A Periodic Review Inventory Model for Deteriorating Items with Price Dependent Demand and Partial Delay in Payment under Inflation

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Research Article

Abstract: The paper studies a period review inventory model for deteriorating items allowing shortage and under price inflation. Demand during a reorder interval is assumed to be price dependent and the inventory manager has the option to pay his dues in two installments within a reorder interval. The optimum stock height to be maintained and the optimum length of a reorder interval are obtained by maximizing the total expected profit over the planning horizon. The sensitivity of the decision variables to change in model parameters has been also investigated through numerical illustration.

Keywords and phrases: Periodic review inventory model; deteriorating items; price dependent demand rate; price inflation; partial delay in payment.

1. Introduction

In classical inventory models it is generally assumed that the inventory manager settles his account with the supplier as soon as the ordered quantity arrives. However, in today's business transactions it is frequently observed that the supplier allows his customer a grace period within which he can repay his dues without having to pay any interest, or may delay the payment beyond the permitted time in which case interest is charged. Since, before settling the account with the supplier, the inventory manager can sell the goods, accumulate revenue and earn interest, it makes economic sense for the manager to delay the settlement of his account to the last day of the permissible settlement period. Goyal [10] first developed an EOQ model under the condition of permissible delay in payments. Shinn *et al.* [5] extended the model by considering quantity discount for freight cost. Recently Aggarwal and Jaggi [7] and Hwang and Shinn [3] extended Goyal's model to consider deterministic inventory model with constant rate of deterioration. Later Jamal *et al.* [4] extended Aggarwal and Jaggi's model to allow for shortages. Pal and Ghosh (2006, 2007) studied deterministic inventory models with quantity dependent permissible delay period. Shah and Shah [2] developed probabilistic inventory model for deteriorating items when delay in payment is permitted. Ghosh (2007) investigated a stochastic inventory model with stock dependent demand under conditions of permissible delay in payments. The above models were developed under the assumption that inflation does not play a significant role on the inventory policy. However, from financial point of view, one may consider an inventory to be a capital investment, and, as such, it should compete with other assets for an organization's limited capital fund. It is, therefore, important to investigate how time-value of money influences various inventory policies. The first study in this direction has been reported by Buzacott (1975), who considered EOQ model with inflation, subject to different types of pricing policies. Misra (1979) developed a discounted-cost model and included internal (company) and external (general economy) inflation rates for various costs associated with an inventory system. Sarker and Pan (1994) surveyed the effects of inflation and the time value of money on order quantity with finite replenishment rate. Some studies were also conducted with variable demand, see, for example, Uthayakumar and Geetha (2009), Maity (2010), Vrat and Padmanabhan (1990), Datta and Pal (1991), Hariga (1995), Hariga and Ben-Daya (1996) and Chung (2003). In this paper, we consider a periodic review inventory model for deteriorating items allowing shortages and under inflation, when demand is price dependent. The inventory manager is allowed to pay his dues in two installments within a reorder interval, which is often observed in real life but has not been studied in literature. This basically amounts to giving the manager a loan without interest during two subsequent time periods, beyond which he has to pay interest. The paper is organized as follows. Section 2, gives the notations used in the model. The model is analyzed in section 3. In section 4, a sensitivity analysis is carried out. Finally, in section 4, a discussion on the model is given.

2. Notations

The following notations are used in the paper:

A=ordering cost; c=cost price per unit; Ic= holding cost per unit per unit time, where $0 < I < 1$;
p=selling price per unit; r=inflation rate; θ= deterioration rate; I_e= interest earned per annum; I_c= interest charged per annum ($I_c > I_e$); T_α= Time when α-fraction of total cost price is to be paid; $T_{1-\alpha}$ = Time when (1-α)-fraction of total cost price is to be paid; T= complete cycle length;
T_1 =time when on hand stock becomes zero;
H = length of the planning period;
$D(t)$=price dependent demand rate and denoted by
$D(t) = a - bpe^{rt}$,
where $a \succ\succ b$ such that $D(t) \geq 0 \ \ \forall t \in (0,T)$.

3. The Model

The inventory policy is to place an order at the beginning of each reorder interval and the order quantity is just sufficient to bring up the stock height to a certain level S. The decision variables of the policy are S and T.
We shall assume that $H/T = n$ is an integer, i.e. the planning horizon is divided into n reorder intervals.
In order to find the inventory level at any arbitrary time point t on the s-th reorder interval, $1 \leq s \leq n$, we note that

$$\frac{dI(t)}{dt} + \theta I(t) = -D_s \quad 0 \leq t \leq T_1$$

$$\frac{dI(t)}{dt} = -D_s \quad T_1 \leq t \leq T,$$

where $D_s = a - bpe^{r(s-1)T}$.

Using the boundary condition $I(T_1) = 0$, we get

$$I_s(t) = \frac{D_s}{\theta}\left(e^{\theta(T_1 - t)} - 1\right) \quad 0 \leq t \leq T_1$$

$$I_s(t) = D_s(T_1 - t) \quad T_1 \leq t \leq T \quad 1 \leq s \leq n.$$

Hence, $S = I_s(0) = \dfrac{D_s}{\theta}\left(e^{\theta T_1} - 1\right)$. We may, therefore, take the decision variables to be T_1 and T.

The different terms in the expression of the expected profit in [0, H] are as follows:

(i) ordering cost in [0,H] = $A + Ae^{rT} + ... + Ae^{r(n-1)T} = A\left(\dfrac{e^{rH} - 1}{e^{rT} - 1}\right)$

(ii) carrying cost in [0,H]= $Ic\int_0^{T_1} I_1(t)dt + Ice^{rT}\int_0^{T_1} I_2(t)dt + ... + Ice^{r(n-1)T}\int_0^{T_1} I_n(t)dt$

$$= \frac{Ic}{\theta^2}\left(e^{\theta T_1} - \theta T_1 - 1\right)\left\{D_1 + D_2 e^{rT} + D_n e^{r(n-1)T}\right\}$$

$$= \frac{Ic}{\theta^2}\left(e^{\theta T_1} - \theta T_1 - 1\right)K(T),$$

where $K(T) = \left\{D_1 + D_2 e^{rT} + D_n e^{r(n-1)T}\right\} = a\dfrac{e^{rH} - 1}{e^{rT} - 1} - bp\dfrac{e^{2rH} - 1}{e^{2rT} - 1}$;

(iii) deteriorating cost in [0,H]= $\theta c\int_0^{T_1} I_1(t)dt + \theta ce^{rT}\int_0^{T_1} I_2(t)dt + ... + \theta ce^{r(n-1)T}\int_0^{T_1} I_n(t)dt$

$$= \frac{\theta c}{\theta^2}\left(e^{\theta T_1} - \theta T_1 - 1\right)\left\{D_1 + D_2 e^{rT} + D_n e^{r(n-1)T}\right\}$$

$$= \frac{\theta c}{\theta^2}\left(e^{\theta T_1} - \theta T_1 - 1\right)K(T);$$

(iv) shortage cost in [0,H]= $-s\left(I_1(T) + e^{rT}I_2(T) + ... + e^{r(n-1)T}I_n(T)\right) = s(T - T_1)K(T)$;

(v) selling price in [0,H] = $pT_1\left\{D_1 + D_2 e^{rT} + D_n e^{r(n-1)T}\right\} + \left\{-pI_0(T) - pe^{rT}I_0(T) - pe^{r(n-1)T}I_n(T)\right\}$

$$= pT_1K(T) + p(T - T_1)K(T) = pTK(T);$$

(vi) purchasing cost in [0,H]

$$= cI_1(0) + ce^{rT}I_2(0) + ... + ce^{r(n-1)T}I_n(0) + \left\{-ce^{rT}I_0(T) - ce^{2rT}I_0(T) - ce^{rnT}I_n(T)\right\}$$

$$= \left\{\frac{c}{\theta}\left(e^{\theta T_1} - 1\right) + ce^{rT}(T - T_1)\right\}K(T);$$

(vii) Interest earned and interest charged in $[0,H]$:
Case1: $T_1 \le T_\alpha \le T_{1-\alpha} \le T$
In this case, the inventory manager earns interest on the goods he sells, and the interest earned is given by

$$I_e\left(PT_1(T_\alpha - T_1) + \left(PT_1 - \alpha\frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right)\right)(T_{1-\alpha}-T_\alpha) + \left(PT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)(T-T_{1-\alpha})\right)$$

$$= I_e K(T)\left\{pT_1(T-T_1) - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\left(T - T_{1-\alpha} + \alpha(T_{1-\alpha}-T_\alpha)\right)\right\}$$

However, the manager does not have to pay any interest to the supplier.

Case 2: $T_\alpha \le T_1 \le T_{1-\alpha} \le T$

(2a) If the total selling price in the interval $(0,T_\alpha)$ is greater than α-fraction of the total cost price, the inventory manager will be able to pay the first installment for settling the account in the s^{th} cycle , $1 \le s \le n$, i.e., the manager pays the first installment at T_α if

$$pe^{r(s-1)T} D_s T_\alpha \ge \alpha c e^{r(s-1)T}\frac{D_s}{\theta}\left(e^{\theta T_1}-1\right)$$

or, $T_1 \le \frac{1}{\theta}\log\left(1+\frac{\theta p T_\alpha}{\alpha c}\right) = T_{20}$, say.

Hence, the manager earns interest, but does not have to pay any interest. His earned interest is given by

$$I_e K(T)\left\{\left(pT_1 - \frac{\alpha c}{\theta}\left(e^{\theta T_1}-1\right)\right)(T_{1-\alpha}-T_1) + \left(pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right)(T-T_{1-\alpha})\right\}$$

(2b) If the total selling price in the interval $(0,T_\alpha)$ is less than α-fraction of the total cost price, the manager will not be able to pay the first installment at T_α. He can pay it only at $T_{1-\alpha}$, and during the intermittent period high interest will be charged on that amount. He can, however, continue to collect revenue on the sold items.
His interest earned is, therefore, given by

$$I_e K(T)\left\{pT_1(T_{1-\alpha}-T_1) + \left(pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right)(T-T_{1-\alpha})\right\},$$

i.e $$pe^{r(s-1)T} D_s T_\alpha \le \alpha c e^{r(s-1)T}\frac{D_s}{\theta}\left(e^{\theta T_1}-1\right)$$

and the interest charged is

$$I_c K(T)\frac{\alpha c}{\theta}\left(e^{\theta T_1}-1\right)(T_{1-\alpha}-T_\alpha).$$

In this case $T_1 \ge T_{20}$.

Case 3: $T_\alpha \le T_{1-\alpha} \le T_1 \le T$

(a) If in the s^{th} cycle the total selling price in the interval $(0,T_\alpha)$ is greater than α-fraction of the total cost price, i.e. $pe^{r(s-1)T} D_s T_{1-\alpha} \ge ce^{r(s-1)T}\frac{D_s}{\theta}\left(e^{\theta T_1}-1\right)$, so that the manager is able to pay the first installment to the supplier at T_α, and the total selling price in $(0,T_{1-\alpha})$ is greater than total cost price i.e. $pe^{r(s-1)T} D_s T_\alpha \ge \alpha c e^{r(s-1)T}\frac{D_s}{\theta}\left(e^{\theta T_1}-1\right)$, then T_1 satisfies

$T_1 \le \frac{1}{\theta}\log\left(1+\frac{\theta p T_\alpha}{\alpha c}\right) = T_{30}$, say, and $T_1 \le \frac{1}{\theta}\log\left(1+\frac{\theta p T_{1-\alpha}}{c}\right) = T_{31}$, say,

i.e $T_1 \le \min(T_{30}, T_{31})$.

In this case, the interest earned in [0, H] is

$$I_e K(T)\left\{pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right\}(T-T_1)\ ,$$

while the interest charged is 0.

(b) In the s^{th} cycle, if the total selling price in the interval $(0,T_\alpha)$ greater than α-fraction of cost price, but the total selling price in $(0,T_{1-\alpha})$ is less than the total cost price, i.e. $T_{30} \le T_1 \le T_{31}$, the manager can pay the first installment in time, but not the second installment. Hence, he will earn interest as well as pay interest to the supplier. The interest earned is given by

$$I_e K(T)\left\{pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right\}(T-T_1)$$

And the interest paid is

$$I_c K(T)\frac{(1-\alpha)c}{\theta}\left(e^{\theta T_1}-1\right)(T-T_1).$$

(c) In the s^{th} cycle, if total selling price in the interval $(0,T_\alpha)$ is less than α-fraction of cost price, i.e. the customer is not able to pay the first installment, and the total selling price in $(0,T_{1-\alpha})$ is less than the total cost price, i.e $pe^{r(s-1)T}D_sT_\alpha \le \alpha c e^{r(s-1)T}\frac{D_s}{\theta}\left(e^{\theta T_1}-1\right)$ and $pe^{r(s-1)T}D_sT_{1-\alpha} \le c e^{r(s-1)T}\frac{D_s}{\theta}\left(e^{\theta T_1}-1\right)$, or, $T_1 \ge \max(T_{30},T_{31})$, then

interest earned in [0, H] $= I_e K(T)\left\{pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right\}(T-T_1)$

interest charged in [0,H] = $I_c K(T)\frac{c}{\theta}\left(e^{\theta T_1}-1\right)\{\alpha(T_1-T_\alpha)+(1-\alpha)(T_1-T_{1-\alpha})\}$

(d)) In the s^{th} cycle, if total selling price in the interval $(0,T_\alpha)$ less than α-fraction of cost and total selling price in $(0,T_{1-\alpha})$ is greater than total cost price, i.e.

$pe^{r(s-1)T}D_sT_\alpha \le \alpha c e^{r(s-1)T}\frac{D_s}{\theta}\left(e^{\theta T_1}-1\right)$ and $pe^{r(s-1)T}D_sT_{1-\alpha} \ge c e^{r(s-1)T}\frac{D_s}{\theta}\left(e^{\theta T_1}-1\right)$, or $T_{31} \le T_1 \le T_{30}$,

Interest earned in [0, H] = $I_e K(T)\left\{pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right\}(T-T_1)$

Interest Charged in [0,H]: $I_c K(T)\frac{\alpha c}{\theta}\left(e^{\theta T_1}-1\right)(T_{1-\alpha}-T_\alpha).$

Thus, the total profit in [0,H] is given by

$$P(T_1,T) = P_1(T_1,T) = I_e K(T)\left\{pT_1(T-T_1) - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)(T-T_{1-\alpha}+\alpha(T_{1-\alpha}-T_\alpha))\right\} + G(T_1,T), \text{ in case 1}$$

$$= P_2^1(T_1,T) = I_e K(T)\left\{\left(pT_1 - \frac{\alpha c}{\theta}\left(e^{\theta T_1}-1\right)\right)(T_{1-\alpha}-T_1) + \left(pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right)(T-T_{1-\alpha})\right\} =$$

$$+ G(T_1,T), \text{ in case 2(a)}$$

$$= P_2^2(T_1,T) = I_e K(T)\left\{pT_1(T_{1-\alpha}-T_1) + \left(pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right)(T-T_{1-\alpha})\right\}$$

$$- I_c K(T)\frac{\alpha c}{\theta}\left(e^{\theta T_1}-1\right)(T_{1-\alpha}-T_\alpha) + G(T_1,T), \text{ in case 2(b)}$$

$$P_3^1(T_1,T) = I_e K(T)\left\{pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right\}(T-T_1) + G(T_1,T)\text{, in case 3(a)}$$

$$= P_3^2(T_1,T) = I_e K(T)\left\{pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right\}(T-T_1) - I_c K(T)\frac{(1-\alpha)c}{\theta}\left(e^{\theta T_1}-1\right)(T-T_1),$$

$$+ G(T_1,T), \quad \text{in case 3(b)}$$

$$= P_3^3(T_1,T) = I_e K(T)\left\{pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right\}(T-T_1) - I_c K(T)\frac{c}{\theta}\left(e^{\theta T_1}-1\right)\times$$
$$\{\alpha(T_1-T_\alpha)+(1-\alpha)(T_1-T_{1-\alpha})\}+G(T_1,T), \quad \text{in case 3(c)}$$
$$= P_3^4(T_1,T) = I_e K(T)\left\{pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right\}(T-T_1) - I_c K(T)\frac{\alpha c}{\theta}\left(e^{\theta T_1}-1\right)(T_{1-\alpha}-T_\alpha)+G(T_1,T),$$

in case 3(d)

where
$$G(T_1,T) = K(T)\left\{pT - \frac{c}{\theta}\left(e^{\theta T_1}-1\right) - ce^{rT}(T-T_1) - s(T-T_1) - \frac{Ic+\theta c}{\theta^2}\left(e^{\theta T_1}-\theta T_1-1\right)\right\} - A\frac{e^{rH}-1}{e^{rT}-1}.$$

In each case, the optimum values of $T_1$and T are obtained by solving the equations

$$\frac{\partial P(T_1,T)}{\partial T_1}=0, \qquad \frac{\partial P(T_1,T)}{\partial T}=0.$$

Theorem 1: $P(T_1,T)$, *is a concave function of* T_1, *for given* T.

Proof: Since $K(T)\ge 0, T-T_{1-\alpha}\ge 0, T_{1-\alpha}-T_\alpha \ge 0,$ we have the following:

Case 1

$$\frac{\partial^2 P_1(T_1,T)}{\partial T_1^2} = \left[I_e K(T)\left\{-2p - c\theta e^{\theta T_1}(T-T_{1-\alpha}+\alpha(T_{1-\alpha}-T_\alpha))\right\} + K(T)\left\{-c\theta e^{\theta T_1} - (Ic+c\theta)e^{\theta T_1}\right\}\right]$$
$$= -K(T)\left[I_e\left\{c\theta e^{\theta T_1}(T-T_{1-\alpha}+\alpha(T_{1-\alpha}-T_\alpha))+2p\right\} + \left\{c\theta e^{\theta T_1} + (Ic+c\theta)e^{\theta T_1}\right\}\right] \le 0,$$

Case 2

$$\frac{\partial^2 P_2^1(T_1,T)}{\partial T_1^2} = -K(T)[I_e\left\{\alpha c\theta e^{\theta T_1}(T_{1-\alpha}-T_1) + \left(p-\alpha ce^{\theta T_1}\right) + \left(p-\alpha ce^{\theta T_1}\right) + \theta ce^{\theta T_1}(T-T_{1-\alpha})\right\}+$$
$$\left\{c\theta e^{\theta T_1} + (Ic+c\theta)e^{\theta T_1}\right\}] \le 0$$
$$\frac{\partial^2 P_2^2(T_1,T)}{\partial T_1^2} = -K(T)\left[I_e\left\{2p + c\theta e^{\theta T_1}(T-T_{1-\alpha})\right\} + I_c\alpha\theta ce^{\theta T_1}(T_{1-\alpha}-T_\alpha) - \left\{c\theta e^{\theta T_1} + (Ic+c\theta)e^{\theta T_1}\right\}\right] \le 0.$$

Case 3

$$\frac{\partial^2 P_3^1(T_1,T)}{\partial T_1^2} = -K(T)\left[I_e\left\{\theta ce^{\theta T_1}(T-T_1) + 2\left(p-ce^{\theta T_1}\right)\right\} + \left\{c\theta e^{\theta T_1} + (Ic+c\theta)e^{\theta T_1}\right\}\right] \le 0$$
$$\frac{\partial^2 P_3^2(T_1,T)}{\partial T_1^2} = -K(T)[I_e\left\{\theta ce^{\theta T_1}(T-T_1) + 2\left(p-ce^{\theta T_1}\right)\right\}$$
$$+ I_c\frac{(1-\alpha)c}{\theta}\left\{\theta^2 e^{\theta T_1}(T-T_1) - \theta e^{\theta T_1} + \theta e^{\theta T_1}\right\} + \left\{c\theta e^{\theta T_1} + (Ic+c\theta)e^{\theta T_1}\right\}] \le 0$$
$$\frac{\partial^2 P_3^3(T_1,T)}{\partial T_1^2} = -K(T)I_e\left\{\theta ce^{\theta T_1}(T-T_1) + 2\left(p-ce^{\theta T_1}\right)\right\}$$
$$+ I_c\left[c\theta e^{\theta T_1}\{\alpha(T_1-T_\alpha)+(1-\alpha)(T_1-T_{1-\alpha})\} + 2ce^{\theta T_1}\right] + \left\{c\theta e^{\theta T_1} + (Ic+c\theta)e^{\theta T_1}\right\} \le 0$$
$$\frac{\partial^2 P_3^4(T_1,T)}{\partial T_1^2} = -K(T)[I_e\left\{\theta ce^{\theta T_1}(T-T_1) + 2\left(p-ce^{\theta T_1}\right)\right\}$$
$$+ I_c\alpha\theta ce^{\theta T_1}(T_{1-\alpha}-T_\alpha) + \left\{c\theta e^{\theta T_1} + (Ic+c\theta)e^{\theta T_1}\right\}] \le 0.$$

Hence, $P(T_1, T)$ is concave in T_1 for a given T.

Theorem 2: *For* $T \le \frac{1}{\theta}\log_e(p/c)$, *optimal* $\mathbf{T_1}$ *is an increasing function in T.*

Proof: If $T \le \frac{1}{\theta}\log_e(p/c)$, then $T_1 \le \frac{1}{\theta}\log_e(p/c)$. Hence we have the following -

Case 1 Optimal T_1 satisfies $\frac{\partial P_1(T_1,T)}{\partial T_1} = 0$, which gives

$$I_e\left\{pT - 2pT_1 - ce^{\theta T_1}\left(T - T_{1-\alpha} + \alpha(T_{1-\alpha} - T_\alpha)\right)\right\} + \left\{-ce^{\theta T_1} + ce^{rT} + s - \frac{Ic+\theta c}{\theta}\left(e^{\theta T_1} - 1\right)\right\} = 0 \quad (1)$$

Differentiating (1) with respect to T, we get

$$I_e\left\{p - 2p\frac{\partial T_1}{\partial T} - c\theta e^{\theta T_1}\frac{\partial T_1}{\partial T}\left(T - T_{1-\alpha} + \alpha(T_{1-\alpha} - T_\alpha)\right) - ce^{\theta T_1}\right\} +$$

$$\left\{-c\theta e^{\theta T_1}\frac{\partial T_1}{\partial T} + cre^{rT} - (Ic+\theta c)e^{\theta T_1}\frac{\partial T_1}{\partial T}\right\} = 0$$

or, $$\frac{\partial T_1}{\partial T} = \frac{cre^{rT} + I_e\left\{p - ce^{\theta T_1}\right\}}{I_e c\theta e^{\theta T_1}\left(T - T_{1-\alpha} + \alpha(T_{1-\alpha} - T_\alpha)\right) + c\theta e^{\theta T_1} + (Ic + c\theta)e^{\theta T_1} + 2p},$$

which is ≥ 0 if $p - ce^{\theta T_1} \ge 0$.

Similarly, we get

Case 2

(a) $$\frac{\partial T_1}{\partial T} = \frac{I_e\left(p - ce^{\theta T_1}\right) + cre^{rT}}{I_e\left\{\left(p - \alpha ce^{\theta T_1}\right) + \left(p - \alpha ce^{\theta T_1}\right) + \theta ce^{\theta T_1}\left(T - T_{1-\alpha}\right)\right\} + c\theta e^{\theta T_1} + (Ic+\theta c)e^{\theta T_1} - I_e\alpha\theta ce^{\theta T_1}\left(T_{1-\alpha} - T_1\right)} \ge 0$$

(b) $$\frac{\partial T_1}{\partial T} = \frac{cre^{rT} + I_e\left(p - ce^{\theta T_1}\right)}{I_e\left(2p + c\theta e^{\theta T_1}\left(T - T_{1-\alpha}\right)\right) + I_c\alpha ce^{\theta T_1}\left(T_{1-\alpha} - T_\alpha\right) + c\theta e^{\theta T_1} + (Ic+\theta c)e^{\theta T_1}} \ge 0$$

Case 3

(a) $$\frac{\partial T_1}{\partial T} = \frac{cre^{rT}}{I_e\left\{c\theta e^{\theta T_1}(T - T_1) + 2\left(p - ce^{\theta T_1}\right)\right\} + ce^{\theta T_1} + (Ic+\theta c)e^{\theta T_1}} \ge 0$$

(b) $$\frac{\partial T_1}{\partial T} = \frac{cre^{rT}}{I_e\left\{c\theta e^{\theta T_1}(T - T_1) + 2\left(p - ce^{\theta T_1}\right)\right\} + ce^{\theta T_1} + (Ic+\theta c)e^{\theta T_1} + I_c(1-\alpha)c\theta e^{\theta T_1}(T - T_1)} \ge 0$$

(c) $$\frac{\partial T_1}{\partial T} = cre^{rT} / [I_e\left\{c\theta e^{\theta T_1}(T - T_1) + 2\left(p - ce^{\theta T_1}\right)\right\} + ce^{\theta T_1} + (Ic+\theta c)e^{\theta T_1} + I_c \times$$

$$\left(c\theta e^{\theta T_1}\left\{\alpha(T_1 - T_\alpha) + (1-\alpha)(T_1 - T_{1-\alpha})\right\} + 2ce^{\theta T_1}\right)] \ge 0$$

(d) $$\frac{\partial T_1}{\partial T} = \frac{cre^{rT}}{I_e\left\{c\theta e^{\theta T_1}(T - T_1) + 2\left(p - ce^{\theta T_1}\right)\right\} + ce^{\theta T_1} + (Ic+\theta c)e^{\theta T_1} + I_c\alpha\theta^2 ce^{\theta T_1}\left(T_{1-\alpha} - T_\alpha\right)} \ge 0.$$

Hence the theorem.

Theorem 3: *$P(T_1,T)$ is a decreasing function of θ and s.*

Proof:

$$\frac{dP_1(T_1,T)}{d\theta} = -\frac{K(T)}{H}\left(Ic\left(\sum_{i=3}^{\infty}\frac{\theta^{i-3}T_1^i}{i!}\right) + c\left(\sum_{i=2}^{\infty}\frac{\theta^{i-2}T_1^i}{i!}\right) + c\left(\sum_{i=2}^{\infty}\frac{\theta^{i-2}T_1^i}{i!}\right)\left\{I_e\left(T - T_{1-\alpha} + \alpha(T_{1-\alpha} - T_\alpha)\right) + 1\right\}\right) \le 0$$

and

$$\frac{dP_1(T_1,T)}{ds} = -\frac{K(T)}{H}(T - T_1) \le 0.$$

Similarly, it can be shown that

$$\frac{dP_i^{(j)}(T_1,T)}{d\theta} \le 0, \quad \frac{dP_i^{(j)}(T_1,T)}{ds} \le 0, \; i, j = 2,3.$$

Hence the theorem.

Numerical Examples and Sensitivity Analysis

Since it is difficult to find optimum values of the decision variables in closed form, we numerically find solutions to the equations $\frac{dP(T_1,T)}{dT_1} = 0$ and $\frac{dP(T_1,T)}{dT} = 0$, for given sets of model parameters, using the statistical software MATLAB.

The following tables show the change in optimal inventory policy with change in some important parameters of the model. We assume that $A =$ ₹ 200, $c =$ ₹ 20, $a = 2000$, $b = 0.1$, $\theta = 0.04$, $H = 2$ years.

Table 1: Showing change in optimum (T_1, T)-values and corresponding profit with change in r for some combinations of (T_α , $T_{1\text{-}\alpha}$) when $p =$ ₹ 25, $s =$ ₹ 0.4, , $I = 0.01$, $\alpha = 0.7$, $I_e =$, $I_c =$

($T_{1\text{-}\alpha}$, T_α)	***R***	**T_1**	***T***	**Profit**	**($T_{1\text{-}\alpha}$, T_α)**	***r***	**T_1**	***T***	**Profit**
(0.07, 0.02)	0.01	0.1279	0.6879	8867.10	(0.1, 0.04)	0.01	0.1330	0.6754	8886.25
	0.03	0.1176	0.3873	8693.70		0.03	0.1230	0.3804	8725.59
	0.05	0.1202	0.3018	8655.44		0.05	0.1247	0.2293	8758.82
	0.07	0.1246	0.2578	8674.69		0.07	0.1247	0.1938	8846.68
	0.10	0.1320	0.2199	8768.73		0.10	0.1247	0.1622	9029.97
(0.7, 0.04)	0.01	0.1326	0.6767	8884.34	(0.1, 0.07)	0.01	0.1247	0.5737	8964.39
	0.03	0.1224	0.3810	8722.42		0.03	0.1247	0.3313	8864.52
	0.05	0.1249	0.2970	8693.77		0.05	0.1246	0.2566	8880.80
	0.07	0.1292	0.2537	8722.05		0.07	0.1246	0.2169	8947.02
	0.10	0.1365	0.2163	8829.20		0.10	0.1246	0.1815	9102.14
(0.7, 0.06)	0.01	0.1070	0.5868	8924.51	(0.1, 0.09)	0.01	0.1603	0.5909	9005.09
	0.03	0.1069	0.3388	8800.59		0.03	0.1602	0.3412	8923.66
	0.05	0.1069	0.2624	8794.59		0.05	0.1602	0.2643	8962.85
	0.07	0.1068	0.2218	8838.81		0.07	0.1601	0.2234.	9054.49
	0.10	0.1067	0.1856	8960.50		0.10	0.1600	0.1869	9251.35

Table 2: Showing change in optimum (T_1, T)-values and corresponding profit with change in I for some values of T_α , $T_{1\text{-}\alpha}$ when $p =$ ₹ 25, $s =$ ₹ 0.4, $r = 0.06$, $\alpha = 0.7$

($T_{1\text{-}\alpha}$, T_α)	***I***	**T_1**	***T***	**Profit**	**($T_{1\text{-}\alpha}$, T_α)**	***I***	**T_1**	***T***	**Profit**
(0.07, 0.02)	0.01	0.1424	0.2537	8448.64	(0.1, 0.04)	0.01	0.1250	0.1590	8662.12
	0.03	0.1358	0.2549	8416.55		0.03	0.1248	0.1663	8621.66
	0.05	0.1297	0.2560	8387.46		0.05	0.1248	0.1731	8582.89
	0.07	0.1241	0.2570	8360.98		0.07	0.1247	0.1797	8545.61
	0.10	0.1166	0.2583	8325.42		0.10	0.1246	0.1892	8492.16
(0.7, 0.04)	0.01	0.1466	0.2467	8503.30	(0.1, 0.07)	0.01	0.1248	0.1908	8734.86
	0.03	0.1398	0.2484	8468.35		0.03	0.1247	0.1969	8700.96
	0.05	0.1336	0.2499	8436.70		0.05	0.1247	0.2027	8668.08
	0.07	0.1280	0.2512	8407.90		0.07	0.1247	0.2084	8636.12
	0.10	0.1203	0.2529	8369.25		0.10	0.1246	0.2166	8589.76
(0.7, 0.06)	0.01	0.1070	0.2040	8593.62	(0.1, 0.09)	0.01	0.1603	0.1856	8898.49
	0.03	0.1070	0.2082	8570.19		0.03	0.1603	0.1957	8841.56
	0.05	0.1069	0.2122	8547.22		0.05	0.1602	0.2053	8787.45
	0.07	0.1068	0.2182	8524.69		0.07	0.1602	0.2145	8735.78
	0.10	0.1068	0.2221	8491.66		0.10	0.1601	0.2276	8662.20

Table 3: Showing change in optimum (T_1, T)-values and corresponding profit with change in p for some values of T_α , $T_{1\text{-}\alpha}$ when $s =$ ₹ 0.4, $r = 0.06$, $\alpha = 0.7$, $I_e = 0.$, $I_c = 0$

($T_{1\text{-}\alpha}$, T_α)	***P***	**T_1**	***T***	**Profit**	**($T_{1\text{-}\alpha}$, T_α)**	***p***	**T_1**	***T***	**Profit**
(0.07, 0.02)	22	0.1400	0.2531	2066.94	(0.1, 0.04)	22	0.1098	0.1741	2167.98
	24	0.1393	0.2540	6310.65		24	0.1197	0.1656	6484.89
	26	0.1388	0.2547	10553.5		26	0.1297	0.1604	10796.7
	28	0.0978	0.1894	14809.8		28	0.1396	0.1586	15099.9
	30	0.1048	0.1853	19099.8		30	0.1496	0.1601	19392.0
(0.7, 0.04)	22	0.1446	0.2455	2320.96	(0.1, 0.07)	22	0.1098	0.1989	2259.69
	24	0.1436	0.2470	6464.10		24	0.1197	0.1948	6566.54
	26	0.1427	0.2482	10606.5		26	0.1297	0.1937	10867.2
	28	0.1419	0.2493	14848.1		28	0.1396	0.1956	15159.9
	30	0.1047	0.1852	18988.7		30	0.1496	0.2001	19443.6
(0.7, 0.06)	22	0.1491	0.2371	2178.59	(0.1, 0.09)	22	0.1410	0.1865	2419.22
	24	0.1026	0.2080	6433.40		24	0.1538	0.1882	6722.95

	26	0.1112	0.2047	10729.5		26	0.1666	0.1943	11012.7
	28	0.1197	0.2036	15021.4		28	0.1794	0.2041	15288.3
	30	0.128	0.2047	19308.1		30	0.1921	0.2172	19550.6

Table 4: Showing change in optimum (T_1, T)-values and corresponding profit with change in I_e for some combinations of (T_α , $T_{1-\alpha}$) when p = ₹ 25, s = ₹ 0.4, r = 0.06, α = 0.7, I =0.2, I_c =0.14

($T_{1-\alpha}$, T_α)	I	T_1	T	Profit	($T_{1-\alpha}$, T_α)	I	T_1	T	Profit
	.03	0.2179	0.2783	8810.23		0.01	0.2175	0.2787	8807.59
	0.05	0.1798	0.2772	8745.34		0.03	0.1247	0.2030	8774.94
(0.07, 0.02)	0.07	0.1562	0.2765	8709.31	(0.1, 0.04)	0.05	0.1247	0.2058	8785.10
	0.10	0.1339	0.2758	8681.59		0.07	0.1246	0.2099	8800.73
	0.12	0.1238	0.2756	8672.79		0.10	0.1246	0.2126	8811.39
	0.03	0.2267	0.2709	8877.31		0.03	0.2306	0.2672	8909.66
	0.05	0.1870	0.2712	8800.72		0.05	0.1247	0.2256	8867.54
(0.7, 0.04)	0.07	0.1622	0.2714	8757.41	(0.1, 0.07)	0.07	0.1247	0.2281	8879.26
	0.10	0.1386	0.2715	8722.77		0.10	0.1247	0.2318	8897.13
	0.12	0.1279	0.2716	8710.83		0.12	0.1246	0.2343	8909.22
	0.03	0.2351	0.2624	8949.09		0.01	0.2388	0.2582	8983.76
	0.05	0.1938	0.2645	8859.60		0.03	0.2165	0.2261	8973.32
(0.7, 0.06)	0.07	0.1679	0.2657	8808.28	(0.1, 0.09)	0.05	0.1720	0.2306	8980.99
	0.10	0.1109	0.2373	8802.34		0.07	0.1605	0.2366	8991.36
	0.12	0.1070	0.2391	8814.66		0.10	0.1602	0.2406	9007.65

Table 5: Showing change in optimum (T_1, T)-values and corresponding profit with change in I_c for some combinations of ($T_{1-\alpha}$, T_α) when p = ₹ 25, s = ₹ 0.4, r = 0.06, α = 0.7, I =0.2, I_e =0.12

($T_{1-\alpha}$, T_α)	I	T_1	T	Profit	($T_{1-\alpha}$, T_α)	I	T_1	T	Profit
	.13	0.1245	0.2749	8679.45		0.13	0.1248	0.2142	8817.97
	0.15	0.1230	0.2762	8666.19		0.15	0.1247	0.2110	8804.89
(0.07, 0.02)	0.17	0.1215	0.2776	8653.15	(0.1, 0.04)	0.17	0.1247	0.2077	8792.10
	0.20	0.1193	0.2794	8634.00		0.20	0.1247	0.2026	8773.49
	0.22	0.1177	0.2806	8621.50		0.22	0.1246	0.1992	8761.48
	.13	0.1283	0.2711	8715.02		0.13	0.1247	0.2343	8909.66
	0.15	0.1274	0.2720	8706.67		0.15	0.1247	0.2343	8867.54
(0.7, 0.04)	0.17	0.1266	0.2729	8698.41	(0.1, 0.07)	0.17	0.1247	0.2344	8879.26
	0.20	0.1252	0.2742	8686.17		0.20	0.1246	0.2345	8897.13
	0.22	0.1243	0.2750	8678.12		0.22	0.1246	0.2344	8909.22
	.13	0.1070	0.2389	8815.98		0.13	0.1602	0.2402	9009.62
	0.15	0.1070	0.2394	8813.33		0.15	0.1602	0.2409	9005.68
(0.7, 0.06)	0.17	0.1069	0.2398	8810.70	(0.1, 0.09)	0.17	0.1602	0.2416	9001.75
	0.20	0.1068	0.2403	8808.08		0.20	0.1601	0.2426	8995.88
	0.22	0.1067	0.2403	8808.08		0.22	0.1600	0.2433	8991.98

From the above tables, we make the following observations:
(i) Optimal T_1 is a non-increasing function of I, I_e and I_c.
(ii) Optimal T is non-decreasing in I and I_c, but is non-increasing in r.
(iii) Maximum profit is a non-increasing function of I and I_c, but a non-decreasing function of p.

Conclusion

The paper studies a periodic review inventory model for deteriorating items when demand is dependent on the selling price and the deterioration rate is constant. The inventory manager has the provision to pay his dues to the supplier in two installments – a proportion α of his dues in the first installment and the remaining in the second installment. Failure to make payment in time imposes an interest on the unpaid amount. Value inflation of money is also taken into account, which is essential when the planning period is sufficiently long.

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