

A New Topology τ^{*g} via g-local Functions in Ideal Topological Spaces

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Abstract: In this paper we introduce and study three different notions via ideals namely g-local function, the set operator Ψ_g and g-compatibility of τ with I . We characterize these new sorts. Several properties of them have been studied and their relationships with other types of similar operators are also investigated.

Keywords: *g-additive space, *g- finitely additive space, g-local function, $(\)^{*g}$ operator, Ψ_g operator, g-compatibility of τ with I

1. Introduction

Ideals topological spaces have been first introduced by K. Kuratowski[2] in 1930. Vaidyanathaswamy [4] introduced local function in 1945 and defined a topology τ . In this paper we introduce and study three different notions via ideals namely g-local function, the set operator Ψ_g , and g-compatibility of τ with I and investigate their relationships with other types of similar operators.

2. Preliminaries

Let (X, τ) be a topological space and $A \subseteq X$. We denote closure of A and interior of A by $cl(A)$ and $int(A)$ respectively.

Definition 2.1: A set A in a topological space (X, τ) is said to be a *generalized closed set* (briefly *g-closed*)[3] if $cl(A) \subseteq \square U$ whenever $A \subseteq \square U$ and U is open in (X, τ) .

The class of g - open set in X will be denoted by $GO(X, \tau)$.

Definition.2.3[2] An ideal I on a non empty set X is a collection of subsets of X which satisfies the following properties:(i) $A \in I, B \in I \Rightarrow A \cup B \in I$

(ii) $A \in I, B \subset A \Rightarrow B \in I$. A topological space (X, τ) with an ideal I on X is called an ideal topological space and is denoted by (X, τ, I) . Let Y be a subset of X . $I_Y = \{I \cap Y / I \in I\}$ is an ideal on Y and by $(Y, \tau/Y, I_Y)$ we denote the ideal topological subspace. Let $P(X)$ be the power set of X , then a set operator $(\)^*$: $P(X) \rightarrow P(X)$ called the local function [7] of A with respect to τ and I is defined as follows: For $A \subset X$, $A^*(I, \tau) = \{x \in X / U \cap A \notin I \text{ for every open set } U$

containing $x\}$ We simply write A^* instead of $A^*(I, \tau)$ in case there is no confusion. A Kuratowski closure operator $cl^*(\)$ for a topology $\tau^*(I, \tau)$, called the τ^* - topology is defined by $cl^*(A) = A \cup A^*$. A set operator $\psi(I, \tau): P(X) \rightarrow P(X)$ is defined as follows: For any $A \subseteq X$, $\psi(I, \tau)(A) = \{x \in X \text{ such that there exists open set } U \text{ such that } U - A \in I\}$. I is said to be compatible with τ , denoted by $I \sim \tau$ if the following holds: for $A \subseteq X$, if for every $x \in A$ there exists open set U such that $U \cap A \in I$ then $A \in I$.

3. g-Local function

In this section we introduce new class of the set operator $(\)^{*g}$ using g-neighbourhood and discuss various properties.

Definition 3.1: Given an ideal space (X, τ, I) , a set operator $(\)^{*g}: P(X) \rightarrow P(X)$, called the g-local function of I with respect to τ is defined as follows.

For $A \subseteq X$, $A^{*g}(I, \tau) = \{x \in X / U_x \cap A \notin I \text{ for every } U_x \in GN(x)\}$ when there is no ambiguity,

we will simply write $A^{*g}(I)$ or $(A)^{*g}$ instead of $A^{*g}(I, \tau)$.

$cl^{*g}(A)$ is defined as $A \cup (A)^{*g}(I)$

Theorem 3.2: Let (X, τ, I) be an ideal topological space and $A, B \subseteq X$.

Then the following statements hold.

- $\phi^{*g} = \phi, A \subseteq B \Rightarrow A^{*g} \subseteq B^{*g}$ and $E^{*g} = \phi$ if $E \in I$
- For another ideal $J, I \subseteq J \Rightarrow A^{*g}(J) \subseteq A^{*g}(I)$
- $A^{*g} \subseteq gcl(A) \subseteq cl(A)$
- $(A^{*g})^{*g} \subseteq (A)^{*g}$

- 5. $(A \cap B)^{*g} \subseteq (A)^{*g} \cap (B)^{*g}$
- 6. $(A \cup B)^{*g} \supseteq A^{*g} \cup B^{*g}$
- 7. $(A)^{*g} = gcl(A)^{*g} \subseteq cl(A)$ if (X, τ) is g -multiplicative.
- 8. If $E \in I$ then $(A \cup E)^{*g} = A^{*g} = (A \setminus E)^{*g}$
- 9. If $U \in \tau$ then $U \cap (A)^{*g} = U \cap (U \cap A)^{*g} \subseteq (U \cap A)^{*g}$

Proof:

(1) and (2) are obvious by definition of g -local function.
 (3) $x \in A^{*g}$ implies $A \cap U \notin I$ for every $U \in GN(x)$ implies $A \cap U \neq \emptyset$ for every $U \in GN(x)$
 implies $x \in gcl(A)$.

$$(4) \quad (A^{*g})^{*g} = \{x \in X / U_x \cap (A)^{*g} \notin I \text{ for every } U_x \in GN(x)\}$$

$$\subseteq \{x \in X / U_x \cap A \notin I \text{ for every } U_x \in GN(x)\}$$

$$= (A)^{*g}.$$

(5) and (6) follow from (1)
 (7) If $x \in X \setminus (A)^{*g}$ then there exists $U \in GO(X)$ such that $A \cap U \in I$ and this implies $U \subseteq X \setminus (A)^{*g}$. Therefore $X - (A)^{*g}$ is the union of g -open sets and hence it is g -open. So $(A)^{*g}$ is g -closed. Therefore $(A)^{*g} = gcl(A)^{*g} \subseteq gcl(gcl(A)) = gcl(A)$ (since (X, τ) is g -multiplicative, $gcl(A)$ is g -closed). But $gcl(A) \subseteq cl(A)$. Hence A^{*g} is a g -closed sub set of $cl(A)$.

(8) $A - E \subseteq A$ implies $(A - E)^{*g} \subseteq A^{*g}$ -----(A)
 Let $x \in A^{*g}$. Suppose $x \notin (A \setminus E)^{*g}$, then there exists $U_x \in GN(x)$ such that $U_x \cap (A \setminus E) \in I$.

Then $E \cup [U_x \cap (A \setminus E)] \in I$. This implies that $E \cup [U_x \cap A] \in I$. So, $U_x \cap A \in I$ which is a contradiction to the fact that $x \in A^{*g}$. So, $A^{*g} \subseteq (A \setminus E)^{*g}$ -----(B)

From (A) and (B) we get $(A \setminus E)^{*g} = A^{*g}$ when $E \in I$.

(9) Let $U \in \tau$, $x \in U \cap (A)^{*g}$ and U_x be a g -open set containing x . Then $U \cap U_x \in GO(X)$ and hence $(U_x \cap U) \cap A \notin I$ which proves $x \in (U \cap A)^{*g}$. Therefore $U \cap (A)^{*g} \subseteq (U \cap A)^{*g}$.

So

$$U \cap (A)^{*g} = U \cap (U \cap (A)^{*g}) \subseteq U \cap (U \cap A)^{*g} \text{ -----(A)}$$

On the otherhand, $U \cap A \subseteq A$ implies $(U \cap A)^{*g} \subseteq A^{*g}$

$$\text{Therefore } U \cap (U \cap A)^{*g} \subseteq U \cap (A)^{*g} \text{ -----(B)}$$

From (A) and (B) it follows that $U \cap (A)^{*g} = U \cap (U \cap A)^{*g}$

Remark 3.3: In general $(A \cup B)^{*g} \neq A^{*g} \cup B^{*g}$ and $(A \cap B)^{*g} \neq A^{*g} \cap B^{*g}$ as seen from examples (3.4) and (3.5).

Example 3.4: Consider Z with cofinite topology and $I = \{\emptyset\}$. $A = \{0, 1, 2, 3, \dots\}$ and $B = \{0, -1, -2, -3, \dots\}$. Then $A^{*g} = X = B^{*g}$ and $A^{*g} \cap B^{*g} = X$. But $A \cap B = \{0\}$ and $(A \cap B)^{*g} = \{0\}$. Therefore $(A \cap B)^{*g} \neq A^{*g} \cap B^{*g}$.

Example 3.5: Let $X = \{a, b, c\}$ and $\tau = \{\{a\}, \{a, c\}\}$. Let $A = \{a\}$ and $B = \{b\}$. The $(A \cup B)^{*g} \neq A^{*g} \cup B^{*g}$, since $A^{*g} = \emptyset$, $B^{*g} = \{b\}$ and $(A \cup B)^{*g} = \emptyset$.

Remark 3.6: In the ideal space (X, τ, I) because $cl^{*g}(A \cup B) \neq cl^{*g}(A) \cup cl^{*g}(B)$ in general, we are not able to define a topology using the operator $cl^{*g}(\)$. To define a topology we need the following definitions.

Definition 3.7: An ideal space (X, τ, I) said to be

- 1. $*g$ - finitely additive if $\left[\bigcup_{i=1}^n A_i \right]^{*g} = \bigcup_{i=1}^n (A_i)^{*g}$ for every finite positive integer n .
- 2. $*g$ - additive if $\left[\bigcup_{\alpha \in \Omega} A_\alpha \right]^{*g} = \bigcup_{\alpha \in \Omega} (A_\alpha)^{*g}$ for every indexing set Ω .
- 3. $*g$ - finitely multiplicative if $\left[\bigcap_{i=1}^n A_i \right]^{*g} = \bigcap_{i=1}^n [A_i]^{*g}$ for every finite positive integer n .
- 4. $*g$ - multiplicative if $\left[\bigcap_{\alpha \in \Omega} A_\alpha \right]^{*g} = \bigcap_{\alpha \in \Omega} [A_\alpha]^{*g}$ for every indexing set Ω .

Remark 3.8:

1. Every $*g$ – additive (resp $*g$ – multiplicative) space is $*g$ – finitely additive (resp $*g$ – finitely multiplicative).
2. $A \subseteq cl^{*g}(A)$
3. If is $*g$ – finitely additive then
 - (a) $cl^{*g}(A \cup B) = cl^{*g}(A) \cup cl^{*g}(B)$ and
 - (b) $cl^{*g}(cl^{*g}(A)) = cl^{*g}(A)$.

Therefore in a $*g$ – finitely additive space, $cl^{*g}(\)$ satisfies Kuratowski closure axioms.

Definition 3.9: Let (X, τ, I) be a $*g$ – finitely additive space. If a $*g$ – closed set A is defined to be one for which $cl^{*g}(A) = A$, then the class of all complements of such sets is a topology on X denoted by τ^{*g} , whose closure operation is given as $cl^{*g}(A) = A \cup (A)^{*g}$.

Example 3.10: Let (X, τ) be an infinite cofinite topological space and $I = \emptyset$. In this space $GC(X) = \{\emptyset, X, A / A \text{ is finite}\}$ and $GO(X) = \{\emptyset, X, A / A^c \text{ is finite}\}$ and $A^{*g} = gcl(A) = A$ if A is finite and $A^{*g} = gcl(A) = X$ if A is infinite. Then (X, τ) is $*g$ – multiplicative but not $*g$ – countably additive and g – additive.

Example 3.11: Let (X, τ) be an indiscrete topological space and $I = \{A \subseteq X / p \notin A\}$ and $GO(X) = \{\text{all subsets}\}$. $A^{*g} = \{p\}$ if $p \in A$ and \emptyset if $p \notin A$. Since $cl^{*g}(A) = A \cup A^{*g} = A$ for all $A \subseteq X$ $\tau^{*g} = \{\text{all subsets}\}$. Then (X, τ) is $*g$ – multiplicative, $*g$ – finitely multiplicative, $*g$ – countably multiplicative, $*g$ – additive, $*g$ – countably additive, and $*g$ – finitely additive.

Example 3.12: Let (X, τ) be an indiscrete topological space, $x_0 \in X, I = \{\emptyset, \{x_0\}\}$ and $GO(X) = \{\text{all subsets}\}$. $A^{*g} = A$ if $x_0 \notin A$ and $A - \{x_0\}$ if $x_0 \in A$.

Since $cl^{*g}(A) = A \cup A^{*g} = A$ and hence $\tau^{*g} = \{\text{all subsets}\}$. Then (X, τ) is $*g$ – multiplicative, $*g$ – finitely multiplicative, $*g$ – countably multiplicative, $*g$ – additive, $*g$ – countably additive, and $*g$ – finitely additive.

Remark 3.13: In a $*g$ – finitely additive space, (X, τ, I) ,

- (1) $\tau^{*g} = \{A \subseteq X / cl^{*g}(X - A) = X - A\}$
- (2) $cl^{*g} \subseteq cl^*(A) \subseteq cl(A)$ and hence $\tau \subseteq \tau^* \subseteq \tau^{*g}$

Thus a new topology τ^{*g} is defined in a $*g$ – finitely additive ideal space (X, τ, I) with the help of g -local function and this topology is finer than τ^* - topology.

Theorem 3.14: Let (X, τ, I) be an ideal space. For $A \subseteq X$, we have the following results.

1. If $I = \{\emptyset\}$ then $A^{*g} = gcl(A)$ and $cl^{*g}(A) = gcl(A)$.
2. If $I = P(X)$ then $A^{*g} = \emptyset$ and $cl^{*g}(A) = A$.

Proof: Obvious from the definition of $(A)^{*g}$

Remark 3.16: In a $*g$ – finitely additive space (X, τ, I) with $I = P(X)$, τ^{*g} is the discrete topology since every subset is $*g$ – open and $*g$ – closed.

Theorem 3.17: If I and J are two ideals in a $*g$ – finitely additive space such that $I \subseteq J$.

Then $\tau^{*g}(I) \subseteq \tau^{*g}(J)$.

Proof: Let A be closed in $\tau^{*g}(I)$ topology

$$I \subseteq J \Rightarrow A^{*g}(J) \subseteq A^{*g}(I).$$

Therefore $cl_J^{*g}(A) \subseteq cl_I^{*g}(A)$

Then $A \subseteq cl_J^{*g}(A) \subseteq cl_I^{*g}(A) = A$ which proves $A \subseteq cl_J^{*g}(A)$ and so A is closed in $\tau^{*g}(J)$.

Definition 3.18: A subset A in an ideal space (X, τ, I) is said to be

1. $*g$ – dense subset in X if $cl^{*g}(A) = X$
2. $*g$ – perfect if $A^{*g} = A$.
3. $*g$ – closed in X if $cl^{*g}(A) = A$

Theorem 3.19: In a $*g$ – finitely additive ideal space (X, τ, I) the following are equivalent

1. $W \in \tau^{*g}$
2. $X - W$ is τ^{*g} - closed
3. $(X - W)^{*g} \subseteq (X - W)$
4. $W \subseteq X - (X \setminus W)^{*g}$

Proof: Obvious.

4. The set operator $\psi_g(I, \tau)$

Definition 4.1: Let (X, τ, I) be an ideal space. A set operator $\psi_g(I, \tau) : P(X) \rightarrow P(X)$ is defined as follows.

For any $A \subseteq X, \psi_g(I, \tau)(A) = \{x \in X / \text{there exists } U \in GN(x) \text{ such that } U - A \in I\}$.

Remark 4.2:

1. Obviously $x \in \psi_g(I, \tau)(A)$ if and only if $x \notin (A^c)^{*g}$. Therefore $\psi_g(I, \tau)(A) = X \setminus (X \setminus A)^{*g}$
2. We denote $\psi_g(I, \tau)$ simply by ψ_g when no ambiguity is present.
3. $\psi(A) \subseteq \psi_g(B)$
4. $\psi(A) \subseteq \psi_g(B)$

Theorem 4.3: For a subset A in an ideal space (X, τ, I) the following results are true.

1. If $I = \{\emptyset\}$ then $\psi_g(A) = g \text{ int}(A)$.
2. If $I = P(X)$ then $\psi_g(A) = X$.

Proof:

1. $\psi_g(A) = X \setminus (X \setminus A)^{*g} = X \setminus gcl(X \setminus A) = g \text{ int}(A)$
2. $\psi_g(A) = X \setminus (X \setminus A)^{*g} = X - \emptyset = X$

The following theorem gives many basic and useful facts for the operator ψ_g .

Theorem 4.4: Let A and B subsets in an ideal space (X, τ, I) .

1. If $A \subseteq B$ then $\psi_g(A) \subseteq \psi_g(B)$.
2. $\psi_g(A \cap B) \subseteq \psi_g(A) \cap \psi_g(B)$.

Proof:

1. $A \subseteq B \Rightarrow X \setminus B \subseteq X \setminus A \Rightarrow (X \setminus B)^{*g} \supseteq (X \setminus A)^{*g} \Rightarrow X \setminus (X \setminus B)^{*g} \subseteq X \setminus (X \setminus A)^{*g} \Rightarrow \psi_g(A) \subseteq \psi_g(B)$

2. Follows from 1.

Theorem 4.5: Let (X, τ, I) be a $*g$ -finitely additive space. Then

1. If $U \in \tau^{*g}$ then $U \subseteq \psi_g(U)$.
2. For every $A \subseteq X$, then $\psi_g(A) \in \tau$.
3. For every $A \subseteq X$, then $\psi_g(A) \subseteq \psi_g(\psi_g(A))$.
4. For every $A \subseteq X$ and $E \in I$ then $\psi_g(A \setminus E) = \psi_g(A) = \psi_g(A \cup E)$.
5. If $A \in GO(X)$ then $A \subseteq \psi_g(A)$.
6. If $A \in \tau$ then $A \subseteq \psi_g(A)$.
7. If $(A \setminus B) \cup (B \setminus A) \in I$ then $\psi_g(A) = \psi_g(B)$.

Proof:

1. $U \in \tau^{*g} \Rightarrow (X \setminus U)^{*g} \subseteq X \setminus U$. Then $\psi_g(U) = X \setminus (X \setminus U)^{*g} \supseteq U$.

2. By theorem (3.2), $(X - A)^{*g}$ is g- closed. Therefore $cl^{*g}[(X \setminus A)^{*g}] \subseteq gcl(X \setminus A)^{*g} = (X \setminus A)^{*g}$

Therefore $(X \setminus A)^{*g}$ is $*g$ - closed and hence $\psi_g(A) = X \setminus (X \setminus A)^{*g} \in \tau^{*g}$.

3. By (2) $\psi_g(A) \in \tau^{*g}$ and hence $\psi_g(A) \subseteq \psi_g(\psi_g(A))$ by (1).

4.

$\psi_g(A \setminus E) = X \setminus [(X \setminus (A \setminus E))]^{*g} = X \setminus [(X \setminus A) \cup E]^{*g}$
 $= X \setminus [X \setminus A]^{*g} = \psi_g(A)$.(by thm (4.3))

$\psi_g(A \cup E) = X \setminus [X \setminus (A \cup E)]^{*g}$
 $= X \setminus [(X \setminus A) \setminus E]^{*g} = X \setminus [X \setminus A]^{*g} = \psi_g(A)$.

5. If $A \in GO(X)$ then $X \setminus A$ is g-closed. Therefore $(X \setminus A)^{*g} \subseteq gcl(X \setminus A) = (X \setminus A)$ and this

implies A is $*g$ - open. So by (1) $A \subseteq \psi_g(A)$.

6. Follows from (5) since $\tau \subseteq GO(X)$.

7. Let $A \setminus B = E$ and $B \setminus A = H$. Then $E \cup H \in I$ implies E and H are in I.

By (4) $B = (A \setminus E) \cup H$ implies

$\psi_g(A) = \psi_g(A \setminus E) = \psi_g[(A \setminus E) \cup H]$ (since $H \in I$)
 $= \psi_g(B)$.

Definition 4.6: In an ideal space (X, τ, I) , we say two subsets A and B are congruent modulo I

(in notation $A \equiv B \text{ mod } I$) if $(A \setminus B) \cup (B \setminus A) \in I$.

Obviously " $\equiv \text{ mod } I$ " is an equivalence relation.

Theorem 4.7 Let A and B are two subsets in $*g$ -finitely additive ideal $*g$ -space (X, τ, I) . If $A \equiv B \text{ mod } I$ then $\psi_g(A) = \psi_g(B)$.

Proof:

It follows from definition of $A \equiv B \text{ (mod } I)$ and by (10) of theorem (4.5).

5. g-compatibility of τ with I

Definition 5.1 Given a space (X, τ, I) , I is said to be

g-compatible with τ , denoted by $I \sim^g \tau$ if the following holds: for $A \subseteq X$, if for every $x \in A$ there exists $U \in GN(x)$ such that $U \cap A \in I$ then $A \in I$.

Remark 5.2

Since $\tau \subseteq GO(x)$, $I \sim^g \tau \Rightarrow I \sim \tau$

The following example shows the existence of this compatibility.

Example 5.3 Let (X, τ) be an indiscrete space,

$p \in X$ and $I = \{A \subseteq X / p \notin A\}$. In this space $I \sim^g \tau$.

Theorem 5.4 If (X, τ, I) is a $*g$ -finitely additive ideal space then the following are equivalent.

- (1) $I \sim^g \tau$

- (2) If A has a cover of g -open set each of whose intersections with A is in I then A is in I .
- (3) For every $A \subseteq X, A \cap A^{*g} = \emptyset \Rightarrow A \in I$
- (4) For every $A \subseteq X, A \setminus A^{*g} \in I$
- (5) For every τ^{*g} -closed subset $A, A - A^{*g} \in I$
- (6) For every $A \subseteq X$, if A contains no non-empty subset $B \subset B^{*g}$ then $A \in I$.

Proof: (1) \Rightarrow (2) Let $I \sim \tau$ and $A = \cup A_\alpha$ where each A_α is g -open and $A \cap A_\alpha \in I$. Then by definition $A \in I$.

(2) \Rightarrow (3) for $A \subseteq X$. Let $A \cap A^{*g} = \emptyset$. So if $x \in A$ then $x \notin A^{*g}$. Therefore there exists $U_x \in GN(x)$ such that $U_x \cap A \in I$. Then $\{U_x / x \in A\}$ is an open cover for A and $U_x \cap A \in I$ hence $A \in I$.

(3) \Rightarrow (4) Let $x \in A - A^{*g}$. Suppose $x \in (A - A^{*g})^{*g}$ then for every $U \in GN(x), U \cap (A - A^{*g}) \notin I$. This implies $U \cap A \notin I$ which implies $x \in A^{*g}$ which is a contradiction.

$\therefore (A - A^{*g}) \cap (A - A^{*g})^{*g} = \emptyset$ and hence $A - A^{*g} \in I$ by (3).

(4) \Rightarrow (5) proof is obvious.

(5) \Rightarrow (1) Let $A \subseteq X$ and for every $x \in A$ there exists $U \in GN(x)$ such that $U \cap A \in I$. Then $A \cap A^{*g} = \emptyset$.

Since (X, τ, I) is $*g$ -finitely additive,

$$(A \cup A^{*g})^{*g} = A^{*g} \cup (A^{*g})^{*g} \subseteq A^{*g} \cup A^{*g} = A^{*g} \subseteq A \cup A^{*g}$$

$\therefore A \cup A^{*g}$ is $*g$ -closed. By (5),

$$(A \cup A^{*g}) - (A \cup A^{*g})^{*g} \in I.$$

But

$$(A \cup A^{*g}) - (A \cup A^{*g})^{*g} = (A \cup A^{*g}) - A^{*g} = A.$$

$\therefore A \in I$

(4) \Rightarrow (6). Let $A \subseteq X$. By (4) $A - A^{*g} \in I$. Let $x \in A \cap A^{*g}$.

Suppose $x \notin (A \cap A^{*g})^{*g}$ then there exists

$U \in GN(x)$ such that $U \cap (A \cap A^{*g}) \in I$. This implies that $U \cap A \in I$ which is a contradiction.

Therefore $A \cap A^{*g} \subseteq (A \cap A^{*g})^{*g}$.

By (6) $A \cap A^{*g} = \emptyset$ and this implies

$$A = A - A^{*g} \in I. \text{ (Since (4) } \Rightarrow \text{ (5).)}$$

(6) \Rightarrow (4). Let $A \subseteq X$, since

$$(A - A^{*g}) \cap (A - A^{*g})^{*g} = \emptyset, \text{ we have}$$

$$(A - A^{*g}) \in I \text{ by (6).}$$

Theorem 5.5 Let (X, τ, I) be an ideal space. Then

$I \sim \tau$ if and only if $\psi_g(A) - A \in I$ for all $A \subseteq X$.

Proof: Necessity: Assume that $I \sim \tau$. Let $A \subseteq X$,

$x \in \psi_g(A) - A$. Then $x \notin A$ and there exists

$U_x \in GN(x)$ such that $U_x - A \in I$. Therefore for

each $x \in \psi_g(A) - A$ there exists $U_x \in GN(x)$ such

that $U_x \cap (\psi_g(A) - A) \in I$. This implies

$$\psi_g(A) - A \in I.$$

Sufficiency: Let $A \subseteq X$ and for each $x \in A$ there

exists $U_x \in GN(x) \ni U_x \cap A \in I$. By definition of

$$\psi_b(A), \psi_g(X - A) = \{x \in X / \exists U_x \in GN(x) \ni U_x \cap A \in I\}$$

$$\therefore A \subseteq \psi_g(X - A) - (X - A) \in I$$

Theorem 5.6 Let (X, τ, I) be $*b$ -finitely additive ideal

space with $I \sim \tau$. Then

$$\psi_g(\psi_g(A)) = \psi_g(A) \forall A \subseteq X.$$

Proof: From theorem (4.1) $\psi_g(A) \subseteq \psi_g(\psi_g(A))$. By

theorem (5.5) $\psi_g(A) - A = E$ for some $E \in I$.

Therefore $\psi_g(A) = A \cup E$.

So, $\psi_g(\psi_g(A)) = \psi_g(A \cup E) = \psi_g(A)$, by theorem

Theorem 5.7 let (X, τ, I) be $*g$ finitely additive ideal

space with $I \sim \tau$. If $U, V \in GO(x)$, and

$$\psi_g(U) = \psi_g(V) \text{ then } U \equiv V \text{ mod } I.$$

Proof: By theorem (4.5) $U \subseteq \psi_g(U)$

$$\therefore U \setminus V \subseteq \psi_g(U) - V = \psi_g(V) - V \in I$$

Therefore $U \equiv V \text{ mod } I$

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