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On Harmonious Colouring of Line Graph of Star Graph Families

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Research Article

Abstract: In this paper we discuss the harmonious coloring and harmonious chromatic number of line graph of Central graph, Middle graph and Total graph of star graph denoted by $L[C(K_{1,n})]$, $L[M(K_{1,n})]$ and $L[T(K_{1,n})]$ respectively.

Keywords: Central graph, Middle graph, Total graph, Line graph, Harmonious coloring, Harmonious chromatic number.

1. Introduction

The first paper on harmonious graph coloring was published in 1982 by Frank Harray and M.J. Plantholt [8].however, the proper definition of this notion is due to J. E. Hopcroft and M.S. Krishnamoorthy [9] in 1983. K. Thilagavathi and J. V. Vivin [11] published a paper "Harmonious coloring of graphs" in 2006. K. Thilagavathi and J. Vernold Vivin, [16] published a paper "On Harmonious coloring of Line graph of central graph of paths" in 2009.

In this paper we discuss about the harmonious chromatic number of line graphs of $C(K_{1,n})$,$M(K_{1,n})$ and $T(K_{1,n})$.

2. Preliminaries

All graphs considered here are undirected. For a given graph G=(V,E) we do a operation on G, by subdividing each edge exactly once and joining all the non adjacent vertices of G,the graph obtained by this process is called Central graph [1,13,14,15] of G denoted by C(G). Let G be a graph with vertex set V(G) and edge set E(G).The Middle graph [10] of G, denoted by M(G) is defined as follows. The vertex set M(G) is V(G)∪E(G).Two vertices x,y in the vertex set M(G) are adjacent in M(G) in case one of the following holds: (i) x,y are in E(G) and x,y are adjacent in G.(ii) x is in V(G), y is in E(G) ,and x,y are incident in G. Let G be a graph with vertex set V(G) and edge set E(G).The total graph [7,10] of G,denoted by T(G) is defined as follows. The vertex set T(G) is V(G)∪E(G).Two vertices x,y in the vertex set T(G) are adjacent in T(G) in case one of the following holds: (i) x,y are in V(G) and x is adjacent to y in G (ii) x,y are in E(G) and x,y are adjacent in G.(ii) x is in V(G), y is in E(G) ,and x,y are incident in G. The Line graph [3,7,10,16] of G denoted by L(G) is the graph with vertices are the edges of G with two vertices of L(G) adjacent whenever the corresponding edges of G are adjacent. A harmonious coloring [2,4,5,6,8,12,16] of a simple graph G is proper vertex coloring such that each pair of colors appears together on at most one edge. The harmonious chromatic number χ_H(G) is the least number of colors in such coloring.

3. Harmonious chromatic number of $L[C(K_{1,n})]$

Theorem 3.1. For any star graph $K_{1,n}$, $\chi_H\left(C\left(K_{1,n}\right)\right)=2n+1$.

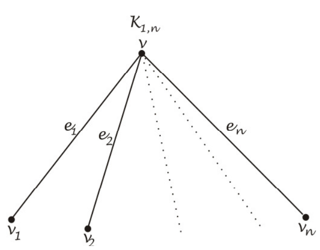

Fig 3(a)

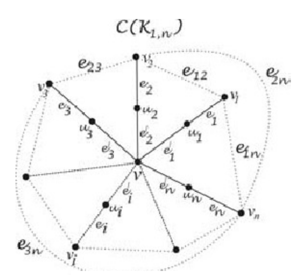

Fig 3(b)

Proof. Let. $V\left(K_{1,n}\right)=\{v_0, v_1, ...v_n\}$. By the definition of central graph, each edge v_0v_i for $1\le i\le n$ of $K_{1,n}$ is subdivided by the vertex u_i in $C\left(K_{1,n}\right)$ and then we have $V\left[C\left(K_{1,n}\right)\right]=\{v_0\}\cup\{u_i / 1\le i\le n\}\cup\{v_i / 1\le i\le n\}$.In $C\left(K_{1,n}\right)$ the vertices v_0 and u_i $1 \le i \le n$ induce a clique on (n+1) vertices in $C(K_{1,n})$. Assign C_0 and C_i $(1 \le i \le n)$ to the vertices v_0 and u_i , $1 \le i \le n$ .Also assign C_{n+i} to the vertices $\{V_i / 1 \le i \le n\}$ since the vertices $\{V_i / 1 \le i \le n\}$ induces a clique on n vertices. Then the above said coloring is harmonic with minimum number of colors.

$$\therefore \chi_H(C(K_{1,n})) = n + 1 + n = 2n + 1.$$

Theorem 3.2. For any star graph $K_{1,n}$, $\chi_H\{L(C(K_{1,n}))\} = \frac{n^2+2n}{2}$, if n is even

$= \frac{n^2+2n+1}{2}$,if nis odd .$\forall n \ge 3$

Proof. Let. $V(K_{1,n}) = \{v_0, v_1, ...v_n\}$. By the definition of central graph, each edge $v_0 v_i$ for $1 \le i \le n$ of $K_{1,n}$ is subdivided by the vertex u_i in $C(K_{1,n})$. Clearly the number of vertices in $C(K_{1,n})$= 2n+1 and the number of edges in $C(K_{1,n}) = \frac{n^2+3n}{2}$. The edges joined by $v_0 u_i$ and $u_i v_i$ are denoted by e_{0i} and e_{ij} respectively. Then by the definition of line graph all the edges of central graph becomes the vertices of $L(C(K_{1,n}))$.(i.e.) $V\{L(C(K_{1,n}))\}$ ={ e_{0i} }∪{ e_{ij} }for $1 \le i \le n$, $1 \le j \le n$. Here K_1 ={ $e_{0i} / 1 \le i \le n$ }, K_2 ={ $e_{ij} / i \ne j. 1 \le i \le n, 1 \le j \le n$ }, K_3 ={ $e_{ij} / i = j. 1 \le i \le n, 1 \le j \le n$ }, moreover $V\{L(C(K_{1,n}))\} = K_1 \cup K_2 \cup K_3$ where each K_1, K_2 and K_3 are distinct. The each vertex set of K_1 and K_3 forms clique on n vertices. Therefore in any harmonious coloring we need $2n$ colors to color the vertices of $K_1 \cup K_3$.

Case(i) If n is even.

The vertex set of K_2 forms a clique on $\binom{n}{2}$ vertices and there exists a vertex v of K_2 ,adjacent with some vertices of K_3 but not with K_1 .In any harmonious coloring $\binom{n}{2} - \frac{n}{2}$ colors are needed to color the vertices of K_2. $\therefore$

$$\chi_H\{L(C(K_{1,n}))\} = 2n + \binom{n}{2} - \frac{n}{2} = \frac{n^2+2n}{2}.$$

Case(ii) If n is odd.

In this case also the vertex set of K_2 forms a clique on $\binom{n}{2}$ vertices and there exists a vertex v of K_2 ,adjacent with some vertices of K_3 but not with K_1 .In any harmonious coloring $\binom{n}{2} - \frac{n-1}{2}$ colors are needed to color the vertices of K_2 .

$$\therefore \chi_H\{L(C(K_{1,n}))\} = 2n + \binom{n}{2} - \frac{n-1}{2} = \frac{n^2+2n+1}{2}.$$

4. Harmonious chromatic number of L[M(K$_{1,n}$)]

Theorem 4.1. For any star graph $K_{1,n}$, $\chi_H(M(K_{1,n})) = n + 2 \ \forall n \ge 3$..

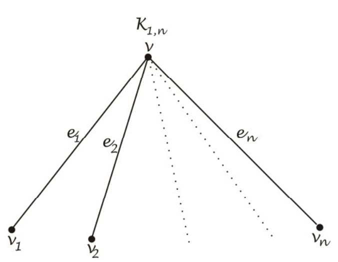

Fig 4(a)

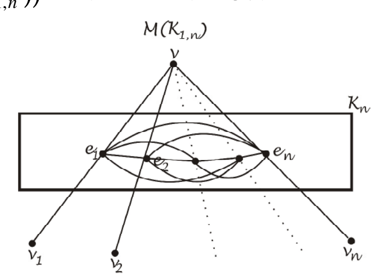

Fig 4(b)

Proof. Let $V(K_{1,n})=\{v_0, v_1, ...v_n\}$. By the definition of middle graph, each edge v_0v_i for $1 \le i \le n$ of $K_{1,n}$ is subdivided by the vertex e_i in $M(K_{1,n})$. we have $V(M(K_{1,n}))= \{V(K_{1,n})\} \cup \{e_i / 1 \le i \le n\}$.In $M(K_{1,n})$ the vertices v_0 and e_i $1 \le i \le n$ induce a clique of order (n+1) in $M(K_{1,n})$.Assign C_0 and C_i ($1 \le i \le n$) to the vertex set $v_0 \cup \{e_i / 1 \le i \le n\}$and C_{n+1} to the pendant vertices v_i ($1 \le i \le n$) for .Then the above said coloring is a harmonious coloring with minimum number of colors. $\therefore$ $\chi_H(M(K_{1,n})) = n+2$.

Theorem 4.2. For any star graph $K_{1,n}$, $\chi_H\{L(M(K_{1,n}))\} = \dfrac{n^2+3n}{2}, \forall n \ge 3$.

Proof. Let $V(K_{1,n})=\{v_0, v_1, ...v_n\}$. By the definition of middle graph, each edge v_0v_i for $1 \le i \le n$ of $K_{1,n}$ is subdivided by the vertex e_i, $1 \le i \le n$ in $M(K_{1,n})$. Clearly the number of vertices in $M(K_{1,n})$= 2n+1 and the edges joined by v_0e_i and e_iv_i are denoted by e_{0i} and e_{ij} respectively. Then by the definition of line graph all the edges of middle graph becomes the vertices of $L(M(K_{1,n}))$(i.e.)$V\{L(M(K_{1,n}))\}=\{e_{0i}\} \cup \{e_{ij}\}$ for $1 \le i \le n, 1 \le j \le n$. Here K_1={ $e_{0i} / 1 \le i \le n$ } K_2={ $e_{ij} / i \ne j.1 \le i \le n, 1 \le j \le n$ }, K_3={ $e_{ij} / i = j.1 \le i \le n, 1 \le j \le n$ },moreover $V\{L(M(K_{1,n}))\}$= $K_1 \cup K_2 \cup K_3$ where each K_1, K_2 and K_3 are distinct. The each vertex set of K_1 and K_3 forms clique on n vertices. Therefore in harmonious coloring we need 2n colors to color the vertices of $K_1 \cup K_3$. And K_2 forms a clique on $\binom{n}{2}$ vertices.

$$\therefore \chi_H\{L(M(K_{1,n}))\} = \binom{n}{2} + 2n = \frac{n^2+3n}{2}.$$

5. Harmonious chromatic number of L[T(K$_{1,n}$)]

Theorem 5.1. For any star graph $K_{1,n}$, $\chi_H(T(K_{1,n})) = 2n+1$, $\forall n \ge 3$.

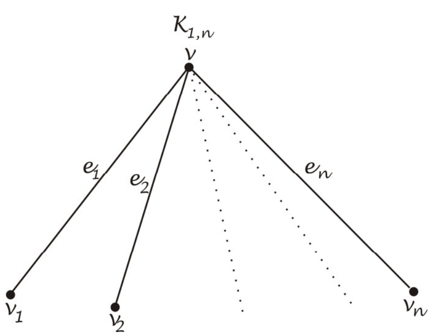

Fig 5(a)

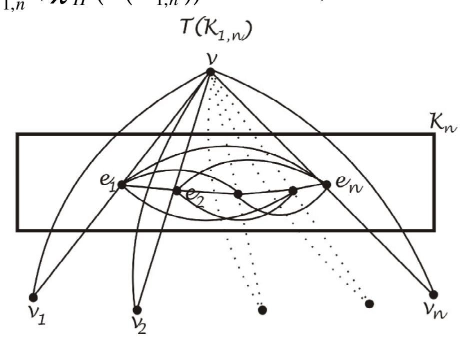

Fig 5(b)

Proof. Let $V(K_{1,n})=\{v_0, v_1, ...v_n\}$. .By the definition of total graph, each edge v_0v_i for $1 \le i \le n$ of $K_{1,n}$ is subdivided by the vertex e_i in $T(K_{1,n})$. we have $V(T(K_{1,n}))= \{v_0\} \cup \{e_i / 1 \le i \le n\} \cup \{v_i / 1 \le i \le n\}$.In $T(K_{1,n})$ the vertices v_0 and e_i, $1 \le i \le n$ induce a clique on (n+1) vertices in $T(K_{1,n})$. Assign C_0 and C_i, $1 \le i \le n$ to the vertex set $v_0 \cup \{e_i / 1 \le i \le n\}$.Also assign C_{n+i} to the vertices $\{v_i / 1 \le i \le n\}$ since the vertices $\{v_i / 1 \le i \le n\}$ induces a clique on n vertices. Then the above said coloring is harmonic with minimum number of colors.

$\therefore \chi_H(T(K_{1,n})) = n+1+n = 2n+1$..

Theorem 5.2. For any star graph $K_{1,n}$, $\chi_H\{L(T(K_{1,n}))\} = \frac{n^2+5n}{2}$, $\forall n \geq 3$.

Proof. Let $V(K_{1,n}) = \{v_0, v_1, ...v_n\}$. .By the definition of total graph, each edge $v_0 v_i$ for $1 \leq i \leq n$ of $K_{1,n}$ is subdivided by the vertex $e_i^{'}$ in $T(K_{1,n})$. we have $V(T(K_{1,n})) = \{v_0\} \cup \{e_i^{'} / 1 \leq i \leq n\} \cup \{v_i / 1 \leq i \leq n\}$ clearly the number of vertices in $T(K_{1,n})$= 2n+1 and the edges joined by $v_0 e_i, v_0 e_i^{'}$, and $e_i^{'} v_i$ are denoted by $e_{0i}, e_{0i}^{'}$ and e_{ij} respectively. Then by the definition of line graph all the edges of total graph becomes the vertices of $L(T(K_{i,n}))$. (i.e.) $V\{L(T(K_{1,n}))\} = \{e_{0i}\} \cup \{e_{0i}^{'}\} \cup \{e_{ij}\}$ for $1 \leq i \leq n, 1 \leq j \leq n$ .. Here $K_1 = \{e_{0i}^{'} / 1 \leq i \leq n\}$ $K_2 = \{\{e_{0i}\} \cup \{e_{ij}\}, i \neq j / 1 \leq i \leq n, 1 \leq j \leq n\}$, K_3 ={ $e_{ij} / i = j . 1 \leq i \leq n, 1 \leq j \leq n$ },moreover $V\{L(T(K_{1,n}))\}$= $K_1 \cup K_2 \cup K_3$ where each K_1, K_2 and K_3 are distinct. The each vertex set of K_1 and K_3 forms clique on n vertices. Therefore in harmonious coloring we need 2n colors to color the vertices of $K_1 \cup K_3$. Also K_2 forms a clique on $\frac{n(n+1)}{2}$ vertices. .

$$\therefore \chi_H\{L(T(K_{1,n}))\} = 2n + \frac{n(n+1)}{2} = \frac{n^2+5n}{2}.,$$

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