A Note on fuzzy Weakly Volterra Spaces

G. Thangaraj*, S. Soundararajan#

*Department of Mathematics, Thiruvalluvar University, Vellore- 632 115, Tamilnadu, INDIA.

#Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi- 635 752, Tamilnadu, INDIA.

*Corresponding Address:

g.thangaraj@rediffmail.com

Research Article

Abstract: In this paper we investigate several characterizations of fuzzy weakly Volterra spaces and study the conditions under which a fuzzy topological space becomes a fuzzy weakly Volterraspace.

Keywords: Fuzzy Baire space, fuzzy D- Baire space, fuzzy σ-Bairespace, fuzzy submaximal space,fuzzy irresolvable space, fuzzy almost resolvablespace, fuzzy P-space, fuzzy Volterra space, fuzzy weakly Volterra space.

2000AMS Classification: 54 A40, 03E72.

1. Introduction

In order to deal with uncertainties, the idea of fuzzy sets and fuzzy set operations was introduced by L.A.Zadehin his classical paper [17] in the year 1965. This inspired mathematicians to fuzzify Mathematical Structures. The first notion of fuzzy topological space had been defined by C.L.Chang[3] in 1968. Since then much attention has been paid to generalize thebasic concepts of general topology in fuzzy settingand thus a modern theory of fuzzy topology has been developed. The conceptsof Volterra spaceshave been studied extensively in classical topology in [5],[6] , [7] and [8]. The concept of Volterra spaces and fuzzy weakly Volterra spaces in fuzzy setting was introducedand studied by the authors in [13]. In this paper we discuss several characterizations of fuzzy weaklyVolterraspaces and study under what conditions a fuzzy topological space becomes a fuzzy weaklyVolterra space ? and fuzzy Baire space, fuzzy σ -Baire space, fuzzy submaximal space , fuzzy hyperconnected space, fuzzy strongly irresolvable Baire space and fuzzy P-space are considered for this work.

2. Preliminaries

Now we introduce some basic notions and results used in the sequel .In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to CHANG.

Definition 2.1: Let λ and μ be any two fuzzy sets in a fuzzy topologicalspaces (X,T). Then we define $\lambda \vee \mu : X \to [0,1]$ as follows : $(\lambda \vee \mu) (x) = \text{Max} \{ \lambda (x), \mu (x) \}$. $\lambda \wedge \mu : X \to [0,1]$ as follows : $(\lambda \wedge \mu) (x) = \text{Min} \{ \lambda (x), \mu (x) \}$.

Definition 2.2: Let(X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). We define

(i). $\text{Int} (\lambda) = \vee \{ \mu / \mu \leq \lambda, \mu \in T \}$,

(ii). $\text{Cl}(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1- \mu \in T\}$.

Lemma 2.1[1] : For afuzzy setλ of a fuzzy topological space X,

(a). $1 - \text{Int} (\lambda)= \text{Cl} (1 - \lambda)$,and

(b). $1 - \text{Cl} (\lambda) = \text{Int} (1 - \lambda)$.

Definition 2.3 [9]: A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy dense set if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$.

Definition 2.4 [9]: A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy nowhere dense set if there exists no non - zero fuzzy open set μ in (X,T) such that $\mu < \text{cl} (\lambda)$. That is, $\text{int cl} (\lambda) = 0$.

Definition 2.5 [2]: Let(X,T)bea fuzzy topological space and λ be a fuzzy set in X. Then λ is called a fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty} \lambda_i$ for each$\lambda_i \in T$.

Definition 2.6 [2]: Let(X,T)bea fuzzy topological space andλ be a fuzzy set in X. Then $\lambda$is called a fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty} \lambda_i$, for each $1 - \lambda_i \in T$.

Definition 2.7 [9]: A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy first category set if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where λ_i's are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category.

Definition 2.8[2]: A fuzzy topological space (X,T) is called a fuzzy submaximal space if for each fuzzy set λ in (X,T) such that $\text{cl} (\lambda) = 1$, then $\lambda \in T$ in (X,T).

Definition 2.9 [12]: Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called a fuzzy σ -nowhere dense set if λ is a fuzzy F_σ- set in (X,T) such that $\text{int} (\lambda) = 0$.

3. Fuzzy Weakly Volterra Spaces

Definition 3.1 [13]: A fuzzy topological space (X,T) is called a fuzzy Volterra space if $\text{Cl}(\wedge_{i=1}^{N} (\lambda_i)) = 1$, where λ_i' s are fuzzy dense and fuzzy G_δ sets in (X,T) .

Definition 3.2 [13]: A fuzzy topological space (X,T) is called a fuzzy weakly Volterra space if $\text{Cl}(\wedge_{i=1}^{N} (\lambda_i)) \neq 0$,where λ_i' s are fuzzy dense and fuzzy G_δ sets in (X,T) .

Theorem 3.1 [12]: In a fuzzy topological space (X,T) , a fuzzy set λ is fuzzy σ - nowhere dense if and only $1 - \lambda$ is a fuzzy dense and fuzzy G_δ -set.

Proposition 3.1: If the fuzzy topological space (X,T) is a fuzzy σ -second category space, then (X,T) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)'s (i = 1 toN) be fuzzy dense and fuzzy G_δ-sets in (X,T). Then,by theorem 3.1, $(1-\lambda_i)$'s are fuzzy σ - nowhere dense sets in (X,T). Let $\mu_\alpha(\alpha = 1$ to ∞) be fuzzy σ - nowhere dense sets in (X,T) in which let us take the first N(μ_α)'s as $(1-\lambda_i)$. Since (X,T) is a fuzzy σ - second category space, $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$.Then $1-[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] \neq 0$. This implies that $\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha) \neq 0$.Thenwe have $cl(\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha)) \neq 0$. Sincecl $(\wedge_{\alpha=1}^{\infty}((1-\mu_\alpha)) \leq cl(\wedge_{\alpha=1}^{N}(1-\mu_\alpha))$, $cl(\wedge_{\alpha=1}^{N}(1-\mu_\alpha)) \neq 0$.Then we have $cl(\wedge_{i=1}^{N}(\lambda_i)) \neq 0$, where λ_i' s are fuzzy dense and fuzzy G_δ sets in (X,T) . Therefore(X, T) is a fuzzy weakly Volterra space.

Definition 3.3 [15] : A fuzzy topological space (X,T) is called a fuzzy almost resolvable space if $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets λ_i' s in (X,T) are such that int (λ_i) = 0.Otherwise (X,T) is called a fuzzy almost irresolvable space.

Proposition 3.2: If the fuzzy topological space (X,T) is a fuzzy almost irresolvable space, then (X,T) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)' s (i = 1 toN) be fuzzy dense and fuzzy G_δ-sets in (X,T).Now cl(λ_i)= 1, implies that int ($1-\lambda_i$) = 0.Since(X,T)is afuzzy almostirresolvable space,$\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, where the fuzzy sets (μ_α)'s in (X,T) are such that int ((μ_α) = 0. Let us take the first N(μ_α)'s as $(1-\lambda_i)$ in (X,T). Now$\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, implies that $1-[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] \neq 0$. Then $\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha) \neq 0$.and hence cl [$\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha)] \neq 0$.Sincecl $(\wedge_{\alpha=1}^{\infty}((1-\mu_\alpha)) \leq cl(\wedge_{\alpha=1}^{N}(1-\mu_\alpha))$, we havecl $[\wedge_{\alpha=1}^{N}(1-\mu_\alpha)] \neq 0$. That is,$cl[\wedge_{i=1}^{N}(\lambda_i)] \neq 0$, where λ_i' s are fuzzy dense and fuzzy G_δ sets in (X,T) . Therefore (X, T) is a fuzzy weakly Volterra space.

Theorem 3.2 [10]: If λ is a fuzzy dense and fuzzy G_δ -setin a fuzzy topological space (X,T),then 1 – λ is a fuzzy first category set in (X,T).

Proposition 3.3: If the fuzzy D-Baire space (X,T) is a fuzzy secondcategory space, then (X,T) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)' s (i = 1 toN) be fuzzy dense and fuzzy G_δ-sets in (X,T).Then ,by theorem 3.2, $(1-\lambda_i)$'s are fuzzy first category sets in (X,T).Since (X,T)is afuzzy D-Bairespace,$(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Since (X,T)is afuzzy second category space,$\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, where (μ_α)'sare fuzzy nowhere dense sets in (X,T).Let us take the first N(μ_α)'s as $(1-\lambda_i)$ in (X,T). Then,$\vee_{i=1}^{N}(1-\lambda_i) \leq \vee_{\alpha=1}^{\infty}(\mu_\alpha)$ and $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, implies that$\vee_{i=1}^{N}(1-\lambda_i) \neq 1$. Then, we have $\wedge_{i=1}^{N}(\lambda_i) \neq 0$ and hence $cl[\wedge_{i=1}^{N}(\lambda_i)] \neq 0$, where λ_i' s are fuzzy dense and fuzzy G_δ sets in (X,T) . Therefore (X, T) is a fuzzy weakly Volterra space.

Proposition 3.4: If the fuzzy topological space (X,T) is not a fuzzyweakly Volterra space, then (X,T) is a fuzzy σ -first category space.

Proof: Let(μ_α)'s (α =1 to ∞) be fuzzy σ - nowhere dense sets in a fuzzy topological space (X,T) which is not a fuzzy weakly Volterra space. Now we claim that $\vee_{\alpha=1}^{\infty}(\mu_\alpha) = 1$.Now assume that $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$. Then, we have $\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha) \neq 0$.Since (μ_α)'sare fuzzy σ - nowhere dense sets in(X,T), by theorem 3.1, $(1-\mu_\alpha)$'sfuzzy dense and fuzzy G_δ-sets in (X,T).Now $\wedge_{\alpha=1}^{\infty}((1-\mu_\alpha) \leq \wedge_{\alpha=1}^{N}(1-\mu_\alpha)$,implies that$\wedge_{\alpha=1}^{N}(1-\mu_\alpha) \neq 0$.Letus denote $\lambda_\alpha = (1-\mu_\alpha)$. Then (λ_α)'s are fuzzy denseandfuzzy G_δ-sets in(X,T).Then,$\wedge_{\alpha=1}^{N}(\lambda_\alpha) \neq 0$.This implies that $cl[\wedge_{\alpha=1}^{N}(\lambda_\alpha)] \neq 0$. But this is a contradiction, since(X,T) is not a fuzzy weakly Volterraspace, $cl[\wedge_{\alpha=1}^{N}(\lambda_\alpha)] = 0$.Hence we must have$\vee_{\alpha=1}^{\infty}(\mu_\alpha) = 1$. Therefore (X,T) is a fuzzy σ - first category space.

Theorem 3.2 [15]: A fuzzy topological space (X,T) is a fuzzy resolvable space if and only if $\vee_{\alpha=1}^{N}(\lambda_\alpha) = 1$, where int [(λ_α)] = 0.

Proposition 3.5: If the fuzzy topological space (X,T) is not a fuzzyweakly Volterra space, then (X,T) is a fuzzy resolvable space.

Proof: Suppose that(X,T) is not a fuzzyweakly Volterra space. Then $cl[\wedge_{\alpha=1}^{N}(\lambda_\alpha)] = 0$, where (λ_α)'s are fuzzy dense and fuzzy G_δ-sets in (X,T).Now $\wedge_{\alpha=1}^{N}(\lambda_\alpha) \leq cl[\wedge_{\alpha=1}^{N}(\lambda_\alpha)]$,implies that $\wedge_{\alpha=1}^{N}(\lambda_\alpha) = 0$.Then $1-\wedge_{\alpha=1}^{N}(\lambda_\alpha) = 1$ and hence we have $\vee_{\alpha=1}^{N}(1-\lambda_\alpha) = 1$……(1).Since (λ_α)'s are fuzzy dense sets in (X,T), cl((λ_α) = 1. Then we have int [1− (λ_α)]= 0…….(2).

From (1) and (2), we have$\vee_{\alpha=1}^{N}(1-\lambda_\alpha) = 1$, where int [1− (λ_α)] = 0.Hence by theorem 3.3,(X,T) is a fuzzy resolvable space.

Theorem 3.3 [10]: If λ is a fuzzy dense and fuzzy open set in a fuzzy topological space, then $1-\lambda$ is a fuzzy nowhere dense set in (X,T).

Proposition 3.6: If the fuzzytopological space (X,T) is a fuzzysecond category fuzzy P-space, , then (X,T) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)' s (i = 1 toN) be fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X,T) is a fuzzy P-space , the

fuzzy G_δ-sets (λ_i)'s (i = 1 toN) are fuzzy open sets in (X,T). Then , (λ_i)' s (i = 1 toN) be fuzzy dense and fuzzy open sets (X,T).By theorem 3.3, $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Since (X,T) is a fuzzy second category space,$\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, where$(\mu_\alpha)$'s are fuzzy nowhere dense sets in(X,T).Letus take the first $N(\mu_\alpha)$'s as $(1-\lambda_i)$ in (X,T). Then,$\vee_{i=1}^{N}(1-\lambda_i) \leq \vee_{\alpha=1}^{\infty}(\mu_\alpha)$ and $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$,impliesthat$\vee_{i=1}^{N}(1-\lambda_i) \neq 1$.
This implies that $\wedge_{i=1}^{N}(\lambda_i) \neq 0$ and hence $cl[\wedge_{i=1}^{N}(\lambda_i)] \neq 0$, where λ_i' s are fuzzy dense and fuzzy G_δ sets in (X,T) . Therefore (X, T) is a fuzzy weakly Volterra space.

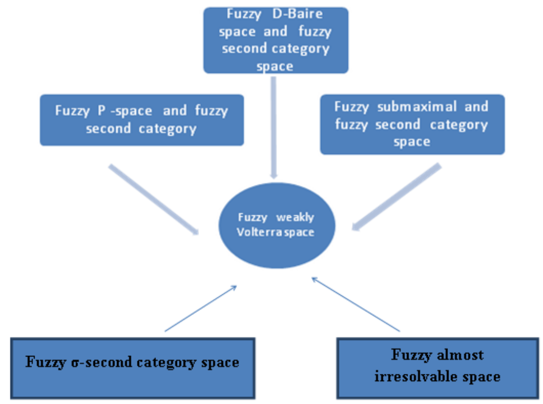
Proposition 3.7: If the fuzzy topological space (X,T) is a fuzzysecond category fuzzy submaximal space, then (X,T) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)' s (i = 1 toN) be fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X,T) is a fuzzy submaximal space , the fuzzy dense sets (λ_i)'s (i = 1 toN) are fuzzy open sets in (X,T). Then , (λ_i)' s (i = 1 toN) be fuzzy dense and fuzzy open sets (X,T).By theorem 3.3, $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Since (X,T) is a fuzzy second category space,$\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, where$(\mu_\alpha)$'s are fuzzy nowhere dense sets in(X,T). Let us take the first $N(\mu_\alpha)$'s as $(1-\lambda_i)$ in (X,T). Then,$\vee_{i=1}^{N}(1-\lambda_i) \leq \vee_{\alpha=1}^{\infty}(\mu_\alpha)$ and $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, implies that$\vee_{i=1}^{N}(1-\lambda_i) \neq 1$. This implies that $\wedge_{i=1}^{N}(\lambda_i) \neq 0$ and hence $cl[\wedge_{i=1}^{N}(\lambda_i)] \neq 0$, where λ_i' s are fuzzy dense and fuzzy G_δ sets in (X,T) . Therefore (X, T) is a fuzzy weakly Volterra space.

Proposition 3.8: If each fuzzy first category set is a fuzzy closed set in a fuzzy second category space (X,T), then (X,T) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)' s (i = 1 toN) be fuzzy dense and fuzzy G_δ-sets in (X,T).Then, by theorem 3.2, $(1-\square_{\square})$'s are fuzzy first category sets in (X,T).By hypothesis, $(1-\square_{\square})$'s are fuzzy closed sets and hence(λ_i)' s are fuzzy open sets in (X,T). Now (λ_i)' s (i = 1 toN) are fuzzy dense and fuzzy open sets in (X,T).Then by theorem 3.3, $(1-\square_{\square})$'s are fuzzy nowhere dense sets in (X,T). Since (X,T) is a fuzzy second category space,$\vee_{\square=1}^{\infty}(\mu_{\square}) \neq 1$, where$(\mu_{\square})$'s are fuzzy nowhere dense sets in(X,T), $\vee_{\square=1}^{\infty}(\mu_{\square}) \neq 1$, where$(\mu_{\square})$'s are fuzzy nowhere dense sets in(X,T).Letus take the first $N(\mu_{\square})$'s as $(1-\lambda_i)$ in (X,T). Then,$\vee_{\square=1}^{N}(1-\lambda_i) \leq \vee_{\square=1}^{\infty}(\mu_{\square})$ and $\vee_{\square=1}^{\infty}(\mu_{\square}) \neq 1$, implies that$\vee_{\square=1}^{N}(1-\lambda_i) \neq 1$. This implies that $\wedge_{i=1}^{N}(\lambda_i) \neq 0$ and hence $cl[\wedge_{i=1}^{N}(\lambda_i)] \neq 0$, where λ_i' s are fuzzy dense and fuzzy G_δ sets in (X,T) . Therefore (X, T) is a fuzzy weakly Volterra space.

Remarks: The relationship among the classes of fuzzy σ-second category spaces, fuzzy almost irresolvable spaces, fuzzy D-Bairefuzzy second category spaces, fuzzy second category and fuzzy P-spaces and fuzzy weakly Volterra spaces can be summarized as follows:



4. Fuzzy weakly Volterra Spaces and functions

In this section by using fuzzy functions, we study under what conditions a fuzzy topological space becomes a fuzzy weakly Volterra space.

Lemma 4.1[3]: Let f : (X ,T) →(Y, S) be a mapping. For fuzzy sets λ and μ of (X ,T) and (Y, S) respectively, the following statements hold :

(1) $ff^{-1}(\mu) \leq \mu$;
(2) $f^{-1}f(\lambda) \geq \lambda$;
(3) $f(1-\lambda) \geq 1 - f(\lambda)$;
(4) $f^{-1}(1-\mu) = 1 - f^{-1}(\mu)$;
(5) If f is one - to - one , then $f^{-1}f(\lambda) = \lambda$;
(6) If f is onto , then $ff^{-1}(\mu) = \mu$;
(7) If f is one - to -one and onto, then $f(1-\lambda) = 1 - f(\lambda)$.

Lemma 4.2 [1] : Let f : (X ,T) →(Y, S) be a mapping and { $\square_{\square}$ } be a family of fuzzy sets of Y. Then,

(a).$f^{-1}(\cup_\alpha [\lambda_j]) = \cup_\alpha [f^{-1}(\square_j)]$,
(b) .$f^{-1}(\cap_{\square} [\lambda_j]) = \cap_{\square} [f^{-1}(\square_j)]$.

Lemma 4.3 [4] : Let f : (X ,T) →(Y, S) be a mapping and { A_j }, j∊ J be a family of fuzzy sets in X. Then,

(a). $f(\cup_{j\in J}[A_j]) = \cup_{j\in J}[f(A_j)]$,
(b).$f(\cap_{j\in J}[A_j]) \leq \cap_{j\in J}[f(A_j)]$.

Definition 4.1 [9]: A function f : (X,T) → (Y,S) froma fuzzy topological space (X,T) into another fuzzy topological space (Y, S) is called somewhat fuzzycontinuous if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$ implies that there exist a fuzzy open set δ in (X ,T) such that $\delta \neq 0$ and $\delta \leq f^{-1}(\lambda)$.

Definition 4.2[9] :A function f : (X,T) → (Y,S) from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is called somewhat fuzzy open if

$\lambda \in$ T and $\lambda \neq 0$ implies that there exists a fuzzy open set η in (Y,S) such that□ $\neq 0$ and $\eta \leq f(\lambda)$.

THEOREM 4.1 [ ]: Ifthefunction f : (X,T) → (Y,S) from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is fuzzy semi-continuous and somewhat fuzzy open function and if λ is a fuzzy nowhere dense set in (Y,S). then f^{-1} (λ) is a fuzzy nowhere dense set in (X,T).

Proposition 4.1: If the function f : (X,T) → (Y,S) from a fuzzy submaximal space (X,T)onto a fuzzy second category space (Y,S) is a fuzzy semi-continuous and asomewhat fuzzy open function, then(X, T) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)' s (i = 1 to ∞) be fuzzy nowhere dense sets in a fuzzy second category space (Y,S) . Then we have $\vee_{\square=1}^{\infty}(\lambda_i) \neq 1_Y$. Now f^{-1} ($\vee_{\square=1}^{\infty}(\lambda_i)$) $\neq f^{-1}(1_Y)$. Then,$\vee_{\square=1}^{\infty} f^{-1}(\lambda_i)$) $\neq 1_X$…(A).Since f :(X,T) → (Y,S) is fuzzy semi-continuous and somewhatfuzzy open function and (λ_i)'sare fuzzy nowhere dense setsin (Y,S),[f^{-1}(λ_i)]'s are fuzzy nowhere denses sets in (X,T). Then from (A),(X,T) is a fuzzy second category space. Hence (X,T) is a fuzzy submaximal and a fuzzy second category space.Therefore, by Proposition 3.7, (X, T) is a fuzzy weakly Volterra space.

Proposition 4.2 : If the function f : (X,T) → (Y,S) from a fuzzy P-space(X,T) onto a fuzzy second category space (Y,S) is a fuzzy semi-continuous and a somewhat fuzzy open function, then(X, T) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)' s (i = 1 to ∞) be fuzzy nowhere dense sets in a fuzzy second category space (Y,S) . Then we have $\vee_{\square=1}^{\infty}(\lambda_i) \neq 1_Y$. Now f^{-1} ($\vee_{\square=1}^{\infty}(\lambda_i)$) $\neq f^{-1}(1_Y)$. Then, $\vee_{\square=1}^{\infty} f^{-1}(\lambda_i)$) $\neq 1_X$…(A). Since f :(X,T) → (Y,S) is fuzzy semi-continuous and somewhat fuzzy open function and (λ_i)'sare fuzzy nowhere dense sets in (Y,S),[f^{-1}(λ_i)]'s are fuzzy nowhere denses sets in (X,T). Then from (A),(X,T) is a fuzzy second category space. Hence (X,T) is a fuzzy P-space and a fuzzy second category space. Therefore, by Proposition 3.6, (X, T) is a fuzzy weakly Volterraspace.

Proposition 4.3: If the function f : (X,T) → (Y,S) from a fuzzy D-Baire space (X,T) onto a fuzzy second category space (Y,S) is a fuzzy semi-continuous and a somewhat fuzzy open function, then(X, T) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)' s (i = 1 to ∞) be fuzzy nowhere dense sets in a fuzzy second category space (Y,S) . Then we have $\vee_{\square=1}^{\infty}(\lambda_i) \neq 1_Y$. Now f^{-1} ($\vee_{\square=1}^{\infty}(\lambda_i)$) $\neq f^{-1}(1_Y)$. Then, $\vee_{\square=1}^{\infty} f^{-1}(\lambda_i)$) $\neq 1_X$…(A). Since f :(X,T) → (Y,S) is fuzzy semi-continuous and somewhat fuzzy open function and (λ_i)'sare fuzzy nowhere dense sets in (Y,S), [f^{-1}(λ_i)]'s are fuzzy nowhere denses sets in (X,T). Then from (A),(X,T) is a fuzzy second category space. Hence (X,T) is a fuzzy P-space and a fuzzy second category space. Therefore, by Proposition 3.3, (X, T) is a fuzzy weakly Volterr aspace.

Theorem 4.2 [10]: If the function f : (X,T) → (Y,S) from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is somewhat fuzzy continuous, 1 -1 and onto function and if λ is a fuzzy nowhere dense set in (X,T) for any fuzzy set λ in (X,T), then f (λ) is a fuzzy nowhere dense set in (Y,S).

Proposition 4.4: If the function f : (X,T) → (Y,S) from a fuzzy second category space (X,T) onto a fuzzy submaximal (Y,S) is a somewhat fuzzy continuous, 1 -1 function , then(Y,S) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)' s (i = 1 to ∞) be fuzzy nowhere dense sets in a fuzzy second category space (X,T) . Then we have $\vee_{\square=1}^{\infty}(\lambda_i) \neq 1_X$. Now f ($\vee_{\square=1}^{\infty}(\lambda_i)$) $\neq f(1_X)$. Then,$\vee_{\square=1}^{\infty} f(\lambda_i)$) $\neq 1_Y$…..(A).Since f :(X,T) → (Y,S) is fuzzy is a somewhat fuzzy continuous, 1 -1 and onto function and if λ_i'are fuzzy nowhere dense sets in (X,T) then [f (λ_i)] ' s are fuzzy nowhere dense sets in (Y,S). Then from (A),(Y,S) is a fuzzy second category space. Hence (Y,S) is a fuzzy submaximal and a fuzzy second category space. Therefore, by Proposition 3.7, (Y,S) is a fuzzy weakly Volterra space.

Proposition 4.5: If the function f : (X,T) → (Y,S) from a fuzzy second category space (X,T) onto a fuzzy D-Bairespace (Y,S) is a somewhat fuzzy continuous, 1 -1 function , then(Y,S) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)' s (i = 1 to ∞) be fuzzy nowhere dense sets in a fuzzy second category space (X,T) . Then we have $\vee_{\square=1}^{\infty}(\lambda_i) \neq 1_X$. Now f ($\vee_{\square=1}^{\infty}(\lambda_i)$) $\neq f(1_X)$. Then,$\vee_{\square=1}^{\infty} f(\lambda_i)$) $\neq 1_Y$…..(A).Since f :(X,T) → (Y,S) is fuzzy is a somewhat fuzzy continuous, 1 -1 and onto function and if λ_i'are fuzzy nowhere dense setsin (X,T) then [f (λ_i)] 's are fuzzy nowhere dense sets in (Y,S). Then from (A),(Y,S) is a fuzzy second category space. Hence (Y,S) is a fuzzy D-Baire space and a fuzzy second category space. Therefore, by Proposition 3.3, (Y,S) is a fuzzy weakly Volterraspace.

Proposition 4.6: If the function f : (X,T) → (Y,S) from a fuzzy second category space (X,T) onto a fuzzy P-space (Y,S) is a somewhat fuzzy continuous, 1 -1 function , then(Y,S) is a fuzzy weakly Volterra space.

Proof: Let(λ_i)' s (i = 1 to ∞) be fuzzy nowhere dense sets in a fuzzy second category space (X,T) . Then we have $\vee_{\square=1}^{\infty}(\lambda_i) \neq 1_X$. Now f ($\vee_{\square=1}^{\infty}(\lambda_i)$) $\neq f(1_X)$.

Then, $\bigvee_{i=1}^{\infty} f(\lambda_i)) \neq 1_Y$…..(A).Since f :(X,T) → (Y,S) is fuzzy is a somewhat fuzzy continuous, 1 -1 and onto function and if λ_i'are fuzzy nowhere dense setsin (X,T) then [f (λ_i)] 's are fuzzy nowhere dense sets in (Y,S). Then from (A),(Y,S) is a fuzzy second category space. Hence (Y,S) is a fuzzy P-space and a fuzzy second category space. Therefore, by Proposition 3.6, (Y,S) is a fuzzy weakly Volterraspace.

References

1. K.K. Azad., On Fuzzy semi continuity, Fuzzy almost continuity and Fuzzy weakly continuity, J. Math. Anal, Appl, 82 (1981), 14 – 32.
2. G.Balasubramanian, Maximal fuzzy topologies, Kybernetika, No. 5, 31 (1995), 459 – 464.
3. C.L. Chang,Fuzzy Topological Spaces, J. Math.Anal.Appl. 24, (1968), 182 - 190.
4. David HFoster, Fuzzy Topological Groups, J.Math. Anal.Appl, 67 (1979),549 – 564.
5. David Gauldand ZbigniewPiotrowski, On Volterra Spaces, Far East J. Math. Sci., 1 (1993), No.2., 209 –214.
6. David Gauld, Sina Greenwood and ZbigniewPiotrowski, On VolterraSpaces-II, Papers on General Topology and Applications, 169 – 173, Ann. New York Acad. Sci.,806, (1996).
7. David Gauld, Sina Greenwood andZbigniewPiotrowski, On Volterra Spaces-III, Toplological Operations, Topology Proc. 23 (1998), 167 – 182.
8. Jiling Cao and David Gauld, VolterraSpaces Revisited, J. Aust.Math.Soc.79 (2005), 61 – 76.
9. G.Thangaraj and G. Balasubramanian, On somewhat fuzzy continuous functions, J. Fuzzy Math., Vol.11, No.2, (2003), 725 –736.
10. G.Thangarajand S.Anjalmose, A note on Fuzzy Bairespaces , Inter. J. Fuzzy Math. and Sys., Vol. 3, No.4, (2013), 269– 274 .
11. G.Thangarajand S.Anjalmose, On fuzzy D-Bairespaces , Ann. Fuzzy Math. And In for.Vol. 9, No. 1, (2013), 1– 7.
12. G.Thangaraj and E.Poongothai, On Fuzzy σ- Bairespaces ,Inter. J.Fuzzy Math.and Sys., Vol. 3, No.4, (2013), 275– 283 .
13. G.Thangarajand S.Soundararajan, On Fuzzy VolterraSpaces, J. Fuzzy Math., Vol.21, No.4, (2013), 895 – 904.
14. G.Thangaraj ,C.Anbazhaganand P.Vivakanandan, On Fuzzy P-Spaces, Weak Fuzzy P-Spaces and Fuzzy AlmostP-Spaces, Gen.Math. Notes ,Vol.18, No.2, October, (2013), 128–139.
15. G.Thangaraj, On Fuzzy Resolvabilityand FuzzyirresolvabilitySpaces in Fuzzy Topological Spaces, News Bull. Cal. Math. Soc., Vol. 31 (4-6), (2008) , 11–14.
16. G.Thangaraj and D.Vijayan, On Fuzzy AlmostResolvable and Fuzzy Almost Resolvable sspaces,(communicated to Inter. J. Statistika andMathematika).
17. L.A. Zadeh , Fuzzy Sets, Information and Control, Vol. 8, (1965) ,338 - 358.