Contra \((\pi p, \mu_y)\)-Continuity on Generalized Topological Spaces

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Abstract: In this paper we introduce a new notion called contra \((\pi p, \mu_y)\) – continuous function on generalized topological space. The properties and characterization of such functions are investigated.

Key words: \(\mu\)-\(\pi\)- space, \(\pi\)-\(\mu\)- space, \(\mu\)-\(\pi\)-\(\alpha\)-\(\beta\)- space, \(\mu\)-\(\pi\)-\(\alpha\)-\(\beta\)- space, \(\mu\)-\(\pi\)- connected, \(\mu\)-Urysohn, \(\mu\)-\(\pi\)- locally indiscrete, contra \((\pi p, \mu_y)\) – continuous, contra \((\pi p, \mu_y)\) – closed, \(\mu\)-\(\pi\)- open.

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1. Introduction

Á.Čsászár [3]-[13] has introduced the notions of generalized topological space, obtain characterizations for generalized continuous functions and associated interior and closure operators. In [5] he introduced characterizations for generalized continuous functions. Also in [3] he investigated the notions of \(\mu\)-\(\alpha\)-open sets, \(\mu\)- semi open sets, \(\mu\)- pre open sets and \(\mu\)-\(\beta\)- open sets in generalized topological space. W.K. Min [15] has introduced and studied the notions of \((\alpha, \mu)\) – continuous functions, \((\sigma, \mu_y)\) – continuous functions, \((\pi, \mu_y)\) – continuous functions, and \((\beta, \mu_y)\) – continuous functions in generalized topological spaces. Also D. Jayanthi [14] has introduced some contra continuous functions on generalized topological spaces such as contra \((\mu_1, \mu_y)\) – continuous functions, contra \((\alpha, \mu_y)\) – continuous, contra \((\sigma, \mu_y)\) – continuous functions, and contra \((\beta, \mu_y)\) – continuous functions. In this paper we introduce contra \((\pi p, \mu_y)\) – continuous functions and investigate their characterizations and relationships among these functions.

2. Preliminaries

We recall some basic concepts and results. Let \(X\) be a nonempty set and let \(\exp(X)\) be the power set of \(X\). \(\mu\in\exp(X)\) is called a generalized topology [5](briefly, GT) on \(X\), if \(\emptyset \in \mu\) and unions of elements of \(\mu\) belong to \(\mu\). The pair \((X, \mu)\) is called a generalized topological space (briefly, GTS). The elements of \(\mu\) are called \(\mu\)-open [3] subsets of \(X\) and the complements are called \(\mu\)-closed sets. If \((X, \mu)\) is a GTS and \(\mathcal{A}\subseteq \mathcal{X}\), then the interior of \(\mathcal{A}\) is the union of all \(\mu\)-open \(\mathcal{A}\in \mathcal{G}\) and the closure of \(\mathcal{A}\) is the intersection of all \(\mu\)-closed sets containing \(\mathcal{A}\).

Note that \(c_\mu(A) = X - i_\mu(X - A)\) and \(i_\mu(A) = X - c_\mu(X - A)\) [5].

Definition 2.1 [5] Let \((X, \mu)\) be a generalized topological space and \(\mathcal{A}\subseteq \mathcal{X}\). Then \(A\) is said to be

(i) \(\mu\)-semi open if \(\mathcal{A}\subseteq c_\mu(i_\mu(A))\).

(ii) \(\mu\)-pre open if \(\mathcal{A}\subseteq i_\mu(c_\mu(A))\).

(iii) \(\mu\)-\(\alpha\)-open if \(\mathcal{A}\subseteq c_\mu(i_\mu(A))\).

(iv) \(\mu\)-\(\beta\)-open if \(\mathcal{A}\subseteq c_\mu(i_\mu(A))\).

(v) \(\mu\)-\(\alpha\)-open [17] if \(A = i_\mu(c_\mu(A))\).

(vi) \(\mu\)-\(\alpha\)-open [2] if there is a \(\mu\)-r-open set \(U\) such that \(U\subseteq A\subseteq \mathcal{C}_\mu(U)\).

Definition 2.2 [2] Let \((X, \mu)\) be a generalized topological space and \(\mathcal{A}\subseteq \mathcal{X}\). Then \(A\) is said to be \(\mu\)-\(\pi\)-closed set if \(c_\mu(A) \subseteq U\) whenever \(\mathcal{A}\subseteq \mathcal{U}\) and \(U\) is \(\mu\)-increasing set. The complement of \(\mu\)-\(\pi\)-closed set is said to be \(\mu\)-open set.

The complement of \(\mu\)-semi open (\(\mu\)-pre open, \(\mu\)-\(\alpha\)-open, \(\mu\)-\(\beta\)-open, \(\mu\)-\(\alpha\)-\(\beta\)-open) set is called \(\mu\)-semi closed (\(\mu\)-pre closed, \(\mu\)-\(\alpha\)- closed, \(\mu\)-\(\beta\)- closed, \(\mu\)-\(\alpha\)-\(\beta\)- closed, \(\mu\)-\(\alpha\)-\(\beta\)- closed) set.

Let us denote the class of all \(\mu\)-semi open sets, \(\mu\)-pre open sets, \(\mu\)-\(\alpha\)-open sets, \(\mu\)-\(\beta\)-open sets, and \(\mu\)-\(\alpha\)-\(\beta\)-open sets on \(X\) by \(\sigma(\mu)\) (\(\sigma\) for short), \(\pi(\mu)\) (\(\pi\) for short), \(\alpha(\mu)\) (\(\alpha\) for short), \(\beta(\mu)\) (\(\beta\) for short) and \(\pi(\mu)\) (\(\pi\) for short)respectively. Let \(\mu\) be a generalized topology on a non empty set \(X\) and \(\mathcal{A}\subseteq \mathcal{X}\). The \(\mu\)-\(\alpha\)-interior (resp. \(\mu\)-\(\alpha\)-closure) of a subset \(S\) of \(X\) denoted by \(c_\mu(S)\) (resp. \(c_\mu(S)\), \(c_\mu(S)\), \(c_\mu(S)\)) is the intersection of \(\mu\)-closed( resp. \(\mu\)-semi closed, \(\mu\)-pre closed, \(\mu\)-\(\alpha\)-closed, \(\mu\)-\(\alpha\)-\(\beta\)-closed, \(\mu\)-\(\alpha\)-\(\beta\)-closed) sets including \(S\). The \(\mu\)-\(\alpha\)-interior (resp. \(\mu\)-\(\alpha\)-\(\beta\)-interior, \(\mu\)-\(\alpha\)-\(\beta\)-interior) of a subset \(S\) of \(X\) denoted by \(i_\mu(S)\) (resp. \(i_\mu(S)\), \(i_\mu(S)\), \(i_\mu(S)\)) is the union of \(\mu\)-\(\alpha\)-open (resp. \(\mu\)-semi open, \(\mu\)-pre open, \(\mu\)-\(\alpha\)-\(\beta\)-open, \(\mu\)-\(\alpha\)-\(\beta\)-open) sets contained in \(S\).

Definition 2.3 [2] A space \((X, \mu)\) is called \(\mu\)-\(\pi\)-\(\alpha\)-\(\beta\) space if every \(\mu\)-\(\pi\)-\(\alpha\)-\(\beta\)- closed set is \(\mu\)-pre closed.

Definition 2.4 [2] Let \((X, \mu)\) be a generalized topological space and let \(x\in X\), a subset \(N\) of \(X\) is said to be \(\mu\)-\(\alpha\)-\(\beta\)-nbhd of \(x\) iff there exists a \(\mu\)-\(\alpha\)-\(\beta\)- open set \(G\) such that \(x\in G\subseteq N\).

Definition 2.5 [2] A function \(f\) between the generalized topological spaces \((X, \mu)\) and \((Y, \mu)\) is called
(i) \((\mu_1, \mu_2, \pi_\alpha)\) - \(\pi\alpha\) - continuous function if \(f^{-1}(A)\in\mu_\pi\)
\(\pi\alpha\) \((X, \mu_\pi, \mu_\pi)\) for each \(A \in (Y, \mu_\pi)\).
(ii) \((\mu_1, \mu_2, \pi_\alpha)\) - irresolute function if \(f^{-1}(A)\in\mu_\pi\)
\(\pi\alpha\) \((X, \mu_\pi, \mu_\pi)\) for each \(A \in (Y, \mu_\pi)\).

**Definition 2.6** [14] Let \((X, \mu_\pi)\) and \((Y, \mu_\pi)\) be GTS’s. Then a function \(f: X\rightarrow Y\) is said to be:

(i) \(\mu\)-\((\mu_\pi, \mu_\pi)\) - continuous if for each \(\mu\)-\open \(\mu\)-\closed \(X\), \(f^{-1}(U)\) is \(\mu\)-\closed \(X\).
(ii) \((\alpha, \mu_\pi)\) - continuous if for each \(\mu\)-\open \(\mu\)-\closed \(X\), \(f^{-1}(U)\) is \(\mu\)-\closed \(X\).
(iii) \((\beta, \mu_\pi)\) - continuous if for each \(\mu\)-\open \(\mu\)-\closed \(X\), \(f^{-1}(U)\) is \(\mu\)-\closed \(X\).

3. Contra \((\pi\alpha, \mu_\pi)\) - continuous functions

**Definition 3.1** Let \((X, \mu_\pi)\) and \((Y, \mu_\pi)\) be GTS’s. Then a function \(f: X\rightarrow Y\) is said to be \(\mu\)-\((\pi\alpha, \mu_\pi)\) - continuous, if for each \(\mu\)-\open \(X\), \(f^{-1}(U)\) is \(\pi\alpha\)-\closed \(X\).

**Theorem 3.2** (i) Every contra \((\mu_\pi, \mu_\pi)\) - continuous function is contra \((\pi\alpha, \mu_\pi)\) - continuous.
(ii) Every contra \((\alpha, \mu_\pi)\) - continuous function is contra \((\pi\alpha, \mu_\pi)\) - continuous.
(iii) Every contra \((\beta, \mu_\pi)\) - continuous function is contra \((\pi\alpha, \mu_\pi)\) - continuous.

Proof: Straightforward. Converse of the above statement is not true as shown in the following examples.

**Remark:** contra \((\pi\alpha, \mu_\pi)\) - continuous and contra \((\alpha, \mu_\pi)\) - continuous, contra \((\beta, \mu_\pi)\) - continuous are independent concepts.

**Example 3.3** Let \(X = \{a, b, c, d\}\). Consider a generalized topology \(\mu_\pi = \{\emptyset, \{a\}, \{a, b, c\}\}\) on \(X\) and define \(f: (X, \mu_\pi)\rightarrow (X, \mu_\pi)\) as follows \(f(a) = d\) \(f(b) = d\) \(f(c) = b\) \(f(d) = a\). Then \(f^{-1}([a]) = \{c, d\}\), \(f^{-1}([a, b, c]) = \{c, d\}\).

We have \(f\) is contra \((\pi\alpha, \mu_\pi)\) - continuous but not contra \((\alpha, \mu_\pi)\) - continuous and contra \((\beta, \mu_\pi)\) - continuous.

**Example 3.4** Let \(X = \{a, b, c\}\). Consider two generalized topologies \(\mu_\pi = \{\emptyset, \{a\}, \{a, b, c\}\}\) on \(X\) and \(Y\) respectively. Define \(f: (X, \mu_\pi)\rightarrow (Y, \mu_\pi)\) as follows \(f(a) = b\), \(f(b) = a\) and \(f(c) = c\). Then \(f^{-1}([c]) = \{c\}\). We have \(f\) is contra \((\pi\alpha, \mu_\pi)\) - continuous but not contra \((\alpha, \mu_\pi)\) - continuous.

**Example 3.5** Let \(X = \{a, b, c\}\). Consider two generalized topologies \(\mu_\pi = \{\emptyset, \{a\}, \{a, b, c\}\}\) on \(X\) and \(Y\) respectively. Define \(f: (X, \mu_\pi)\rightarrow (Y, \mu_\pi)\) as follows \(f(a) = b\), \(f(b) = c\) and \(f(c) = c\). Then \(f^{-1}([c]) = \{b, c\}\). We have \(f\) is contra \((\pi\alpha, \mu_\pi)\) - continuous but not contra \((\alpha, \mu_\pi)\) - continuous, contra \((\beta, \mu_\pi)\) - continuous and contra \((\sigma, \mu_\pi)\) - continuous.

**Example 3.6** Let \(X = \{a, b, c, d\}\). Consider a generalized topology \(\mu_\pi = \{\emptyset, \{a\}, \{a, b, c\}\}\) on \(X\) and \(Y\) respectively. Define \(f: (X, \mu_\pi)\rightarrow (X, \mu_\pi)\) as follows \(f(a) = d\), \(f(b) = a\) and \(f(c) = d\). Then \(f^{-1}([a]) = \{b\}\), \(f^{-1}([a, b, c]) = \{b\}\). We have \(f\) is contra \((\pi\alpha, \mu_\pi)\) - continuous and contra \((\beta, \mu_\pi)\) - continuous but not contra \((\alpha, \mu_\pi)\) - continuous.
Definition 3.11 A generalized topological space $(X, \mu_x)$ is called (i) $\mu$-$\pi$ locally indiscrete if every $\mu$-$\pi$ open set is $\mu$-closed.

(ii) $T_{\pi\mu}$-space if every $\mu$-$\pi$ closed set is $\mu$-pre closed.

(iii) $\mu$-$\pi$ space if every $\mu$-$\pi$ closed set is $\mu$-closed.

Theorem 3.12 Let $(X, \mu_x)$ and $(Y, \mu_y)$ be two GTS’s.

(i) If a function $f$: $(X, \mu_x)$→$(Y, \mu_y)$ is a $(\mu, \mu_y)$-continuous and $(X, \mu_x)$ is $\mu$-$\pi$ locally indiscrete then $f$ is contra $(\tau_\pi, \mu_y)$-continuous.

(ii) If a function $f$: $(X, \mu_x)$→$(Y, \mu_y)$ is a contra $(\tau_\pi, \mu_y)$-continuous and $(X, \mu_x)$ is $\mu$-$\pi$ $\Gamma_{1/2}$ space then $f$ is contra $(\tau_\pi, \mu_y)$-continuous.

(iii) If a function $f$: $(X, \mu_x)$→$(Y, \mu_y)$ is contra $(\tau_\pi, \mu_y)$-continuous and $(X, \mu_x)$ is $\mu$-$\pi$ space then $f$ is contra $(\mu_x, \mu_y)$-continuous.

(iv) If a function $f$: $(X, \mu_x)$→$(Y, \mu_y)$ is contra $(\tau_\pi, \mu_y)$-continuous and $(X, \mu_x)$ is $T_{\pi\mu}$ space then $f$ is contra $(\beta, \mu_y)$-continuous.

Proof: (i) Let $V$ be an $\mu$-open set in $Y$. By assumption $f^1(V)$ is $\mu$-$\pi$ open in $X$. Since $X$ is $\mu$-$\pi$ locally indiscrete, $f^1(V)$ is $\mu$-closed in $X$. Hence $f$ is contra $(\mu_x, \mu_y)$-continuous.

(ii) Let $V$ be an $\mu$-open set in $Y$. By assumption $f^1(V)$ is $\mu$-$\pi$ closed in $X$. Since $X$ is $\mu$-$\pi$-$\Gamma_{1/2}$ space then $f^1(V)$ is $\mu$-pre closed in $X$. Hence $f$ is contra $(\tau_\pi, \mu_y)$-continuous.

(iii) Let $V$ be an $\mu$-open set in $Y$. By assumption $f^1(V)$ is $\mu$-$\pi$ closed in $X$. Since $X$ is $\mu$-$\pi$-$\Gamma_{1/2}$ space then $f^1(V)$ is $\mu$-pre closed in $X$. But every $\mu$-pre closed set is $\mu$-$\beta$ closed set. Therefore $f^1(V)$ is $\mu$-$\beta$ closed set in $X$. Hence $f$ is contra $(\beta, \mu_y)$-continuous.

Theorem 3.13 Let $(X, \mu_x)$ and $(Y, \mu_y)$ be two GTS’s and a function $f$: $X$→$Y$ then the following are equivalent.

(i) The function $f$ is $(\mu_x, \mu_y)$-$\pi$-continuous.

(ii) The inverse of each $\mu$-open set is $\mu$-$\pi$ open.

(iii) For each $x$ in $(X, \mu_x)$, the inverse of every $\mu$-nbhd of $f(x)$ is $\mu$-$\pi$ nbhd of $x$.

(iv) For each $x$ in $(X, \mu_x)$ and every $\mu$-open set $U$ containing $f(x)$ there exist a $\mu$-$\pi$ open set $V$ containing $x$ such that $f(V) \subseteq U$.

(v) $c_{\pi\mu}(A) \subseteq c_\mu(f(A))$, for every subset $A$ of $X$.

Proof: (i) $\Rightarrow$ (ii) Straightforward.

(ii) $\Rightarrow$ (iii) Let $x \in X$. Assume that $V$ be a $\mu$-nbhd of $f(x)$, there exists a $\mu$-open set $U$ in $Y$ such that $f(x) \in U \subseteq V$. Consequently $f^1(U)$ is $\mu$-$\pi$ open in $X$ and $x \in \mu$-$\pi$ $\subseteq f^1(V)$. Then $f^1(V)$ is $\mu$-$\pi$ $\subseteq V$.

(iii) $\Rightarrow$ (iv) Let $x \in X$ and $U$ be a $\mu$-nbhd of $f(x)$. Then by assumption $V = f^1(U)$ is a $\mu$-$\pi$ nbhd of $x$ and $f(V) = f(f^1(U)) \subseteq U$.

(iv) $\Rightarrow$ (v) Let $A$ be a subset of $X$, $f(x) \in c_{\pi\mu}(f(A))$. Then there exists a $\mu$-open subset $V$ of $Y$ containing $f(x)$ such that $V \cap A = \emptyset$. Since $X$ is $\mu$-$\pi$ $\subseteq V$. Hence $f(U) \cap A = \emptyset$ and $f(V) \cap A = \emptyset$.

Consequently $x \in c_{\pi\mu}(A)$ and $f(x) \in c_{\pi\mu}(f(A))$. Hence $f(c_{\pi\mu}(A)) \subseteq c_\mu(f(A))$.

(v) $\Rightarrow$ (vi) Let $f$ be a $\mu$-closed subset of $Y$. Since $c_\mu(F) = F$ and by (vi) $f(c_{\pi\mu}(f(A))) \subseteq c_\mu(f(f^1(A)))$. Thus $f(c_{\pi\mu}(f(A))) \subseteq c_\mu(f(A))$.

This implies $c_{\pi\mu}(f(A)) \subseteq c_\mu(f(A))$ and $f^1(F)$ is $\mu$-$\pi$ closed.

Theorem 3.14 A function $f$: $(X, \mu_x)$→$(Y, \mu_y)$ is $(\mu_x, \mu_y)$ $-\pi$-continuous if and only if $f^1(U)$ is $\mu$-$\pi$ open in $X$, for every $\mu$-open set $U$ in $Y$.

Theorem 3.15 Let $(X, \mu_x)$ and $(Y, \mu_y)$ be two GTS’s. If a function $f$: $X$→$Y$ is contra $(\tau_\pi, \mu_y)$-continuous and $Y$ is $\mu$-regular then $f$ is $(\mu_x, \mu_y)$ $-\pi$-continuous.

Proof: Let $x$ be an arbitrary point of $X$ and $V$ be an $\mu$-open set of $Y$ containing $f(x)$. Since $Y$ is $\mu$-regular there exist an $\mu$-open set $W$ in $Y$ containing $f(x)$ such that $c_\mu(W) \subseteq V$. Since $f$ is contra $(\tau_\pi, \mu_y)$-continuous, by theorem 3.10 (iii) there exist a $\mu$-$\pi$ open set $U$ of $X$ containing $x$ such that $f(U) \subseteq c_{\pi\mu}(W)$. Then $f(U) \subseteq c_{\pi\mu}(W) \subseteq V$. Hence $f$ is $(\mu_x, \mu_y)$ $-\pi$-continuous. Hence $f$ is $(\mu_x, \mu_y)$ $-\pi$-continuous.

Definition 3.16 Let $f$: $(X, \mu_x)$→$(Y, \mu_y)$ be a function on GTS’s. Then the function $f$ is said to be

(i) $\mu$-$\pi$ open, if the image of each $\mu$-$\pi$ open set in $X$ is a $\mu$-$\pi$ open set in $Y$.

(ii) $\mu$-$\pi$ closed, if the image of each $\mu$-$\pi$ closed in $X$ is $\mu$-$\pi$ closed in $Y$.

Definition 3.17[1] A GTS $(X, \mu_x)$ is said to be $\mu_x$-connected if $X$ is not the union of two disjoint non-empty $\mu$-open subsets of $X$. 
Definition 3.18 A GTS $(X, \mu_x)$ is said to be $\mu_x$, $\pi$ connected if $X$ is not the union of two disjoint non empty $\mu_x$, $\pi$ open subsets of $X$.

**Theorem 3.19** Let $f: (X, \mu_x) \rightarrow (Y, \mu_y)$ be a $(\mu_x, \mu_y)$, $\pi$ continuous surjection and if $(X, \mu_x)$ is $\mu_x$, $\pi$ connected then $(Y, \mu_y)$ is $\mu_y$, $\pi$ connected.

**Proof:** Let $f$ be a $(\mu_x, \mu_y)$, $\pi$ continuous function of a $\mu_x$, $\pi$ connected space $X$ onto $Y$. If possible let $Y$ be $\mu_y$ disconnected. Let $A$ and $B$ form a disconnected of $Y$. Then $A$ and $B$ are $\mu_y$ open and $Y=A\cup B$ and $\cap A B=\emptyset$.

Since $f$ is $(\mu_x, \mu_y)$, $\pi$ continuous surjection function, $X=f^{-1}(Y)=f^{-1}(A\cup B)=f^{-1}(A)\cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non empty $\mu_x$, $\pi$ open sets in $X$. Also $f^{-1}(A)\cap f^{-1}(B)=\emptyset$. Hence $X$ is not $\mu_x$, $\pi$ connected. This is a contradiction. Therefore $Y$ is $\mu_y$, $\pi$ connected.

**Definition 3.20** A GTS $(X, \mu_x)$ is said to be

(i) $\mu_x$, $\pi$ $T_1$ if for each pair of distinct points $x$ and $y$ in $X$, there exist two disjoint $\mu_x$, $\pi$ open sets $U$ and $V$ in $X$ such that $x \in U$, $y \in V$ and $y \in V$, $x \in V$.

(ii) $\mu_x$, $\pi$ $T_2$ if for each pair of distinct points $x$ and $y$ in $X$, there exist disjoint $\mu_x$, $\pi$ open sets $U$ and $V$ containing $x$ and $y$ respectively.

**Definition 3.21** A GTS $(X, \mu_x)$ is said to be $\mu_x$-Urysohn space if for each pair of distinct points $x$ and $y$ in $X$, there exists $\mu_x$ open sets $U$ and $V$ such that $x \in U$ and $y \in V$.

**Theorem 3.22** Let $(X, \mu_x)$ and $(Y, \mu_y)$ be GTS’s. If the following three assumptions are satisfied

(i) for each pair of distinct points $x$ and $y$ in $X$ there exists a function $f$ of $X$ into $Y$ such that $f(x) \neq f(y)$.

(ii) $(Y, \mu_y)$ is a $\pi$-Urysohn space.

(iii) $f$ is a $\pi$, $\mu_y$ connected at $x$ and $y$.

Then $(X, \mu_x)$ is $\mu_x$, $\pi$ $T_2$.

**Proof:** Let $x$ and $y$ be any distinct points in $X$. By assumption (i) there exists a function $f: X \rightarrow Y$ such that $f(x) \neq f(y)$. Let $a = f(x)$ and $b = f(y)$. Since $Y$ is a $\mu_y$, $\pi$-Urysohn space then there exists $\mu_y$ open sets $V$ and $W$ containing $a$ and $b$ respectively such that $c_{\mu_y}(V) \cap c_{\mu_y}(W) = \emptyset$.

Since $f$ is $\pi$, $\mu_y$ –continuous at $x$ and then there exists $\mu_x$, $\pi$ open sets $A$ and $B$ containing $x$ and $y$ respectively such that $f(A) \subseteq c_{\mu_y}(V)$ and $f(B) \subseteq c_{\mu_y}(W)$. Then $f(A) \cap f(B) = \emptyset$. So $A \cap B = \emptyset$. Hence $X$ is $\mu_x$, $\pi$ $T_2$.

For a map $f: (X, \mu_x) \rightarrow (Y, \mu_y)$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of $f$ and is denoted by $G(f)$.

**Theorem 3.23** Let $(X, \mu_x)$ and $(Y, \mu_y)$ be GTS’s. Let $f: X \rightarrow Y$ be a map and $g: X \times X$ the graph function of $f$ defined by $g(x) = (x, f(x))$ for every $x \in X$.

If $g$ is contra $(\mu_y, \mu_y)$- continuous then $f$ is contra $(\mu_y, \mu_y)$-continuous.

**Proof:** Let $U$ be an $\mu_y$-open set in $Y$. Then $X \times U$ is an $\mu_x$-open set in $X \times Y$. Since $g$ is contra $(\mu_y, \mu_y)$-continuous then $f^{-1}(U) = g^{-1}(X \times U)$ is $\mu_x$-closed in $X$. Hence $f$ is contra $(\mu_y, \mu_y)$-continuous.

**Definition 3.24** The graph $G(f)$ of a map $f: (X, \mu_x) \rightarrow (Y, \mu_y)$ is said to be contra $\mu_x$, $\mu_y$-closed if for each $(x, y) \in (X \times Y)$, $G(f)$, there exist an $\mu_x$, $\mu_y$ open set $U$ in $X$ containing $x$ and a $\mu_y$ closed set $V$ in $Y$ containing $y$ such that $(U \times V) \cap G(f) = \emptyset$.

**Lemma 3.25** Let $G(f)$ be the graph of a map $f: (X, \mu_x) \rightarrow (Y, \mu_y)$ between GTS’s. For any subset $A \subseteq X$ and $B \subseteq Y$, $f(A) \cap B = \emptyset$ if and only if $(A \times B) \cap G(f) = \emptyset$.

**Proposition 3.26** The following properties are equivalent for the graph $G(f)$ of a map $f$ in GTS’s.

(i) $G(f)$ is contra $(\mu_y, \mu_y)$ –closed.

(ii) For each $(x, y) \in (X \times Y)$, $G(f)$, there exist an $\mu_x$, $\mu_y$ open set $U$ in $X$ containing $x$ and a $\mu_y$-closed $V$ in $Y$ containing $y$ such that $f(U) \cap V = \emptyset$.

**Theorem 3.27** Let $(X, \mu_x)$ and $(Y, \mu_y)$ be two GTS’s. If $f: X \rightarrow Y$ is contra $(\mu_y, \mu_y)$ –continuous and $Y$ is $\mu_y$-Urysohn space, then $G(f)$ is contra $(\mu_y, \mu_y)$ –closed in $X \times Y$.

**Proof:** Let $(x, y) \in (X \times Y) G(f)$. It follows that $f(x) \neq y$ and $Y$ is $\mu_y$-Urysohn space then if for each distinct points $x$ and $y$ in $X$ there exists $\mu_x$ open sets $B$ and $C$ such that $f(x) \notin B$ and $y \notin C$ and $c_{\mu_x}(B) \cap c_{\mu_x}(C) = \emptyset$. Since $f$ is contra $(\mu_y, \mu_y)$ –continuous then there exists an $\mu_x$, $\mu_y$ closed set $A$ in $X$ containing $x$ such that $f(A) \cap c_{\mu_x}(B)$. Therefore $f(A) \cap c_{\mu_x}(B) = \emptyset$ and $G(f)$ is contra $(\mu_y, \mu_y)$ –closed in $X \times Y$.

**Theorem 3.28** Let $(X, \mu_x)$ and $(Y, \mu_y)$ be two GTS’s. Let $f: X \rightarrow Y$ have a contra $(\mu_y, \mu_y)$ closed graph. If $f$ is injective then $X$ is $\mu_x$, $\pi$ $T_1$.

**Proof:** Let $x_1$ and $x_2$ be any two distinct points of $X$. We have $(x_1, f(x_2)) \in (X \times Y) G(f)$ and there exist an $\mu_x$, $\pi$ open set $U$ in $X$ containing $x_1$ and a $\mu_y$-closed set $V$ in $Y$ containing $x_2$ such that $f(U) \cap F = \emptyset$.

Hence $U \cap f^{-1}(F) = \emptyset$. Therefore we have $x_2 \notin U$. This implies that $X$ is $\mu_x$, $\pi$ $T_1$.

**Theorem 3.29** Let $(X, \mu_x)$, $(Y, \mu_y)$, and $(Z, \mu_z)$ be GTS’s. Let $f: X \rightarrow Y$ be surjective, $(\mu_x, \mu_y)$ –$\pi$ irresolute and $\mu_x$, $\mu_z$ $\pi$ connected and $g: Y \rightarrow Z$ be any function. Then $g \circ f$ is contra $(\mu_x, \mu_z)$ –$\pi$ continuous if and only if $g$ is contra $(\mu_x, \mu_z)$ –continuous.

**Proof:** Suppose $g \circ f$ is contra $(\mu_x, \mu_z)$ –continuous. Let $F$ be any $\mu_x$-open set in $Z$. Then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\mu_x$, $\pi$-closed in $X$. Since $f$ is $\mu_x$, $\pi$-closed and surjective, $f^{-1}(g^{-1}(F)) = g^{-1}(F)$ is $\mu_x$, $\pi$-closed in $X$.
in Y and we obtain that \( g \) is contra \((\pi_\mu, \mu_y)\) – continuous.

Conversely, suppose \( g \) is contra \((\pi_\mu, \mu_y)\) – continuous. Let \( V \) be \( \mu \)-open in \( Z \). Then \( g^{-1}(V) \) is \( \mu\)-\( \pi \)-closed in \( Y \). Since \( f \) is \((\mu_x, \mu_y)\) – irresolute, \( f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) \) is \( \mu\)-\( \pi \)-closed in \( X \) and so \( g \circ f \) is contra \((\pi_\mu, \mu_y)\) – continuous.

References
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