Use of Restricted Least Squares Estimator to Study the Growth of Manufactured Food Products in Haryana

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Research Article

Abstract: This article studies the growth of manufactured food products in Haryana from 2001 onwards. For this purpose an extension of Cobb-Douglas production function is used.

Keywords: Ordinary Least Squares Estimator, Restricted Least Squares Estimator, Cobb-Douglas Production Function.

1. Introduction

Haryana is among the developed states in India. Mainly aggregarian, Haryana has achieved impressive growth rates in industrial and infrastructural development. over the years numerous large and small manufacturing companies and units have developed in Haryana during past four decades. Several other avenues have also opened up, prominent among them is manufactured food products. The following article is devoted to study the growth of manufactured food products in Haryana from 2001 onwards; the data has been obtained from http://statesofindia.cmie.com. For this purpose Cobb-Douglas production function (Cobb and Douglas [1]) is used as it is a well recognized technique to study the relation between input and output variables. Owing to constant returns to scale production, attempts have been made in this paer to estmate the coefficients in the model using restricted least squares (see; e.g. Elias [2], Gujarati and Sangeetha [3], Zellner and Richard [4]). The plan of the paper is as follows: in section 2 the model and estimator has been described. In section 3 the study of growth of manufactured food products in Haryana is carried out and lastly in section 4 the conclusions are made.

2. The Model and Estimators

The usual Cobb-Douglas production function consists of the inputs from Labour and Captals. However an extension of this may be defined as

$$Y_t = A\, L_t^{\beta_1} K_t^{\beta_2} e^{CT_t} e^{U_t} \quad ; \; t = 1, 2, \ldots\ldots n \tag{2.1}$$

This includes the technological progress represented by the time T_t as well as the capital K_t and labour L_t as input variable. Here Y_t is the production, A is total factor productivity and C represent the rate of annual growth in production as a consequence to technological progress. In order to estimate the production function using ordinary least squares estimator, we need to create a linear model, which may be written as

$$lnY_t = ln\, A + \beta_1\, ln\, L_t + \beta_2\, ln\, K_t + CT_t + U_t \tag{2.2}$$

For convenience, we can rewrite (2.2) as

$$y = X\beta + u \tag{2.3}$$

where y is an $n \times 1$ vector of n observations on the response variable, X is an $n \times p$ full column rank matrix of n observations on p explanatory variables, β is a $p \times 1$ unknown coefficients vector and u is an $n \times 1$ vector of disturbances.

Using principle of least squares, the following estimator

$$b = (X'X)^{-1}X'y \tag{2.4}$$

is obtained which is well known to be the unbiased with variance co-variance matrix

$$V(b) = \sigma^2 (X'X)^{-1} \tag{2.5}$$

If prior information on some parameters of the model in the form of linear restriction

$$r = R\beta \tag{2.6}$$

is available, where r is $q \times 1$ vector and R is a $q \times p$ $(q < p)$ matrix of full row rank with known elements, the least squares estimator of β using (2.3) and (2.6) is given by

$$b_R = b + (X'X)^{-1}R'(R(X'X)^{-1}R')^{-1}(r - Rb) \tag{2.7}$$

which is also unbiased with variance covariance matrix

$$V(b_R) = V(b) - \sigma^2(X'X)^{-1}R'(R(X'X)^{-1}R')^{-1}R(X'X)^{-1} \tag{2.8}$$

3. Growth of Manufactured Food Products in Haryana

In order to study the growth of manufactured food products in Haryana using the data from http://statesofindia.cmie.com for this purpose the following predictive model

$$y_t = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon \tag{3.1}$$

is obtained where $y_t = lnY_t$, $\beta_0 = ln\, A$, $x_1 = ln\, L_t$, $x_2 = ln\, K_t$, $x_3 = T_t$, $\beta_3 = C$ and $\varepsilon = U_t$

Using equation (3.1) the following regression equation is obtained

$$\hat{y}_t = 8.80 - 0.496\, x_1 + 0.705 x_2 + 0.0468\, x_3 \tag{3.2}$$

The well known Cobb-Douglas production assumes the constant returns to scale. Hence here we assume that the sum of coefficients of $\ln L$ and $\ln K$ to be constant which for convenience is taken to be unity i.e. $\beta_1 + \beta_2 = 1$
Using this restriction, the following regression equation is obtained which is uses restricted least squares estimator given by (2.7)

$$\hat{y}_t = 0.4983 + 0.2544\, x_1 + 0.7455\, x_2 + 0.0382 x_3 \qquad (3.3)$$

The equation (3.2) shows that coefficient of x_1 is negative, from which it is concluded that an increase in labour may decrease the productions and vice versa which is clearly not true. However, this is corrected when we use the restricted least squares estimator. Now the coefficient of both x_1 and x_2 in equation (3.3) are positive which seems justified. It is also noted from equation (3.3) that while increase in labour has little effect, the increase in capital has large effect on production. Also the production satisfies an annual increment of about 3 percent during the period of study as result of technological progress. Moreover the restriction $\beta_1 + \beta_2 = 1$ agrees with the assumption of the Cobb-Douglas production function.

4. Conclusion

The above study clearly concludes that the growth of manufactured food products in Haryana is inevidently characterized by constant returns to scale over the period under study. As far as theoretical justification are concerned, it is concluded that restricted least squares estimator instead of ordinary least squares estimator should be used in estimation of production function. It is also pertinent to mention that the restricted least squares estimator is more efficient than the ordinary least squares estimator and it is observed that the coefficient of x_1 is 5 percent more while that of coefficient of x_2 is almost 95 percent efficient than that of ordinary least squares estimator.

References

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