An EOQ Model for Deteriorating Items with Quadratic Time Dependent Demand rate Under Permissible Delay in Payment

Santosh P. Gite

Department of Statistics, University of Mumbai, Mumbai, Maharashtra, INDIA.

Corresponding Adrress:

santgite@yahoo.com

Research Article

Abstract: In this paper, I developed here an economic order quantity model (EOQ) for deteriorating items with quadratic time dependent demand rate and instantaneous replenishment. Shortages are not allowed. Deterioration rate is assumed as variable. The results are illustrated with the help of a numerical example.

Keywords: EOQ model, Quadratic demand, Deterioration and Shortages etc.

1. Introduction

Demand is the crucial factor in the inventory management. In the classical inventory models the demand rate is assumed as constatnt. In reality , demand for physical goods may be stock dependent, time dependent and price dependent etc. Silver and Meal (2) have developed model under time varying demand.Donaldson (1) discussed , for the first time , the inventory policies under linear, time dependent demand. Ritchie(3), Sachan(13) and Dave and Patel (5) have studied models with time dependent demand without backlogging and shortages. Deb and Chaudhuri(6) developed inventory lot-sizing problem with linearly increasing time-varying demand under shortages. Subsequent contribution in this direction came from researchers like Goyal(15) and Hariga(12). Teng and chang(7) studied Economic order quantity (EOQ) models for deteriorating items under price and stock dependent demand. Ghare and Schrader(4) have established inventory model with an exponentially decaying. Further, Covert and Philip(8) have relaxed assumption of constatnt deteriorating rate. They have used two parameter weibull distribution to represents the distribution of time to deterioration. Recently, the condition of permissible delay in payment has drawn the attention of researchers. While developing in an inventory model it is generally assumed that the payment is made to the supplier (wholesaler)for the items immediately after reciving consignment. It is observed that quite unrealistic in a present real life situation. While dealing with day to day life , it is observed that supplier is allowed a fixed period period for settle account. There is no intrest charged on the outstanding amount if it is paid within the permissible delay period but beyond this period , intrest is charged by the wholesaler. During this fixed period of permissible delay in payments, retailer can sell the item, invest the revenues in an intrest earning account and earn intrest. Goyal (9) first studied an EOQ model under the condition of permissible delay in payments. Agarawal and Jaggi (10) , Hwang and Shinn (12) extended Goyals model to consider deterministic inventory model with constant rate of deterioration. P. Chu, K.J. Chung and S.P. Lan (14) have developed Economic order quantity of deteriorating items under permissible delay in payment. In this paper, I have made attempt to develop an EOQ model for deteriorating item under permissible delay in payment. All the above models are developed with constant , Time dependent and stock dependent inventory models. Here it is assumed that quadratic time dependent rate with variable rate of deterioration.

2. Assumptions and Notations

To develop the proposed mathematical model, the following notations and assumptions are used in this paper.

1. The demand rate $R(t) = a+bt+ct^2$ at a time (t>0) is a continuous function of time where a, b & c are all constant.
2) A variable function $\theta = \alpha t$ of the on hand inventory deteriorates per unit of time where, $0 < \alpha < 1$.
3) A is the ordering cost per order.
4) P is the purchasing cost per item.
5) h is per unit holding cost excluding interest charges per unit per year.
6) I_e is the interest earned per year.
7) I_r is the interest charged per stocks per year.
8) M is the permissible delay in settling the accounts, $0 < M < 1$.
9) T is the time interval between two successive orders.
10) Replenishment rate is infinite and instantaneous.
11) Lead time is zero.

12) Shortages are not allowed.
13) There is no repair or replacement of deteriorated unit in the given cycle.
14) When M ≤ T, the account is settled at time M = T and retailers start paying for the interest charges on the items in stock with rate I_r. When M ≥ T , the account is settled at M =T and retailer does not need to pay interest charge.
15) $K_1(T)$ and $K_2(T)$ are the total variable costs for case(1) and case(2) respectively.
16) The retailer can accumulate revenue and earn interest after the customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier.That is the retailer can accumulate revenue and earn interest during the period T to M with rate I_r, under the condition of trade credit.

3. Mathematical formulation

Let I (t) be the inventory level at time t (0 ≤ t ≤ T). The inventory is depleted partly to meet the demand and partly for deterioration during the period of inventory. The differential equations for the instantaneous state over (0 ,T) are given by

$$\frac{dI(t)}{dt}+\alpha t I(t)=-R(t) \quad , \qquad 0\le t\le T$$

$$\frac{dI(t)}{dt}+\alpha t I(t)=-(a+bt+ct^2) \quad , \qquad 0\le t\le T \qquad (1)$$

The solution of the equation (1) is, Since α is very small using taylor series expansions (ignoring α^2 and higher powers). After applying the boundary condition I(0) = Q, I(T) = 0, t =T. The solution of equation is

$$I(t)=\Big[\ a(T-t)+\frac{b}{2}(T^2-t^2)+\frac{C}{3}(T^3-t^3)+\frac{a\alpha t^3}{3}+\frac{b\alpha t^4}{8}+\frac{c\alpha t^5}{15}+\frac{a\alpha T^3}{6}+\frac{b\alpha T^4}{8}+\frac{c\alpha T^5}{10}$$
$$-\frac{a\alpha T t^2}{2}-\frac{b\alpha T^2t^2}{4}-\frac{c\alpha t^2T^3}{6}\Big] \qquad (2)$$

The initial order quantity at t = 0 is,

$$I(0)=Q=aT+\frac{bT^2}{2}+\frac{cT^3}{3}+\frac{a\alpha T^3}{6}+\frac{b\alpha T^4}{8}+\frac{c\alpha T^5}{10} \qquad (3)$$

The total demand during one cycle is

$$\text{Td}=\int_0^T R(t)dt = aT+\frac{bT^2}{2}+\frac{cT^3}{3} \qquad (4)$$

Number of deteriorated units is

$$\text{Du}=Q-Td$$
$$=\frac{a\alpha T^3}{6}+\frac{b\alpha T^4}{8}+\frac{c\alpha T^5}{10} \qquad (5)$$

The cost of stock holding for one cycle is,

$$\text{Hc}=h\int_0^T I(t)dt$$
$$=h\left[\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}+\frac{a\alpha T^4}{12}+\frac{b\alpha T^5}{15}+\frac{2c\alpha T^6}{45}\right], \qquad (6)$$

Case I:

Let M < T: Since, the interest is payable during the time (T – M) , the interest payable in any cycle, denoted by IP, is given by

$$\text{IP}=PI_r\int_M^T I(t)dt$$
$$=PI_r\left[aT(T-M)-\frac{a(T^2-M^2)}{2}+\frac{bT^2(T-M)}{2}-\frac{b(T^3-M^3)}{6}+\frac{cT^3(T-M)}{3}-\frac{c(T^4-M^4)}{12}\right. \qquad (7)$$
$$\left.+\frac{a\alpha(T^4-M^4)}{12}+\frac{b\alpha(T^5-M^5)}{40}+\frac{c\alpha T^5}{10}(T-M)-\frac{a\alpha T(T-M)}{6}-\frac{b\alpha T^2(T-M)}{12}-\frac{c\alpha T^3(T-M)}{18}\right]$$

The interest earned, denoted by IE_1, is , therefore

$IE_1 = PI_e \int_0^T tR(t)dt$

$$= PI_e T\left(\frac{a}{2} + \frac{bT}{3} + \frac{cT^2}{4}\right) \quad (8)$$

Therefore, the total average cost per unit per unit time in this case is given by

$K_1(T) = \frac{A + Hc + Du + IP - IE_1}{T}$

$$= \frac{1}{T}\Big\{A + h\left[\frac{aT^2}{2} + \frac{bT^3}{3} + \frac{cT^4}{4} + \frac{a\alpha T^4}{12} + \frac{b\alpha T^5}{15} + \frac{2c\alpha T^6}{45}\right] + p\left[\frac{a\alpha T^3}{6} + \frac{b\alpha T^4}{8} + \frac{c\alpha T^5}{10}\right]$$
$$+ PI_r\Big[aT(T-M) - \frac{a(T^2 - M^2)}{2} + \frac{bT^2(T-M)}{2} - \frac{b(T^3 - M^3)}{6} + \frac{cT^3(T-M)}{3} - \frac{c(T^4 - M^4)}{12}$$
$$+ \frac{a\alpha(T^4 - M^4)}{12} + \frac{b\alpha(T^5 - M^5)}{40} + \frac{c\alpha T^5}{10}(T-M) - \frac{a\alpha T(T-M)}{6} - \frac{b\alpha T^2(T-M)}{12} - \frac{c\alpha T^3(T-M)}{18}\Big] - PI_e T\left(\frac{a}{2} + \frac{bT}{3} + \frac{cT^2}{4}\right)\Big\} \quad (9)$$

The optimum values of T (say T_1), which minimize the total average cost per unit per unit time, can be obtained by solving following equation.

$\frac{dK_1(T)}{dT} = 0$ and $\frac{d^2K_1(T)}{dT^2} > 0$

After simplification, yields the following equation.

$$-A - h\left[aT + bT^2 + cT^3 + \frac{a\alpha T^3}{3} + \frac{b\alpha T^4}{3} + \frac{12c\alpha T^5}{45}\right] - p\left[\frac{a\alpha T^2}{2} + \frac{b\alpha T^3}{2} + \frac{c\alpha T^4}{2}\right] -$$
$$pI_r\Big[aT - aM + \frac{bT^2}{2} - bMT + cT^3 - cMT^2 + \frac{4a\alpha T^3}{12} + \frac{5b\alpha T^4}{40} + \frac{6c\alpha T^5}{12} - \frac{2a\alpha T}{6} + \frac{a\alpha M}{6} \quad (10)$$
$$- \frac{3b\alpha T^2}{12} - \frac{2b\alpha MT}{12} - \frac{4c\alpha T^3}{18} + \frac{3c\alpha MT^2}{18}\Big] + PI_e\left[\frac{a}{2} + \frac{2bT}{3} + \frac{3cT^2}{4}\right] = 0$$

Above equation (10) is nonlinear in T. The optimum values of T (say T_1) can be obtained by Newton-raphson method.

The EOQ in this case is as follows

$$q_0(T_1) = aT_1 + \frac{bT_1^2}{2} + \frac{cT_1^3}{3} + \frac{a\alpha T_1^3}{6} + \frac{b\alpha T_1^4}{8} + \frac{c\alpha T_1^4}{10} \quad (11)$$

Case 2:

M>T: In this case, the customer earns interest on the sales revenue up to the permissible delay period and no interest is payable during this period for the items kept in stock. Interest earned up to T is in this case, denoted by IE_T, is given by

$IE_T = pI_e \int_0^T tR(t)dt$

$$= pI_e\left[\frac{aT^2}{2} + \frac{bT^3}{3} + \frac{cT^4}{4}\right] \quad (12)$$

Interest earned during (M-T) i.e. up to the permissible delay period is

$IE_{pd} = pI_e(M - T)\int_0^T R(t)dt$

$$= pI_e(M - T)\left[aT + \frac{bT^2}{2} + \frac{cT^3}{3}\right] \quad (13)$$

Combine equation (11) and (12), the total interest earned during the cycle is, denoted by IE_2

$IE_2 = IE_T + IE_{pd}$

$$= pI_eT\left[aM + \frac{bMT}{2} + \frac{cMT^2}{3} - \frac{aT}{2} - \frac{bT^2}{6} - \frac{cT^3}{12}\right] \quad (14)$$

Then the total average cost per unit time is

$$K_2(T) = \frac{A + Hc + Du - IE_2}{T}$$

$$= \frac{1}{T}\left\{A + h\left[\frac{aT^2}{2} + \frac{bT^3}{3} + \frac{cT^4}{4} + \frac{a\alpha T^4}{12} + \frac{b\alpha T^5}{15} + \frac{2c\alpha T^6}{45}\right] + p\left[\frac{a\alpha T^3}{6} + \frac{b\alpha T^4}{8} + \frac{c\alpha T^5}{10}\right]\right.$$

$$\left. - PI_e\left[\frac{bMT}{2} + aM + \frac{cMT^2}{3} - \frac{aT}{2} - \frac{bT^2}{6} - \frac{cT^3}{12}\right]\right\} \quad (15)$$

The optimum values of T (say T_2), which minimize the total average cost per unit per unit time, can be obtained by solving following equation.

$$\frac{dK_1(T)}{dT} = 0 \quad \text{and} \quad \frac{d^2K_1(T)}{dT^2} > 0$$

After simplification, yields the following equation .

$$-A - h\left[aT + bT^2 + cT^3 + \frac{a\alpha T^3}{3} + \frac{b\alpha T^4}{3} + \frac{2c\alpha T^5}{15}\right] -$$

$$p\left[\frac{a\alpha T^2}{2} + \frac{b\alpha T^3}{2} + \frac{c\alpha T^4}{2}\right] + pI_e\left[aM + bMT + cMT^2 - aT - \frac{bT^2}{2} - \frac{cT^3}{3}\right] = 0 \quad (16)$$

Above equation (16) is nonlinear in T. The optimum values of T (say T_1) can be obtained by Newton-raphson method. The EOQ in this case is

$$q_0(T_2) = aT_2 + \frac{bT_2^2}{2} + \frac{cT_2^3}{3} + \frac{a\alpha T_2^3}{6} + \frac{b\alpha T_2^4}{8} + \frac{c\alpha T_2^4}{10} \quad (17)$$

Above equation (10) is nonlinear in T. The optimum values of T (say T_1) can be obtained by Newton-raphson method.

Case 3:

T=M: for T=M, the cost functions $K_1(T)$ and $K_2(T)$ become identical. It is obtained on substituting T=M either in Eq.(9) or Eq. (14) thus:

$$C(M) = \frac{1}{M}\left\{A + h\left[\frac{aM^2}{2} + \frac{bM^3}{3} + \frac{cM^4}{4} + \frac{a\alpha M^4}{12} + \frac{b\alpha M^5}{15} + \frac{2c\alpha M^6}{45}\right]\right.$$

$$\left. + p\left[\frac{a\alpha M^3}{6} + \frac{b\alpha M^4}{8} + \frac{c\alpha M^5}{10}\right] - PI_e\left[\frac{aM}{2} + \frac{bM^2}{3} + \frac{cM^3}{4}\right]\right\} \quad (18)$$

The EOQ in this case is

$$q_0(M) = aM + \frac{bM^2}{2} + \frac{cM^3}{3} + \frac{a\alpha M^3}{6} + \frac{b\alpha M^4}{8} + \frac{c\alpha M^4}{10} \quad (19)$$

Now in order to find the economic operating policy, the following steps are to be followed:

- Step 1 : Obtain T_1^* from eq.(10) . If $T_1^* \geq M$, find $K_1(T_1^*)$ from eq. (9).
- Step 2 : Determine T_2^* from eq.(16). If $T_2^* < M$, Evaluate $K_2(T_2^*)$ from Eq. (15).
- Step 3: If $T_1^* < M$ and $T_2^* \geq M$ then, obtain K(M) from Eq. (18).
- Step 4: Compare $K_1(T_1^*)$, $K_2(T_2^*)$ and K(M) and take the minimum.

4. Numerical Examples

Ex.1): (Case I and Case II): Minimum average cost is $K_1(T_1^*)$

The parameter values of the inventory system as follows

A= Rs. 100 per order, $I_p = 0.15$ year, $I_c = 0.12$ per year, h = Rs. 0.12 per year, p = Rs. 30 per unit , $\alpha = 0.1$, M=0.40 yr, b =150 units/yr, c = 15 units/yr, a =1000 units/yr.

Solving eq. (10) , we have, $T_1^* = 0.531$ year and the minimum average cost is
$K_1(T_1^*) =$ Rs. 502.39.
Again Solving eq.(16) ,we have, $T_2^* = 0.31$ year and the minimum average cost is
$K_2(T_2^*) =$ Rs. 554.28.
Here $T_1^* > M$ and $T_2^* < M$, Now $K_1(T_1^*) < K_2(T_2^*)$,
Hence the minimum average cost in this case is $K_1(T_1^*) =$ Rs. 502.39 where the optimal cycle length is $T_1^* = 0.531$.
The economic order quantity is given by $q_0(T_1^*) = 541.21$.

Ex.2): (Case I and Case II): Minimum average cost is $K_2(T_2^*)$
The parameter values of the inventory system as follows
A= Rs. 100 per order, $I_p = 0.15$ year, $I_c = 0.12$ per year, h = Rs. 0.12 per year, p = Rs. 30 per unit , $\alpha = 0.1$, M=0.45 yr, b =150 units/yr, c = 15 units/yr, a =1000 units/yr.
Solving eq. (10) , we have, $T_1^* = 0.564$ year and the minimum average cost is
$K_1(T_1^*) =$ Rs. 496.39 year.
Again Solving eq.(16) ,we have, $T_2^* = 0.31$ year and the minimum average cost is
$K_2(T_2^*) =$ Rs. 464.28 year
Here $T_1^* > M$ and $T_2^* < M$, Now $K_1(T_1^*) > K_2(T_2^*)$,
Hence the minimum average cost in this case is $K_2(T_2^*) =$ Rs. 464.28 year where the optimal cycle length is $T_2^* = 0.31$ year.
The economic order quantity is given by $q_0(T_2^*) = 340.83$.

5. Conclusion

In this paper, I have made attempt to develop EOQ model for deteriorating items under permissible delay in payment. Deterioration rate is assumed as variable. This model can be used where the demand of product rises rapidly to peak in the mid season and then falls rapidly as the season wanes out. The different type of demand can be better represented by the functional form $R(t)= a+bt+ct^2$, $a \geq 0$, $b \neq 0$, $c \neq 0$. for b>0, c>0, we can call it accelerated growth in demand . It happens to the seasonal products towards the beginning of the seasons. For b<0,c<0 , we can call it accelerated decline demand. Such situation arises to the seasonal product towards the end of product.

Refrences

1. Donaldson W.A.: Inventory replenishment policy for a linear trend in demand: An analytical solution. Oper. Res. Q. 28, 663-670(1977).
2. Silver, E.A.: A simple inventory replenishment decision rule for linear trend in demand. J.Oper. Res. Soc. 30, 71-75(1979).
3. Ritchie, E.: The EOQ for linear increasing demand. A simple optimum solution. J. Oper. Res. Soc.35, 949-952(1984).
4. Ghare, P. M. and Schrader, G.F.: A model for exponentially decaying inventories. J. Ind. Eng.14, 238-243(1963).
5. U. Dave and Patel L.K.: (T,Si) policy inventory model for deteriorating items with time proportional demand. Journal of Operational Research Society ,32, (1981) 137-142.
6. Deb , M. and Chaudhuri , K.S :An EOQ model for items with finite rate of production and variable rate of deterioration , OPSEARCH , 23 , 175-181.
7. Teng J.T. and Chang C.T : Economic production quantity models for deteriorating items with price and stock dependent demand. Computer and Operation Research,32, 297-308.
8. Covert , R.P. and Philip, G.C: An EOQ model for items with Weibull distribution deterioration. AIIE Transaction, 5, 323-326.
9. S.K. Goyal: EOQ under conditions of permissible delay in payments, Journal of Operation Research Society, 36, (1985) 335-338.
10. Aggarwal, S.P. and C. K. Jaggi: Ordering policies of deteriorating items under permissible delay in payments, Journal of operation research society, 46(1995) ,658-662.
11. S.W. Shinn and H.P. Hwang: Joint price and lot size determination under condition of permissible delay in payment and quantity discounts for freight cost, 91(1996), 528 – 542, European Journal of Operation Research.
12. Hariga, M.A. : The inventory replenishment problem with a linear trend in demand. Comput. Ind. Eng.,24(1993), 143-150.
13. Sachan, R.S.: On (T, S_i) policy inventory model for deteriorating items with time proportional demand. J.Operat. Res. Soc.,35(1984),1013-1019.
14. P. Chu, K.J. and Chung, S.P. Lan: Economic order quantity of deteriorating items under permissible delay in payments, Computer & Operations Research 25, (1998) 817–824.
15. S.K. Goyal: On improving replenishment policies for linear trend in demand, Engineering cost and production economics , 10 (1985),73-76.