$ is μ-pre closed in X. But every μ- pre closed set is μ-β closed set. Therefore $f^{-1}(V)$ is μ-β closed set in X. Hence f is contra (β, μ_y)- continuous

Theorem 3.13 Let (X, μ_x) and (Y, μ_y) be two GTS's and a function f: X→Y then the following are equivalent.

(i) The function f is (μ_x, μ_y)- πrα continuous.
(ii) The inverse of each μ-open set is μ-πrα open.
(iii) For each x in (X, μ_x), the inverse of every μ-nbhd of f(x) is μ-πrα nbhd of x.
(iv) For each x in (X, μ_x) and every μ-open set U containing f(x) there exist a μ-πrα open set V containing x such that $f(V)\subseteq U$.
(v) $f(c_{\pi p}(A))\subseteq c_\mu(f(A))$, for every subset A of X.
(vi) $c_{\pi p}(f^{-1}(B))\subseteq f^{-1}(c_\mu(B))$, for every subset B of Y.

Proof: (i) ⇒(ii) Straight forward.
(ii)⇒(iii) Let x∈X. Assume that V be a μ-nbhd of f(x), there exists a μ-open set U in Y such that $f(x)\in U\subseteq V$. Consequently $f^{-1}(U)$ is μ-πrα open in X and $x\in f^{-1}(U)\subseteq f^{-1}(V)$. Then $f^{-1}(V)$ is μ-πrα nbhd of x.
(iii)⇒(iv) Let x∈X and U be a μ-nbhd of f(x). Then by assumption $V= f^{-1}(U)$ is a μ-πrα nbhd of x and $f(V)= f(f^{-1}(U))\subseteq U$.
(iv)⇒(v) Let A be a subset of X, $f(x)\notin c_\mu(f(A))$. Then there exists a μ- open subset V of Y containing f(x) such that $V\cap f(A)=\emptyset$. Then by (iv) there exists a μ-πrα open set such that $f(x)\in f(U)\subseteq V$.Hence $f(U)\cap f(A)= \emptyset$, which implies $U\cap A= \emptyset$. Consequently $x\notin c_{\pi p}(A)$ and $f(x)\notin f(c_{\pi p}(A))$. Hence $f(c_{\pi p}(A))\subseteq c_\mu(f(A))$.
(v)⇒(vi) Let A be a subset of Y.
By (v) we obtain $f(c_{\pi p}(f^{-1}(A))\subseteq c_\mu(f(f^{-1}(A))$.
Thus $f(c_{\pi p}(f^{-1}(A))\subseteq c_\mu(A)$.
This implies $(c_{\pi p}(f^{-1}(A))\subseteq f^{-1}(c_\mu(A))$.
(vi)⇒(i) Let F be a μ-closed subset of Y. Since $c_\mu(F) = F$ and by (vi) $f(c_{\pi p}(f^{-1}(F))\subseteq c_\mu(f(f^{-1}(F))\subseteq c_\mu(F) = F$.
This implies $c_{\pi p}(f^{-1}(F))\subseteq f^{-1}(F)$ and so $f^{-1}(F)$ is μ-πrα closed.

Theorem 3.14 A function f: $(X, \mu_x) \to(Y, \mu_y)$ is (μ_x, μ_y) – πrα continuous if and only if $f^{-1}(U)$ is μ-πrα open in X, for every μ- open set U in Y.

Theorem 3.15 Let (X, μ_x) and (Y, μ_y) be two GTS's.
If a function f: X→Y is contra $(\pi p, \mu_y)$ –continuous and Y is μ-regular then f is (μ_x, μ_y) – πrα continuous.
Proof: Let x be an arbitrary point of X and V be an μ- open set of Y containing f(x).Since Y is μ-regular there exist an μ-open set W in Y containing f(x) such that $c_\mu(W)\subseteq V$. Since f is contra $(\pi p, \mu_y)$ – continuous, by theorem 3.10 (iii) there exist a μ-πrα open set U of X containing x such that $f(U)\subseteq c_\mu(W)$. Then $f(U)\subseteq c_\mu(W) \subseteq V$.Hence f is (μ_x ,μ_y) –πrα continuous. Hence f is (μ_x, μ_y) –πrα continuous.

Definition 3.16 Let f: $(X, \mu_x)\to(Y, \mu_y)$ be a function on GTS's. Then the function f is said to be

(i) μ-πrα open, if the image of each μ-πrα open set in X is a μ-πrα open set in Y.
(ii) μ-πrα closed, if the image of each μ-πrα closed in X is μ-πrα closed in Y.

Definition 3.17[1] A GTS (X, μ_x) is said to be μ_x-connected if X is not the union of two disjoint non empty μ-open subsets of X.

Definition 3.18 A GTS (X, μ_x) is said to be μ_x-πrα connected if X is not the union of two disjoint non empty μ-πrα open subsets of X.

Theorem 3.19 Let f: $(X, \mu_x) \to (Y, \mu_y)$ be a (μ_x, μ_y)-πrα continuous surjection and if (X, μ_x) is μ_x- πrα connected then (Y, μ_y) is μ_y - connected.

Proof: Let f be a (μ_x, μ_y)- πrα continuous function of a μ_x-πrα connected space X onto Y. If possible let Y be μ_y- disconnected. Let A and B form a disconnected of Y. Then A and B are μ- open and $Y= A\cup B$ and $A\cap B = \emptyset$.

Since f is (μ_x, μ_y)-πrα continuous surjection function, $X = f^{-1}(Y) = f^{-1}(A\cup B) = f^{-1}(A)\cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non empty μ-πrα open sets in X. Also $f^{-1}(A)\cap f^{-1}(B) =\emptyset$. Hence X is not μ-πrα connected. This is a contradiction. Therefore Y is μ_y-connected.

Definition 3.20 A GTS (X, μ_x) is said to be

(i) μ-πrα T_1 if for each pair of distinct points x and y in X, there exist two disjoint μ-πrα open sets U and V in X such that $x\in U$, $y\notin U$ and $y\in V$, $x\notin V$.

(ii) μ-πrα T_2 if for each pair of distinct points x and y in X, there exist disjoint μ-πrα open sets U and V containing x and y respectively.

Definition 3.21 A GTS (X, μ) is said to be μ-Urysohn space if for each pair of distinct points x and y in X, there exists μ- open sets U and V such that $x\in U$ and $y\in V$ and $c_\mu(U)\cap c_\mu(V) =\emptyset$.

Theorem 3.22 Let (X, μ_x) and (Y, μ_y) be GTS's. If the following three assumptions are satisfied

(i) for each pair of distinct points x and y in X there exists a function f of X into Y such that $f(x) \neq f(y)$.

(ii) (Y, μ_y) is a μ_y-Urysohn space.

(iii) f is a contra $(\pi p, \mu_y)$ continuous at x and y.

Then (X, μ_x) is μ- πrα T_2.

Proof: Let x and y be any distinct points in X. By assumption (i) there exists a function f: X→ Y such that $f(x) \neq f(y)$. Let $a = f(x)$ and $b= f(y)$. Since Y is a μ_y- Urysohn space then there exists μ-open sets V and W containing a and b respectively such that $c_\mu(V)\cap c_\mu(W) = \emptyset$.

Since f is contra $(\pi p, \mu_y)$ –continuous at x and y then there exists μ-πrα open sets A and B containing x and y respectively, such that $f(A) \subseteq c_\mu(V)$ and $f(B)\subseteq c_\mu(W)$. Then $f(A)\cap f(B) = \emptyset$. So $A\cap B =\emptyset$. Hence X is μ-πrα T_2.

For a map f: $(X, \mu_x)\to(Y, \mu_y)$, the subset $\{(x, f(x));x\in X\}\subset X\times Y$ is called the graph of f and is denoted by $G_\mu(f)$.

Theorem 3.23 Let (X, μ_x) and (Y, μ_y) be GTS's. Let f: X→Y be a map and g: X→ X×Y the graph function of f defined by g(x) = (x, f(x)) for every $x\in X$.

If g is contra $(\pi p, \mu_y)$- continuous then f is contra $(\pi p, \mu_y)$- continuous.

Proof: Let U be an μ- open set in Y. Then X×U is an μ- open set in X×Y. Since g is contra $(\pi p, \mu_y)$ –continuous then $f^{-1}(U) = g^{-1}(X\times U)$ is μ-πrα closed in X. Hence f is contra $(\pi p, \mu_y)$ – continuous.

Definition 3.24 The graph $G_\mu(f)$ of a map f: $(X, \mu_x) \to (Y, \mu_y)$ between GTS's is said to be contra $(\pi p, \mu_y)$-closed if for each $(x, y) \in (X\times Y)\backslash G_\mu(f)$, there exist an μ-πrα open set U in X containing x and a μ- closed set V in Y containing y such that $(U\times V)\cap G_\mu(f) = \emptyset$.

Lemma 3.25 Let $G_\mu(f)$ be the graph of a map f: $(X, \mu_x) \to (Y, \mu_y)$ between GTS's. For any subset $A\subseteq X$ and $B\subseteq Y$, $f(A)\cap B = \emptyset$ if and only if $(A\times B)\cap G_\mu(f) = \emptyset$

Proposition 3.26 The following properties are equivalent for the graph $G_\mu(f)$ of a map f in GTS's.

(i) $G_\mu(f)$ is contra $(\pi p, \mu_y)$ – closed.

(ii) For each $(x, y) \in (X\times Y)\backslash G_\mu(f)$, there exist an μ-πrα open set U in X containing x and a μ-closed V in Y containing y such that $f(U)\cap V = \emptyset$.

Theorem 3.27 Let (X, μ_x) and (Y, μ_y) be two GTS's. If f: X→Y is contra $(\pi p, \mu_y)$ – continuous and Y is μ_y- Urysohn space, then $G_\mu(f)$ is contra $(\pi p, \mu_y)$ – closed in X×Y.

Proof: Let $(x, y)\in (X\times Y)\backslash G_\mu(f)$. It follows that $f(x)\neq y$ and since Y is μ_y- Urysohn space then if for each distinct points x and y in X there exists μ-open sets B and C such that $f(x)\in B$ and $y\in C$ and $c_\mu(B)\cap c_\mu(C) =\emptyset$. Since f is contra $(\pi p, \mu_y)$ – continuous then there exists an μ-πrα closed set A in X containing x such that $f(A)\subset c_\mu(B)$. Therefore $f(A)\cap c_\mu(C) = \emptyset$ and $G_\mu(f)$ is contra $(\pi p, \mu_y)$ – closed in X×Y.

Theorem 3.28 Let (X, μ_x) and (Y, μ_y) be two GTS's. Let f: X→Y have a contra $(\pi p, \mu_y)$ – closed graph. If f is injective then X is μ-πrα T_1.

Proof: Let x_1 and x_2 be any two distinct points of X. We have $(x_1, f(x_2))\in(X\times Y)\backslash G_\mu(f)$ and there exist an μ-πrα open set U in X containing x_1 and a μ-closed set V in Y containing x_2 such that $f(U)\cap F = \emptyset$.

Hence $U\cap f^{-1}(F) = \emptyset$. Therefore we have $x_2\notin U$. This implies that X is μ-πrα T_1.

Theorem 3.29 Let (X, μ_x), (Y, μ_y) and (Z, μ_z) be GTS's. Let f: X→Y be surjective, (μ_x, μ_y) –πrα irresolute and μ-πrα closed and g: Y→Z be any function. Then g ∘f is contra $(\pi p, \mu_y)$ – continuous if and only if g is contra $(\pi p, \mu_y)$ – continuous.

Proof: Suppose g ∘f is contra $(\pi p, \mu_y)$ – continuous. Let F be any μ-open set in Z. Then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is μ-πrα closed in X. Since f is μ-πrα closed and surjective, $f(f^{-1}(g^{-1}(F)) = g^{-1}(F)$ is μ-πrα in Y and we obtain that g is contra (πp, μ_y) – continuous.

Conversely, suppose g is contra (πp, μ_y) – continuous. Let V be μ-open in Z. Then $g^{-1}(V)$ is μ-$\pi r\alpha$ closed in Y. Since f is (μ_x, μ_y) – irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is μ-$\pi r\alpha$ closed in X and so $g\circ f$ is contra (πp, μ_y) – continuous.

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