Application of Fuzzy Relations for Selection of Crop Pattern

M. S. Bapat[*], P. N. Kamble[#], S. N. Yadav[†]

[*]Willingdon College, Sangli - 416415, Maharashtra, INDIA.

[#]Dr. Babasaheb Ambedkar Marathwada University, Aurangabad - 431004, Maharashtra, INDIA.

[†]Research Scholar, Shivaji University, Kolhapur – 416414, Maharashtra, INDIA.

Corresponding Addresses:

[*]msbapt@gmail.com, [#]kamble.prakash69@gmail.com, [†]sunil_snehank@yahoo.com

Research Article

Abstract: The purpose of this paper is to develop an application of multi-objective decision making model for farming system. The information about the crops provided by the farmers is represented by means of fuzzy relations. The crop pattern based on multi-objective decision making is obtained by means of fuzzy relations. The crops are ranked according to their profitability. Fuzzy majority is represented by a fuzzy quantifier, and applied in the aggregation, by means of OWA operator. The weights of OWA are calculated by the fuzzy quantifier.

Key words: Multi-objective decision making; fuzzy relation; fuzzy majority; farming system.

1. Introduction

When in an environment in which the goals, constraints, information and consequences of available actions are not precisely known the uncertainty is of a qualitative nature. In this situation, use of fuzzy set theory might provide the flexibility needed to represent the uncertainty resulting from the lack of knowledge. It can be used to design a decision process. Several authors have provided results on decision making by means of fuzzy set theory [8]. The different fuzzy relations are obtained frommultiple objectives and then fused it into a single fuzzy relation [7]. A collective fuzzy relationis obtained by aggregation a set of "individual" fuzzy relations using OWA operator [5] guided by a relative linguistic quantifier [6].

2. Preliminaries

Let $D=\{a_1,...,a_i,...,a_n\}$ be a finite set of decision actions evaluated by attributes $\Omega=\{k_1,...,K_j,...,K_m\}$ with weights $W=\{w_1,...,w_j,...,w_m\}$. Let a set of all fuzzy relations be $P^{(k)}$ where $P^{(k)}=(p_{ij}^{(k)})_{n\times n}$, and $p_{ij}^{(k)}$ represents the intensity of decision action a_i over decision action a_j with respect to k^{th} attribute.

Definition 2.1 An OWA operator of dimension m is a mapping $\phi: R^m \to R$ with an associated weight vector $w=(w_1,...,w_j,...,w_m)^T$ such that

$$\sum_{k=1}^{m} w_k = 1 \text{ and } \phi(a_1,a_2...,a_m)=\sum_{k=1}^{m} w_k b_k,$$

where b_k is the kth largest of $\{a_1,a_2,...,a_m\}$. In [6], Yager suggested a way to compute the weights (i.e., $w_k, k=1,2,\cdots,m$) of the OWA operator using linguistic quantifiers, which, in the case of a non-decreasing proportional quantifier Q, is given by this expression:

$$w_k = Q\left(\frac{k}{m}\right) - Q\left(\frac{(k-1)}{m}\right), \forall k$$

being the membership function of a non-decreasing proportional quantifier Q, as follows:

$$Q(x)=\begin{cases} 0, & 0 \le x < a, \\ \dfrac{x-a}{b-a}, & a \le x \le b, \\ 1, & b < x \le 1, \end{cases}$$

With $a,b \in [0,1]$. When it is used a fuzzy linguistic quantifier Q to compute the weights of the OWA operator ϕ, it is symbolized by ϕ_Q. Using an OWA operator ϕ_Q, we derive a collective relation, $P^c=(p_{ij}^c)_{n\times n}$ indicating the global information between every pair of decision actions according to majority of attributes, which is represented by Q. In this case,

$$P_{ij}^c = \phi_Q(p_{ij}^{(1)},...,p_{ij}^{(k)},...,p_{ij}^{(m)}) = \sum_{k=1}^{m} w_k q_{ij}^{(k)},$$

Where $q_{ij}^{(k)}$ is the kth largest value in the set $\{p_{ij}^{(1)},...,p_{ij}^{(m)}\}$.

Aggregation phase: Using the concept of fuzzy majority represented by a linguistic quantifier and applied in the aggregation operations by mean of OWA operators [5], a collective fuzzy relation is obtained from all individual fuzzy relations. Exploitation phase: Using the concept of fuzzy majority the choice degrees of decision actions are used i.e., the quantifier guided dominance degree is used. These choice degrees will act over the collective relations supplying a selection set of decision actions.

3. The Decision Process

In this section we will deal with choosing the decision action(s) which is(are) to be desirable. For that reason, we have a set of m individual fuzzy relations. These individual relations have to be fused into a single fuzzy relation by aggregation procedures. Then, selection is made by aggregation and exploitation. The aggregation phase defines a collective fuzzy relation. This indicates the global information between every ordered pair of decision actions. The exploitation phase transforms the global information about the decision actions into a global ranking of them, supplying a selection set of decision actions.

4. Case study and Research Methodology

The crisp data of farming system regarding crop pattern issystematically taken under the experts' supervision. This crisp data is considered for the purpose of computational results and is analyzed to know the best decision action to select the best decision action. A decision situation in this model is characterized by the following components

Crop Selection Decision System:

The problem we will deal is that of choosing the best profitable crop among a finite set, D = {a_1, a_2,…,a_n}(n ≥2)ofdecision actions(crops) evaluated by attributes $\Omega = \{k_1,...,K_j,...,K_m\}$.The decision actions will be classified from the best to worst,

5. Results and Discussions

The Multi-objectiveDecision Making problem has been solved by using by fuzzy relation approach as mentioned. The solution has been presented in Table-2.For a primary data of a typical farming system on kharif crops in Sangli district the following results were observed.

Decision actions

Jawar (a_1), Soyabean (a_2), Groundnut (a_3), Maize (a_4), Moong (a_5), Ghewada (a_6).

Attributs

Total production (k_1), Preparation of soil and sowing (k_2), Nutrients (k_3), Good quality seed (k_4), Protection from weeds (k_5), Spraying of pesticides (k_6), Harvesting (k_7), Threshing (k_8), Storage (k_9), Marketing (k_{10}), Effect on soil fertility (k_{11}), Production cost (k_{12}), Net profit (k_{13}).

Table 1: Characteristics of crisp values

Sr. No.	Decision Action (D)	Attributes (Ω)												
		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}
1	a_1	9-10	9	1.32	1.25	12.0	1.0	4.0	7.0	4.0	1.6	0.2	41.17	4.8
2	a_2	6.5-7	9	2.46	2.70	5.5	2.0	4.0	4.5	2.8	9.5	0.9	33.91	3.1
3	a_3	7.5-9	12	2.04	7.50	7.0	1.0	2.5	4.0	5.6	1.9	0.8	44.03	3.0
4	a_4	7-7.5	9	5.29	1.50	6.5	1.0	4.0	5.0	3.0	1.4	0.2	36.39	3.3
5	a_5	6.5-7.5	9	1.50	1.00	5.0	1.0	4.0	1.56	1.04	0.35	0.6	24.45	4.0
6	a_6	6.2-7.0	9	1.50	2.70	6.0	2.0	4.0	2.56	2.0	0.68	0.7	30.38	3.1

Decision Making

We find the intensityof the decision action a_i over a_j for attribute k_j , p_{ij}^k by using the formula

$$p_{ij}^k = \frac{u_i^k}{u_i^k + u_j^k}, i \neq, j.k = 1, 2, ..., 13.$$

There are thirteen relations $P^{(1)},...,P^{(13)}$. They are listed below:

$$P^1 = \begin{pmatrix} - & 0.58 & 0.53 & 0.55 & 0.58 & 0.59 \\ 0.41 & - & 0.45 & 0.48 & 0.49 & 0.50 \\ 0.46 & 0.55 & - & 0.53 & 0.54 & 0.56 \\ 0.45 & 0.52 & 0.47 & - & 0.51 & 0.52 \\ 0.42 & 0.51 & 0.46 & 0.49 & - & 0.51 \\ 0.41 & 0.49 & 0.44 & 0.48 & 0.59 & - \end{pmatrix} \quad P^2 = \begin{pmatrix} - & 0.5 & 0.43 & 0.5 & 0.5 & 0.5 \\ 0.5 & - & 0.43 & 0.5 & 0.5 & 0.5 \\ 0.57 & 0.57 & - & 0.57 & 0.57 & 0.57 \\ 0.5 & 0.5 & 0.43 & - & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.43 & 0.5 & - & 0.5 \\ 0.5 & 0.5 & 0.43 & 0.5 & 0.5 & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & 0.34 & 0.4 & 0.2 & 0.46 & 0.46 \\ 0.64 & - & 0.53 & 0.3 & 0.61 & 0.61 \\ 0.61 & 0.47 & - & 0.27 & 0.57 & 0.57 \\ 0.8 & 0.69 & 0.73 & - & 0.78 & 0.78 \\ 0.54 & 0.39 & 0.43 & 0.22 & - & 0.22 \\ 0.54 & 0.39 & 0.43 & 0.22 & 0.5 & - \end{pmatrix} \quad P^4 = \begin{pmatrix} - & 0.33 & 0.15 & 0.5 & 0.57 & 0.31 \\ 0.68 & - & 0.26 & 0.68 & 0.73 & 0.5 \\ 0.85 & 0.74 & - & 0.85 & 0.88 & 0.74 \\ 0.5 & 0.33 & 0.15 & - & 0.57 & 0.33 \\ 0.43 & 0.27 & 0.12 & 0.43 & 0.5 & 0.27 \\ 0.68 & 0.5 & 0.26 & 0.68 & 0.73 & - \end{pmatrix}$$

$$P^5 = \begin{pmatrix} - & 0.33 & 0.5 & 0.5 & 0.5 & 0.33 \\ 0.66 & - & 0.66 & 0.66 & 0.66 & 0.5 \\ 0.5 & 0.33 & - & 0.5 & 0.5 & 0.33 \\ 0.5 & 0.33 & 0.5 & - & 0.5 & 0.33 \\ 0.5 & 0.33 & 0.5 & 0.5 & - & 0.33 \\ 0.66 & 0.5 & 0.66 & 0.66 & 0.66 & - \end{pmatrix} \quad P^6 = \begin{pmatrix} - & 0.69 & 0.63 & 0.65 & 0.71 & 0.67 \\ 0.31 & - & 0.44 & 0.46 & 0.52 & 0.48 \\ 0.37 & 0.56 & - & 0.52 & 0.58 & 0.54 \\ 0.35 & 0.54 & 0.48 & - & 0.57 & 0.52 \\ 0.29 & 0.48 & 0.42 & 0.43 & - & 0.45 \\ 0.33 & 0.52 & 0.46 & 0.48 & 0.55 & - \end{pmatrix}$$

$$P^7 = \begin{pmatrix} - & 0.5 & 0.62 & 0.5 & 0.5 & 0.5 \\ 0.5 & - & 0.62 & 0.5 & 0.5 & 0.5 \\ 0.38 & 0.38 & - & 0.38 & 0.38 & 0.38 \\ 0.5 & 0.5 & 0.62 & - & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.62 & 0.5 & - & 0.5 \\ 0.5 & 0.5 & 0.62 & 0.5 & 0.5 & - \end{pmatrix} \quad P^8 = \begin{pmatrix} - & 0.61 & 0.64 & 0.58 & 0.82 & 0.73 \\ 0.39 & - & 0.53 & 0.47 & 0.74 & 0.64 \\ 0.36 & 0.47 & - & 0.44 & 0.72 & 0.62 \\ 0.42 & 0.53 & 0.56 & - & 0.76 & 0.66 \\ 0.18 & 0.26 & 0.28 & 0.24 & - & 0.38 \\ 0.26 & 0.36 & 0.38 & 0.33 & 0.62 & - \end{pmatrix}$$

$$P^9 = \begin{pmatrix} - & 0.59 & 0.42 & 0.57 & 0.79 & 0.66 \\ 0.41 & - & 0.33 & 0.48 & 0.73 & 0.58 \\ 0.58 & 0.66 & - & 0.65 & 0.84 & 0.74 \\ 0.43 & 0.52 & 0.35 & - & 0.74 & 0.5 \\ 0.21 & 0.27 & 0.16 & 0.26 & - & 0.34 \\ 0.33 & 0.42 & 0.26 & 0.4 & 0.66 & - \end{pmatrix} \quad P^{10} = \begin{pmatrix} - & 0.63 & 0.46 & 0.54 & 0.82 & 0.70 \\ 0.37 & - & 0.33 & 0.41 & 0.73 & 0.58 \\ 0.54 & 0.66 & - & 0.58 & 0.84 & 0.74 \\ 0.46 & 0.59 & 0.42 & - & 0.79 & 0.66 \\ 0.18 & 0.27 & 0.16 & 0.21 & - & 0.34 \\ 0.3 & 0.42 & 0.26 & 0.33 & 0.66 & - \end{pmatrix}$$

$$P^{11} = \begin{pmatrix} - & 0.18 & 0.2 & 0.5 & 0.25 & 0.22 \\ 0.82 & - & 0.53 & 0.82 & 0.6 & 0.56 \\ 0.8 & 0.47 & - & 0.8 & 0.57 & 0.53 \\ 0.5 & 0.18 & 0.2 & - & 0.25 & 0.22 \\ 0.75 & 0.4 & 0.43 & 0.75 & - & 0.46 \\ 0.77 & 0.44 & 0.47 & 0.77 & 0.54 & - \end{pmatrix} \quad P^{12} = \begin{pmatrix} - & 0.55 & 0.48 & 0.53 & 0.63 & 0.58 \\ 0.45 & - & 0.44 & 0.48 & 0.58 & 0.53 \\ 0.52 & 0.56 & - & 0.55 & 0.64 & 0.59 \\ 0.47 & 0.52 & 0.45 & - & 0.6 & 0.55 \\ 0.37 & 0.42 & 0.36 & 0.40 & - & 0.45 \\ 0.42 & 0.47 & 0.41 & 0.45 & 0.55 & - \end{pmatrix}$$

$$P^{13}=\begin{pmatrix} - & 0.61 & 0.61 & 0.59 & 0.55 & 0.61 \\ 0.39 & - & 0.5 & 0.48 & 0.43 & 0.5 \\ 0.39 & 0.5 & - & 0.48 & 0.43 & 0.49 \\ 0.41 & 0.52 & 0.52 & - & 0.45 & 0.52 \\ 0.45 & 0.57 & 0.57 & 0.55 & - & 0.56 \\ 0.39 & 0.5 & 0.51 & 0.58 & 0.44 & - \end{pmatrix}.$$

Method I Simple Average Fusion Method

By using simple average

$$P_{ij}^{C}=\frac{1}{13}\sum_{i=1,j=1}^{6,13}a_{ij},\ i\neq j$$

we can fuse the relations to single relation as shown below.

$$P_{ij}^{C}=\begin{pmatrix} - & 0.495 & 0.42 & 0.395 & 0.59 & 0.527 \\ 0.502 & - & 0.465 & 0.48 & 0.601 & 0.462 \\ 0.528 & 0.532 & - & 0.547 & 0.62 & 0.57 \\ 0.48 & 0.48 & 0.45 & - & 0.58 & 0.51 \\ 0.41 & 0.40 & 0.38 & 0.43 & - & 0.41 \\ 0.47 & 0.46 & 0.43 & 0.48 & 0.57 & - \end{pmatrix}$$

We exploit fused relations by using the same method. Theranking of the decision actions acting over the collective fuzzy relation supply the following values.

	a_1	a_2	a_3	a_4	a_5	a_6
X	0.485	0.502	0.559	0.5	0.406	0.482

where $X=\frac{1}{5}\sum_{j}^{6}a_{ij},\ i\neq j.$

Clearly the maximal set is: $\{a_3\}$. Therefore, the selection set of decision actions for selection procedure is the singleton $\{a_3\}$. Ranking of crops for their profitability is $\langle a_3,a_2,a_4,a_1,a_6,a_5\rangle$.

Method II Quantifier Guided Fusion Method (weighted average)

Most of the decision actions satisfies at least half of the attributes. With the help of these fuzzy quantifiers we can order the decision actions in the following way.

Using the fuzzy majority criteria with the fuzzy quantifier "at least half", with the pair (0,0.5), and corresponding OWA operator with the weighting vector, $W=\left[\frac{2}{13},\frac{2}{13},\frac{2}{13},\frac{2}{13},\frac{2}{13},\frac{2}{13},\frac{1}{13},0,0,0,0,0,0\right]$.

The collective fuzzy relation $P^{c}=\left(p_{ij}^{c}\right)_{n\times n}$ indicating the intensity between every pair of alternatives to the majority of attributes, which is represented by Q. In this case,

$P_{ij}^{c}=\phi_Q(p_{ij}^{(1)},...,p_{ij}^{(k)},...,p_{ij}^{(m)})=\sum_{k=1}^{m}w_k q_{ij}^{(k)}$, where $q_{ij}^{(k)}$ is the kth largest value in the set $\{p_{ij}^{(1)},...,p_{ij}^{(m)}\}$. Therefore, the collective fuzzy relation is:

$$P_{ij}^{C}=\begin{pmatrix} - & 0.6131 & 0.5798 & 0.5759 & 0.7129 & 0.6537 \\ 0.6083 & - & 0.5529 & 0.5967 & 0.6921 & 0.5783 \\ 0.6190 & 0.6190 & - & 0.6560 & 0.7359 & 0.6583 \\ 0.5437 & 0.5614 & 0.5606 & - & 0.6959 & 0.6075 \\ 0.5306 & 0.4890 & 0.4845 & 0.5391 & - & 0.4929 \\ 0.5937 & 0.5021 & 0.5198 & 0.5890 & 0.6390 & - \end{pmatrix}$$

Exploitation Process:
We use the fuzzy quantifier "most" with the pair (0.3, 0.8) and (0.7, 1), i.e., the corresponding OWA operator with the weighting vector $W = [w_1, w_2, w_3, w_4, w_5]$.
Calculation of W:

$$w_k = Q\left(\frac{k}{m}\right) - Q\left(\frac{(k-1)}{m}\right) \forall k$$

being the membership function of a non-decreasing proportional quantifier Q, as follows:

$$Q(x) = \begin{cases} 0, & 0 \le x < a, \\ \frac{x-a}{b-a}, & a \le x \le b, \\ 1, & b < x \le 1, \end{cases}$$

$$Q(x) = \begin{cases} 0, & 0 \le x < 0.3, \\ \frac{x-0.3}{0.5}, & 0.3 \le x \le 0.8, \\ 1, & x \ge 0.8, \end{cases}$$

With $m = 5, k = 1, 2, 3, 4, 5. W = \left[0, \frac{1}{5}, \frac{2}{5}, \frac{2}{5}, 0\right]$. The quantifier guided choice degrees of decision actions acting over the collective fuzzy relation give the following values.

	a_1	a_2	a_3	a_4	a_5	a_6
Q^{GDD_I}	0.63148	0.6261	0.68056	0.61488	0.50724	0.54394

These values represents the dominance that one decision actions has over the "most" decision actions according to "at least half" of the attributes.

Clearly the maximal set is: $X^{QGDD} = \{a_3\}$. Therefore, the selection set of decision actions for selection procedure is the singleton $\{a_3\}$. Ranking of crops for their profitability is $\langle a_3, a_1, a_2, a_4, a_6, a_5 \rangle$.
Similarly for the fuzzy quantifier "most" with the pair (0.7, 1) and $m = 5,\ k = 1, 2, 3, 4, 5.$ $W = \left[0, 0, 0, \frac{1}{3}, \frac{2}{3}\right]$. The quantifier guided choice degrees of decision acting over the collective fuzzy relation givethe following values.

	a_1	a_2	a_3	a_4	a_5	a_6
Q^{GDD_I}	0.6736	0.6158	0.6839	0.6366	0.5081	0.6225

These values represents the dominance that one decision action has over the "most" decision actions according to "at least half" of the attributes.

Clearly the maximal set is: $X^{QGDD} = \{a_3\}$. Therefore, the selection set of decision actions for selection procedure is the singleton $\{a_3\}$. rankingof crops for their profitability is $\langle a_3, a_1, a_4, a_6, a_2, a_5 \rangle$.

6. Conclusions

In this paper we developed the application of MODM problem in farming system, where the information supplied by the group of experts (farmers) is modeled in terms of fuzzy relations. These fuzzy relations are then fused into single fuzzy relation. The concept of fuzzy majority for the aggregation and exploitation of the information in decision making is used.A quantifier guided choice degree of decision actions is used to quantify the dominance that one decision action has over all others in a fuzzy majority sense.

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