A Fuzzy Economic Replacement Decision Model

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Abstract: In every field of our real life situations, we deal with a replacement problem, when some items such as machines, medical equipment, military tank, electric bulb etc. or workers need to replace due to their decreased efficiency, failure or break down. To get a more realistic view of a replacement problem, here, we consider that the analytic hierarchy process (AHP) for economic Replacement of component. In this Paper, we use Fuzzy AHP method for solving replacement problem in fuzzy environment. Linguistic values are used to assess the ratings and the weights for key components. These linguistic ratings can be expressed in trapezoidal fuzzy numbers. Euclidian distance method is used to calculate the distance between two trapezoidal numbers. Finally, a closeness coefficient of each alternative is defined to determine the ranking order of all alternatives (Key Components).

1 Introduction

The standard replacement policy is a basic and well-known policy in maintenance optimization. It is concerned with the question which component is replaced firstly. System service life can be extended if a suitable maintenance policy has been adopted. According to the contents, maintenance is classified in to two types, preventive maintenance (PM) and corrective maintenance (CM). The former concerns with the activities, e.g. adjusting the operation parameters, repairing or replacing elements before the system break-down etc. The latter deals with the necessary repair or replacement of component as it fail. The advantage of PM is that the system can always be kept in an available condition as the situation needs. However, the costs are sometimes much higher as the replacement of component is taken [57]. In this paper, the maintenance is considered only for replacing elements no matter which of PM or CM is chosen. This situation occurs frequently in the maintenance of some profit-oriented system, for example, vehicles, machine tool systems, etc. The studies mentioned above deals with the PM policy about the component which has already been decided. For system, the choice of key components which should be replaced preventively is another issue. The main purpose of this paper is to give the replacement Orders of key components in a system. The selection of key components follows five criteria, once the key components have been chosen, the order of replacement is decided based on the Analytic Hierarchy Process (AHP) [1]. From the literature it can be concluded that in replacement policy the classical concept of “Optimality” may not always be the most appropriate policy. Over all speaking, we conclude that replacement policy involve several and different types criteria, combination of different decision models, group decision making and various forms of uncertainty. It is difficult to find out the replacement orders of key components in a system. In essential, the replacement of key components problem is a group decision-making under multiple criteria. Under many conditions, crisp data are inadequate to model real-life situations, since human judgments including preferences are often vague and cannot estimate his orders with an exact numerical value. A more realistic approach may be to use linguistic assessments instead of numerical values. In other words, the ratings and the weights of the criteria in the problem are assessed by means of linguistic variables. Considering the fuzziness in the decision data and group decision making process, linguistic variables are used to assess the weights of all criteria and the ratings of each alternative with respect to each criterion. We can convert the decision matrix into a fuzzy decision matrix and construct a weighted-normalized fuzzy decision matrix once the decision-maker’s fuzzy ratings have been pooled. In the concept of AHP we give the orders of key components, then Euclidian distance method is applied to calculate the distance between two fuzzy ratings. In this paper, we use Fuzzy AHP method for solving replacement problem in fuzzy environment. Linguistic values are used to assess the ratings and the weights for key components. These linguistic ratings can be expressed in trapezoidal fuzzy numbers. Euclidian distance method is used to calculate the distance between two trapezoidal numbers. Finally, a closeness coefficient of each alternative is defined to determine the ranking order of all alternatives (Key Components). The analytic hierarchy process (AHP) is a structured technique for organizing and analyzing complex decisions. Based on mathematics and psychology, it was developed by Thomas L. Saaty in the 1970s and has been extensively studied and refined since then. It has particular application in group decision making and is used around the world in a wide variety of decision situation in fields such as government, business, industry, healthcare, and education. Rather than prescribing a "correct" decision, the AHP helps decision makers find one that best suits their goal and their understanding of the problem. It provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions.

Users of the AHP first decompose their decision problem into a hierarchy of more easily comprehended sub-problems, each of which can be analyzed independently. The elements of the hierarchy can relate to any aspect of the decision problem tangible or intangible, carefully measured or roughly estimated, well- or poorly-understood anything at all that applies to the decision at hand. Once the hierarchy is built, the decision makers systematically evaluate its various elements by comparing them to one another two at a time, with respect to their impact on an element above them in the hierarchy. In making the comparisons, the decision makers can use concrete data about the elements, but they typically use their judgments about the elements' relative meaning and importance. It is the essence of the AHP that human judgments, and not just the underlying information, can be used in performing the evaluations. The AHP converts these evaluations to numerical values that can be processed and compared over the entire range of the problem. A numerical weight or priority derived for each element of the hierarchy, allowing diverse and often incommensurable elements to be compared to one another in a rational and consistent way. This capability distinguishes the AHP from other decision making techniques. In the final step of the process, numerical priorities are calculated for each of the decision alternatives. These numbers represent the alternatives' relative ability to achieve the decision goal, so they allow a straightforward consideration of the various courses of action.

In this Paper, we use Fuzzy AHP method for solving replacement problem in fuzzy environment. Linguistic values are used to assess the ratings and the weights for key components. These linguistic ratings can be expressed in trapezoidal fuzzy numbers. Euclidian distance method is used to calculate the distance between two trapezoidal numbers. Finally, a closeness coefficient of each alternative is defined to determine the ranking order of all alternatives (Key Components). The standard replacement policy is a basic and well-known policy in maintenance optimization. It is concerned with the question which component is replaced firstly. The maintenance is considered only for replacing elements no matter which of PM or CM is chosen. This situation occurs frequently in the maintenance of some profit-oriented system for example, Vehicles, machine tool system etc.

2. Components for Economic Replacement model

System performance can be kept as good as possible if great care is taken in its maintenance during its operation. Mean-while, the life cycle of the system is extended and the efficiency promoted. To achieve this goal the manner of how to maintain the system in a normal condition becomes important. Thus, taking periodical replacement for some components in a system should be considered. The selection of key components and the replacement priority/orders are presented in [57]. A system consists of many subsystems, each carries out a specific function, a typical example of the machinery relationship between functions and components can be represented by a matrix with elements ‘1’ denoting relation exists and ‘0’ otherwise, see table. 1 it is better to design a system when it fails, all components fail simultaneously. But this is very difficult to achieve for the real system, thus replacement of component should be taken.

Table 1: Matrix relationship between Functions and Components

Functions	Components				
	C_1	C_2	C_3	…..	C_m
A_1	1	0	0	…..	1
A_2	1	0	**1**	.....	0
A_3	0	1	0	.....	1
…..	…..	…..	…..	…..	…..
A_m	1	0	1	.....	0

A positive trapezoidal fuzzy number (PTFN) $\tilde{n}$ can be defined as (n_1, n_2, n_3, n_4), The membership function $\mu_{\tilde{n}}(x)$ is defined as:

$$\mu_{\tilde{n}}(x) = \begin{cases} 0 & x < n_1 \\ x - n_1/(n_2 - n_1) & n_1 \le x \le n_2 \\ 1 & n_2 \le x \le n_3 \\ x - n_4/(n_3 - n_4) & n_3 \le x \le n_4 \\ 0 & x > n_4 \end{cases} \quad (1)$$

3 Euclidean distance method

Let $\tilde{m} = (m_1, m_2, m_3, m_4)$ and $\tilde{n} = (n_1, n_2, n_3, n_4)$ be two trapezoidal fuzzy numbers. Then the distance between them can be calculated by using the Euclidean distance method as;

$$d_v(\tilde{m}, \tilde{n}) = \sqrt{1/4\left[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2 + (m_4 - n_4)^2\right]}$$
(2)

The Euclidean distance method is an effective and simple method to calculate the distance between trapezoidal fuzzy numbers. According to the Euclidean distance method, two trapezoidal fuzzy numbers $\tilde{m}$ and $\tilde{n}$ are identical if and only if $d_v(\tilde{m}, \tilde{n}) = 0$.

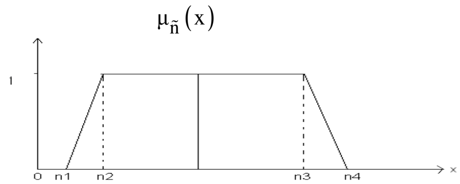
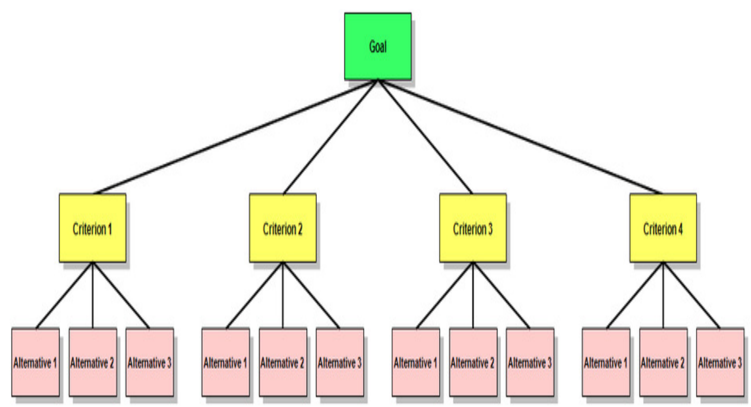


Fig.6.2: *Trapezoidal fuzzy number* $\tilde{n}$

4 Proposed Methodology

A systematic approach to extend the AHP is proposed to solve the replacement of key component in mechanical system under fuzzy environment. In this paper the important weights of various criteria (key components) and the ratings of qualitative criteria are considered as linguistic variables. Because linguistic assessments merely approximate the subjective judgment of decision-makers, we can consider linear trapezoidal membership functions to be adequate for capturing the vagueness of these linguistic assessments [33]. These linguistic variables can be expressed in positive trapezoidal fuzzy numbers, [Linguistic variables for importance weight- Low, Very Low, Medium, Medium Low, High, Medium High, Very High], [Linguistic variables for rating- Poor, Very poor, Medium Poor, Fair, Medium Good, good, Very Good]. The importance of weight of each criterion can be by either directly assigning or indirectly using pairwise comparison.



In fact, replacement of key components is a group decision making problem, which may be described by means of the following sets:

(i) a set of K decision-makers called $E = \{D_1, D_2, ..., D_k\}$;

(ii) a set of m possible functions called $A = \{A_1, A_2, ..., A_m\}$;

(iii) a set of n criteria, $C = \{C_1, C_2, ..., C_n\}$, with which function performances are measured;

(iv) a set of performance ratings of $A_i (i = 1, 2, ..., m)$ with respect to criteria $C_j (j = 1, 2, ..., n)$, called $X_{ij} = \{x_{ij}, i = 1, 2, ..., m, j = 1, 2, ..., n\}$.

Assume that a decision group has K decision makers, and the fuzzy rating of each decision-maker $D_k (k = 1, 2, ..., K)$ can be represented as a positive trapezoidal fuzzy number $\tilde{R}_k (k = 1, 2, ..., K)$ with membership function $\mu_{\tilde{R}}(x)$.

A good aggregation method should be considered the range of fuzzy rating of each decision-maker. It means that the ranges of all decision-makers fuzzy ratings. Let the fuzzy ratings of all decision-makers be trapezoidal fuzzy numbers $\tilde{R}_k = (a_k, b_k, c_k, d_k), k = 1, 2, ..., K$. Then the aggregated fuzzy rating can be defined as;

$$\tilde{R} = (a, b, c, d), k = 1, 2, ..., K \quad (3)$$

Where

$$a = \min_k (a_k) \quad b = 1/k \sum_{k=1}^{K} b_k \quad c = 1/k \sum_{k=1}^{K} c_k \quad d = \max_k (d_k)$$

Let the fuzzy rating and important weight of the k^{th} decision maker be $\tilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk}, d_{ijk})$ and $\tilde{w}_{ijk} = (w_{ijk}, w_{ijk}, w_{ijk}, w_{ijk})$ $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$, respectively. Hence, the aggregated fuzzy ratings $(\tilde{x}_{ij})$ of alternatives with respect to each criterion can be calculated as:

$$\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}) \quad (4)$$

Where;

$$a_{ij} = \min_k (a_{ijk}) \quad b_{ij} = 1/k \sum_{k=1}^{K} b_{ijk} \quad c_{ij} = 1/k \sum_{k=1}^{K} c_{ijk}$$

$$d_{ij} = \max_k (d_{ijk})$$

The aggregated fuzzy weights $(\tilde{w}_j)$ of each criterion can be calculated as:

$$\tilde{w}_{j1} = (w_{j1}, w_{j2}, w_{j3}, w_{j4}) \quad (5)$$

Where;

$$w_{j1} = \min_k (a_{jk1}) \quad w_{j2} = 1/k \sum_{k=1}^{K} b_{jk2} \quad w_{j3} = 1/k \sum_{k=1}^{K} c_{jk3}$$

$$w_{j4} = \max_k (d_{jk4})$$

As stated above, a replacement of key components problem can be concisely expressed in matrix format as follows;

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1j} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2j} & \cdots & \tilde{x}_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \tilde{x}_{i1} & \tilde{x}_{i2} & \cdots & \tilde{x}_{ij} & \cdots & \tilde{x}_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mj} & \cdots & \tilde{x}_{mn} \end{bmatrix}$$

$$\tilde{w} = [\tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \tilde{w}_4],$$

Where; $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ and

$\tilde{w}_{j1} = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$

$i = 1, 2, ..., m$ and $j = 1, 2, ..., n$ can be approximated by positive trapezoidal fuzzy numbers. To avoid complexity of mathematical operations in a process, the linear scale transformation is used here to transform the various criteria scales in to comparable scales. The set of criteria can be divided into benefit criteria (the larger the rating, the greater the performance) and cost criteria (the smaller the rating, the greater the performance). Therefore, the normalized fuzzy-decision matrix can be represented as

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n} \quad (6)$$

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{d_j^+}, \frac{b_{ij}}{d_j^+}, \frac{c_{ij}}{d_j^+}, \frac{d_{ij}}{d_j^+} \right)$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{d_{ij}}, \frac{a_j^-}{a_{ij}} \right)$$

$$d_j^+ = \max_i (d_{ij})$$

$$a_j^- = \min_i (a_{ij})$$

The normalization method mentioned above is designed to preserve the property in which the elements $\tilde{r}_{ij}, \forall_{ij}$ are standardized (normalized) trapezoidal fuzzy numbers.

Considering the different importance of each criterion, the weighted normalized fuzzy decision matrix is constructed as

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n}, \ i = 1, 2, \ldots\ldots, m, \ j = 1, 2, \ldots\ldots, n$$

(7)

Where $\tilde{v}_{ij} = \tilde{r}_{ij} (.) w_j$

According to the weighted normalized fuzzy decision matrix, normalized positive trapezoidal fuzzy numbers can also approximate the elements $\tilde{v}_{i,j}, \forall_{i,j}$. Then, the fuzzy positive-ideal solution (FPIS, A^+) and fuzzy negative-ideal solution (FNIS, A^-) can be defined as

$$A^+ = (\tilde{v}_1^+, \tilde{v}_2^+, \tilde{v}_3^+, \tilde{v}_4^+) \quad (8)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \tilde{v}_3^-, \tilde{v}_4^-) \quad (9)$$

Where

$\tilde{v}_j^+ = \max_i (v_{ij})$, and $\tilde{v}_j^- = \min_i (v_{ij})$,

$i = 1, 2, ..., m$ and $j = 1, 2, ..., n$

The distance of each alternative (component) from A^+ and A^- can be currently calculated as

$$d_i^+ = \sum_{j=1}^{n} d_v (v_{ij}, \tilde{v}_j^+), \quad i = 1, 2, ..., m$$

(10)

$$d_i^- = \sum_{j=1}^{n} d_v (v_{ij}, \tilde{v}_j^-), \quad j = 1, 2, ..., n$$

(11)

Where $d_v(.,.)$ is the distance between two fuzzy numbers.

5 Numerical Example

We consider an example motivated by a real-life system to demonstrate the practical use of proposed solution. A Power loom system in a forming machine consists of five components to carry out five functions. After preliminary screening, three functions

(A_1, A_2, A_3) remain for further evolution. A committee of three decision-makers D_1, D_2, and D_3 has been formed to select the most suitable key component. Five benefit criteria are considered:

(1) Motor oil (C_1) (2) Pipe (C_2) (3) Release valve (C3) (4) Loom Motor (C_4) (5) Motor Belt (C5)

The proposed method is currently applied to solve this problem, the computational procedure of which is summarized as follows:

Step 1: Three decision-makers use the linguistic weighting variables to assess the importance of the criteria. The importance weights of the criteria determined by these three decision-makers are shown in table.

Step 2: three decision-makers use the linguistic rating variables to evaluate the ratings of functions with respect to each component. The ratings of the five functions by the decision-makers under the Various criteria.

Table 2: Importance weight of criteria from three decision-makers

Criteria	Decision-makers		
	D_1	D_2	D_3
C_1	H	VH	VH
C_2	VH	H	H
C_3	VH	H	VH
C_4	VH	H	VH
C_5	H	VH	H

Step 3: The linguistic evaluation shows in tables 6.3 and 6.4 are converted into trapezoidal fuzzy numbers to construct the fuzzy-decision matrix and determine the fuzzy weight of each criterion, as in table.

Step 4: The normalized fuzzy-decision matrix is constructed as in table

Table: Ratings of the five Functions by decision-makers under Various Criteria.

Criteria	Functions	Decision-makers		
C_1	A_1	MG	MG	MG
	A_2	G	G	G
	A_3	VG	VG	G
C_2	A_1	MG	MG	VG
	A_2	G	G	VG
	A_3	G	VG	VG
C_3	A_1	VG	G	VG
	A_2	VG	G	VG
	A_3	G	G	G
C_4	A_1	G	G	G
	A_2	VG	VG	G
	A_3	G	G	VG
C_5	A_1	VG	G	VG
	A_2	G	VG	G
	A_3	VG	G	VG

Table 3: Priority with respect to

$$\begin{array}{c|ccccc} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \hline A_1 & (0.118, 0.119, 0.113, 0.111) & (0.4, 0.0243, 0.8, 1) & (0.199, 0.200, 0.201, 0.200) & (0.013, 0.015, 0.012, 0.013) & (0.301, 0.322, 0.358, 0.321) \\ A_2 & (0.390, 0.392, 0.389, 0.387) & (0.011, 0.010, 0.011, 0.12) & (0.49, 0.052, 0.053, 0.051) & (0.037, 0.032, 0.035, 0.038) & (0.390, 0.492, 0.489, 0.490) \\ A_3 & (0.034, 0.035, 0.036, 0.037) & (0.092, 0.091, 0.093, 0.092) & (0.015, 0.017, 0.017, 0.016) & (0.5, 0.03, 0.04, 0.05) & (0.146, 0.149, 0.148, 0.147) \end{array}$$

According to the Decision Table of Priority the replacement orders of key Components are:

$$A_2 > A_1 > A_3$$

6 Concluding Remarks

Fuzzy Economic Replacement problem adhere to uncertain and imprecise data, and fuzzy set theory is adequate to deal with them. In a replacement decision process, the use of linguistic variables in replacement decision problems is highly beneficial when performance values cannot be expressed by means of numerical values. Due to the decision-makers experience, feel and subjective estimates often appear in the replacement of key component in a system, an extension version of AHP in a fuzzy environment is proposed in this Paper.

According to the Priority with respect to the criteria and alternatives we can determine not only the ranking order but also the assessment status of all key components. Significantly, the proposed method provides more objective information for Economic replacement of key component in a system.

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