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Oil Well Model and Rate Allocation Problem

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Research Article

Abstract: The oil well and rate allocation problem which refers to allocating production rate and lift gas rate of a well to achieve certain operational goals are described in this paper. These goals vary with the field and time. In some petroleum fields, especially mature fields, oil production can be constrained by fluid handling capacities of facilities. For such fields, rate allocation can be an effective way to increase the oil rate or reduce the production cost. The objective of this paper is to maximize the total production of oil, so that by using some properties of rate allocation problem, we reformulate the problem in the form of Linear Programming (LP) model and Mixed Integer Linear Programming (MILP) model. This problem was solved by using branch and bound method.

Key words: Oil Well, Rate Allocation, Gas Lift, LP Model, MILP Model, Bound and Branch Method etc.

1. Introduction

The mechanism of Gas Lift is elaborated. An appropriate amount of gas lift increases the oil rate, while excessive gas lift injection will reduce the oil rate. To determine the optimal gas lift rate, the usual practice is to allocate the gas lift to a well according to a gas lift performance curve (Nishikiori, Redner, Doty and Schmidt [9]). A gas lift performance curve is a plot of oil rate versus gas lift rate for oil well. When the gas supply is unlimited, the optimal gas lift rate is the one corresponding to the maximum oil rate on the Performance Curve. When the gas supply is limited, the gas lift is usually allocated using some optimization algorithm (Ray and Sarker [10]). The earliest gas lift allocation method is simple heuristic method based on the concept of Equal Slope, which states that at the optimal solution, the slope of the gas lift performance curve should be equal for all wells (Camponogara and Codes [4]). In addition to the equal slope method,Nishikiori, Redner, Doty and Schmidt et al. [9], also applied a formal optimization algorithm, a Quasi - Newton method, to the gas lift allocation problem. Both the equal slope method and the Quasi - Newton method relies on derivative information to verify optimality. Therefore, tend to get trapped in local optima. This limitation can be serious when some gas lift performance curves are not concave. Buitrago [3], addressed this problem by proposing a stochastic algorithm that uses a heuristic method to calculate the descent direction. However, as suggested by their results, their method is not good for handling constraints. Gas lift optimization based on performance curves is simple and easy to implement. However, this approach ignores flow interactions among wells in the optimization process. Dutta - Roy and Kattapuram [5], analyzed a gas lift optimization problem with two wells sharing a common flow line and pointed out that when flow interactions are significant, nonlinear optimization tools are needed to obtain satisfactory results. They applied the Sequential Quadratic Programming (SQP) method to a linearly constrained gas lift optimization problem with 13 wells results showed that the SQP method does perform better than methods based on performance curves. General Rate Allocation (GRA), this problem refers to allocating production rates and gas lift rate of single well to achieve certain operational goals. Linear programming seems to be the most popular for this kind of problems. First method (Attra, Wise and Black [2]), applied linear programming to maximize daily income from multi reservoir, case subject to well producing capacities, reservoir injection requirements, compressor capacity limits, gas lift requirements and sale contracts. Second method (Lo and Holden [7]), proposed a linear programming model to maximize daily oil rate by allocating well rates subject to multiple flow rate constraints. Third method (Fang and Lo [6]), applied LP to allocate both gas lift and production rates. The last method proximate gas lift performance curves by a piecewise linear curve and then formulated the rate allocation problem as an LP problem. The LP method is very efficient for the rate allocation problem, however, suffers from the fact that the objective and constraint functions in an LP problem have to be linear. Due to this limitation, the gas lift performance curves in the third method have to be concave.

2. The Problem

Rate allocation refers to the problem of optimally adjusting producing rates and gas lift rates of production wells to achieve certain operational goals. These goals vary with the field and time. In some petroleum fields, especially mature fields, oil production can be constrained by fluid handling capacities of facilities. For such fields, rate allocation can be an effective way to increase the oil rate or reduce the production cost. For example, if the oil production in a field is constrained by the gas processing capacity of the separation units, closing or reducing production rates of wells with the highest Gas Oil Ratio (GOR) will increase the total oil rate, in addition, reducing the gas lift rate of certain wells may increase the overall oil production by utilizing the gas processing capacity more efficiently.

In this Paper, we address the following rate allocation problem. The objective function is the total oil rate. An oil, gas, water, or liquid flow rate constraints can be put on any production well or network node. In abstract form, the optimization problem can be expressed as:

$$Maximize \sum_{i=1}^{n_w} (q^w_{0,i}) \quad (1)$$

$$Subject\ to\ \ q^n_{p,j} \le Q^n_{p,j}\ ,\ j\ \varepsilon\ \Omega^n,\ P\ \varepsilon(0,g,W,i) \quad (2)$$

(Phase$g$ includes both the formulation gas and gas lift, superscript n and w denotes nods and wells, respectively).

Where, $q^w_{o,i}$is the oil rate of well i,

$q^n_{p,j}$is the flow rate of phase $p$in node$j$,

$Q^n_{p,,j}$is the flow rate limit of phase p for node$j$,

Ω^n is the set of all nodes.

For gathering system with a tree like structure, the flow rate of network node$j$ is the sum of the flow rates of wells connected to node j.

Physically, the control variables for the rate allocation problem are the well chokes and gas lift rates of wells and the flow rate of each well, $q^w_{p,,j}$, is a nonlinear function of the control variables.

Fang and Lo et al. [6], made the following assumptions:

1) The well performance information can be evaluated individually for each well

by ignoring flow interactions among wells.

2) The gas oil ratio and water cut for a well remain constant for varying oil rate.

3) The gas lift performance curves are concave.

With the above assumptions, they reformulated the rate allocation problem to a linear programming problem and solved it by the simplex algorithm. The method was found to be very efficient.

The objective and constraint functions of the rate allocation problem (1) and (2) are linear combinations of the oil, water and gas lift rates of individual wells. Further, when the well performance approximated by piecewise linear performance curves, the water, formation gas and gas lift rates of a well can be regarded as functions of the oil rate of that well. Therefore, if we regard the oil rate as the control variables, denoted as X, equations (1) and (2) become an optimization problem whose objective and constraint functions are the sums of functions of one variable, which can be expressed as:

$$Maximize \sum_{j=1}^{n_w} f_j(X_i) \quad (3)$$

$$Subject\ to\ \sum h_{ij}(X_j) \le b_i\ ,\ i=1,...,m \quad (4)$$

$$X_j \ge 0\ ,\ j=1,...,n_w \quad (5)$$

Where,f_j denotes the objective functions,

h_{ij} denotes the j^{th} function involved in the i^{th} constraint,

b_i denotes the limit of the constraint,

m denotes the number of constraints.

Optimization problems of the form of equations (3), (4) and (5) are piecewise linear problems, which can be solved by linear optimization techniques. This is demonstrated in the following;

First, a point [x, f (x)] on a piecewise linear curve defined by a set of discrete points [x_i , f (x_i)], $i = o,\ldots, r$. Can be expressed as follows:

$$X = \sum_{j=0}^{r} \lambda_j\ x_j \quad (6)$$

$$f\ (X) = \sum_{j=0}^{r} \lambda_j\ f\ (x_j) \quad (7)$$

$$\sum_{j=0}^{r} \lambda_j = 1,\ \lambda_j \ge 0 \quad (8)$$

No more than two $\lambda_j$can be positive and they must be adjacent. (9)

Suppose for well j, each of its well performance curves are defined by $r_j+ 1$ discrete points with $x_{jo}= o$ and $x_{jrj}= q_{oj}^{max}$,

Whereq_{oj}^{max} is the maximum oil rate for well j.

Then for all functions $f_j(x_j)$and$g_{ij}\ (x_j)$, we can write:

$$f_j\ (X_j) = \sum_{k=0}^{r_j} \lambda_{jk}\ f_{jk}\ ,\ \ f_{jk} = f_j(X_{jk}) \quad (10)$$

$$g_{ij}\left(X_j\right) = \sum_{k=0}^{r_j} \lambda_{jk}\ g_{ijk}\ , g_{ijk} = g_{ij}\left(X_{jk}\right),\ i=1,...,m \quad (11)$$

$$X_j = \sum_{k=0}^{r_j} \lambda_{jk}\ X_{jk} \quad (12)$$

$$\sum_{k=0}^{r_j} \lambda_{jk} = 1\ , \lambda_{jk} \ge 0, \qquad \forall\ \ j,k \quad (13)$$

For a given$j$, no more than two λ_{jk} can be positive and they must be adjacent (14) Substituting equations [(10) - (14)] into equations [(3), (4) and (5)], we obtain the following problem (A).

$$Maximize\ Z = \sum_{j=1}^{n_w} \sum_{k=0}^{r_j} f_{ik}\ \ \lambda_{jk} \quad (15)$$

$$Subject\ to = \sum_{j=1}^{n_w} \sum_{k=0}^{r_j} g_{ijk}\ \ \lambda_{jk} \le b_i\ \ ,i=1,...,m \quad (16)$$

$$\sum_{k=0}^{r_j} \lambda_{jk} = 1 , \ j = 1,...,n_w \tag{17}$$

$$\lambda_{jk} \geq 0, \qquad \forall \ j, k \tag{18}$$

For a given j, no mere than two λ_{jk} can be positive and they must be adjacent (19)

Without constraint equation (19), we would have a general linear programming problem. It can be shown that if all $f_j(x_j)$ are concave functions and $g_{ijk}$are convex functions, then, the optimal solution of an LP problem defined by equations [(15) to (18)] automatically satisfies constraint equation (19). However, in general cases, constraint equation (19) has to be enforced explicitly.

3. Mixed Integer Linear Programming (MILP) Models for Rate Allocation Problem

This model enforces constraint equation (18), explicitly and is suitable for rate allocation problems with performance curves of arbitrary shapes. The disadvantage of this method is that even for a rate allocation problem with moderate size this model can contain a large number of binary variables and constraints.

To enforce the constraint that for certain j at most two consecutive coefficients λ_{jk}, are non-zero, we introduce a binary variable y_{jk},$k = o,..., r_j - 1$, which can be equal to 1 only if $x_{jk} \leq x_j \leq x_{j(k+1)}$and 0 otherwise. Problem (A) is then formulated as a Mixed Integer linear Programming Problem (MILP) to obtain the following problem (B).

$$Maximize \ Z = \sum_{j=1}^{n_w} \sum_{k=0}^{r_j} f_{ik} \ \lambda_{jk} \tag{20}$$

$$Subject \ to = \sum_{j=1}^{n_w} \sum_{k=0}^{r_j} g_{ijk} \ \lambda_{jk} \leq b_i \ , i = 1,...,m \tag{21}$$

$$\sum_{k=0}^{r_j} \lambda_{jk} = 1 , \ j = 1,...,n_w \tag{22}$$

$$\lambda_{j0} \leq y_{i0}, \qquad \forall \ j \tag{23}$$

$$\lambda_{jk} \leq y_{j(k-1)} + y_{jk} \ , k = 1,...,r_j - 1, \quad \forall \ j \tag{24}$$

$$\lambda_{jrj} \leq y_{j(r_j - 1)}, \qquad \forall \ j \tag{25}$$

$$\sum_{k=0}^{r_j - 1} y_{jk} = 1, \qquad \forall \ j \tag{26}$$

$$\lambda_{jk} \geq 0, \qquad \forall \ j, k \tag{27}$$

$$y_{ik} \in (0,1) \ , \ k = 1,...,r_j - 1, \qquad \forall \ j \tag{28}$$

Several similar reformulations exist and their computational performance can be different.

In principle, a MILP problem can be solved by enumeration. However, complete enumeration is computationally infeasible as soon as the number of integer variables in a MILP problem exceeds 20 or 30. So we need some strategies to cut the number of necessary enumerations. An effective method for this purpose is the Branch and Bound method (Mavrotas and Diakoulaki [8]).

Branch and Bound is a general search method for optimization problems over a search space that can represents as leaves of a tree. A fundamental idea behind that is to divide and conquer. Consider a general optimization problem.

$$Z = \max \left\{ C^T X : X \in S \right\} \tag{29}$$

Where,

X is the control variable and S denotes its feasible set,

Let, $S = S_1 U ... S_k$be a decomposition of S into smaller sets,

Let, $Z^k \ \max \{ C^T X : X \in S_k \}$ for K = 1,..., k .Then $Z = \max_k z^k$.

Based on this idea, branch and bound recursively divides an optimization problem into several sub problems and there by forms an enumeration tree, with each node in the tree representing a sub problem. The enumeration tree is constructed implicitly and the bound information on sub problems, are used to prune the tree.

Next, we present a branch and bound method for solving problem (B). The algorithm maintains a lower bound -Z and upper bound +Z,(-Z ≤ Z ≤ + Z)and a stack of active search nodes representing sub problems to be examined.

4. Algorithm of Branch and Bound Method

Step1.**Initialization**: Let -Z = -∞ and +Z = +∞. Put problem (B) into the stack of active search nodes.

Step2.**Choosing a node**: If the stack of active nodes is empty, the entire tree has been enumerated and the research ends. If the stack of active nodes contains several nodes, pop out the node on top of the stack. This node represents a MILP sub problem generated in Step I or 6.

Step3.**Optimizing**: Solve the LP relaxation of the sub problem selected in Step 2. Denote its optimal value as$\hat{Z}$.

Step4.**Bounding**: If the node selected in Step 2 is the root node, update +Z = $\hat{Z}$. If the solution in step 3 is a feasible solution to problem (B), update - Z= max (- Z,$\hat{Z}$) and store the corresponding feasible solution.

Step5.**Pruning**: The following conditions allow us to prune the tree and thus enumerate a large number of solutions implicitly.

a) **Pruning by optimality**: If +Z - (-Z)≤εz ,Where $\varepsilon_z$is a prescribed tolerance. Then the feasible solution corresponds to - Z can be regarded as the option solution of problem (B) and to search ends.

b) **Pruning by feasibility**: The solution from step 3 is a feasible solution to problem (B). There is no need to divide further the sub problem represented by current node.

c) **Pruning by bound**: $\hat{Z} \leq -Z$. The upper bound off the sub problem represented by current searching node is below the lower bound. There is no need to divide further the sub problem represented by current node.

d) **Pruning by infeasibility**: The problem examined has no feasible solution. There is no need to divide further the sub problem represented by current node. If condition (a) is met, stop. If condition (b), (c) or (d) is met, go to step 2 to backtrack, otherwise, go to step 6 to branch.

Step6.**Branching**: To reach this step, the solution from step 3 must not be a feasible solution to problem (B), some binary variable y_{jk} has fractions value .Suppose variabley_{jk} , $1 \leq J \leq n_w$, $K \in$ [o … r_j -1], has a fractional value. The MILP sub problem from step 2 (the parent sub problem) can be divided into two sub problems. The first sub problem is comprised of the parent sub problem plus a constrain of y_{jk} = 0. The second sub problem is comprised of the parent sub problem plus a constraint of y_{jk}=1. Add the two sub problems on top of the stack of active nodes in a prescribed order (such as the first sub problem goes first and the second sub problem goes second)

Step7. Continue the search by going to step 2.

The purpose of step 3 is to compute an upper bound for the MILP sub problem from step 2. The bund information is used in step 5 to prune the enumeration tree. The upper bound of a MILP sub problem is often obtained by solving relaxed problem , a problem that has an optimal value no worse than that of the MILP sub problem. One such relaxation for a MILP problem is its linear programming relaxation that allows the integer variables in a MILP problem to take real values. For example, an LP relaxation of problem (4B) is an LP problem that replaces constraint equation (28) with the following linear constraints.

$$0 \leq y_{ik} \leq 1 \,,\; k = 1,\dots, r_j - 1, \qquad \forall\; j \tag{30}$$

5. Numerical Example

The numerical example is taken from Ministry of oil and minerals in Yemen (Annual Bulletin [1] and Statistics Department Report [11]). The problem is to optimize oil production from a set of 56 wells with 22,500 (MSCF/ d) of available gas. Wells 47-56 cannot flow without gas lift. In this study, the method hereby described using NETSO programme which was written in Fortran 77 language and developed on a silicon graphics origin 200 for workstation.This problem is solved by using both of MILP method and branch and bound method for reducing the mixed integer problem to a sequence of linear programming problems. The results for MILP method are shown in table (1).

Table 1: Gas injection and oil production rates for a set of 56 wells obtained from the MILP method

Well No	Oil Rate STB/d	Gas Rate MSCF/d	Well No	Oil Rate STB/d	Gas Rate MSCF/d	Well No	Oil Rate STB/d	Gas Rate MSCF/d
1	386	672	20	391	975	39	207	301
2	626	450	21	455	772	40	27	98
3	605	521	22	214	370	41	372	0
4	280	0	23	944	0	42	200	0
5	281	0	24	1,680	1,030	43	337	797
6	333	157	25	487	0	44	397	0
7	836	235	26	105	120	45	83	0
8	276	268	27	353	0	46	50	14
9	1,568	1,295	28	1,044	0	47	441	3,042
10	233	0	29	184	0	48	483	2,466
11	957	1,048	30	308	0	49	232	1,418
12	510	800	31	354	0	50	0	0
13	108	0	32	654	131	51	0	0
14	302	186	33	211	208	52	0	0
15	648	598	34	209	0	53	267	1,484
16	361	460	35	216	195	54	0	0
17	892	0	36	204	108	55	0	0
18	1,213	282	37	64	0	56	452	1,770
19	310	0	38	282	157	-	-	-

Table (2) compares the performance of the MILP method with the equal slope method and the Ex - In method results, for the equal slope method and the Ex - In method are taken from Buitrago et al.[3]. The MILP method outperforms both the equal slope method and the Ex In method for this example. Specifically, using the same amount of gas lift, the MILP method allocates 64% more oil than the equal slope method. To allocate 21,265 STB/d oil, the MILP method requires 37.0% less gas lift than the equal slope method. To allocate 21,790 STB/d oil, the MILP method requires 16.6% less gas lift then the EX – In method.

Table 2: Gas lift allocation results obtained from different methods

Different Methods	Equal slope	Ex-In	MILP 1	MILP 2	MILP 3
Gas Lift rate (MSCF/d)	22,508	20,454	**22,500**	14,175	17,040
Oil Rate (STB/d)	21,265	21,790	22,632	**21,265**	**21,790**

1) Allocate all available gas lift of **22,500** MSCF/d.
2) Minimize gas lift rate while keeping the oil rate at **21,265** STB/d.
3) Minimize gas lift rate while keeping the oil rate at **21,790** STB/d.

6. Conclusions

This Paper describes the oil well and rate allocation problem which refers to allocating production rate and lift gas rate of a well to achieve certain operational goals. These goals vary with the field and time. For such field rate allocation can be an effective way to increase the oil rate or reduce the production cost. The objective of this Paper is to maximize the total production of oil, so that by using some properties of rate allocation problem, we reformulate the problem in the form of linear programming model and mixed integer linear programming model. This problem was solved by using branch and bound method.The different results obtained from different methods are compared. Theperformance of the MILP method is compared with the equal slope method and the Ex- In method. The results showed that the MILP method out performs both the equal slope method and Ex-In method. Specifically, using the same amount of gas lift, the MILP method allocates 64% more oil than the equal slope method. To allocate 21,265 STB/d oil, the MILP method requires 37% less gas lift than the equal slope method. To allocate 21,790 STB/d oil, the MILP method requires 16.6% less gas lift than the Ex-In method.

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