On Fuzzy Almost Resolvable and Fuzzy Almost Irresolvable Spaces

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Research Article

Abstract: In this paperthe concepts of fuzzy almost resolvable and fuzzy almost irresolvable spaces are introduced and Characterizations of fuzzy almost resolvable spaces and levels of fuzzy irresolvability are studied. Several examples are given to illustrate the concepts introduced in this paper.

Keywords: Fuzzy resolvable, fuzzy irresolvable, fuzzy almost resolvable and fuzzy almost irresolvable, fuzzy first category and fuzzy submaximal and fuzzy Baire spaces.

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1. Introduction

The fuzzy concept has invaded almost all branches of Mathematics ever since the introduction of fuzzy set by L. A. ZADEH [12]. The theory of fuzzy topological was introduced and developed by C.L.CHANG [3]. Since then much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. E.HEWIT [5] introduced the concepts of resolvability and irresolvability intopological spaces. A.G.El'kin [4] introduced open hereditarily irresolvable spaces in the classical topology. The concepts of almost resolvable spaces was introduced by RICHARD BOLSTEIN [8] asa generalization of resolvable spaces of E.HEWIT. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. In [6] EL. NASCHIE showed that the notion of fuzzy topology might be relevant to Quantum Particle Physics in connection with String theory. In this paper we introduce the concepts of fuzzy almost resolvable and fuzzy almost irresolvable spaces. Also we discuss several characterizations of fuzzy almost resolvable spaces and study inter-relations between fuzzy submaximal, fuzzy openhereditarily irresolvable, fuzzy irresolvable and fuzzyalmost irresolvable spaces.

Several examples are given to illustrate the concepts introduced in this paper.

2. Preliminaries

By a fuzzy topological space we shall mean a non -empty set X together with a fuzzy topology T (in the sense of Chang) and denote it by (X,T).

Definition 2.1: Let $\lambda$and μ be any two fuzzy sets in (X,T). Then we define

$\lambda \vee \mu : X \to [0,1]$ as follows : $(\lambda \vee \mu)(x) = \text{Max}\{ \lambda(x), \mu(x)\}$. Also we define

$\lambda \wedge \mu : X \to [0,1]$ as follows: $(\lambda \wedge \mu)(x) = \text{Min}\{ \lambda(x), \mu(x)\}$.

Definition 2.2 : Let (X,T) be afuzzy topological space and λ be any fuzzy set in (X,T). We defineint(λ) $=\vee\{ \mu / \mu \leq \lambda, \mu\in T \}$ and cl $(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1-\mu \in T \}$.

For any fuzzy set in a fuzzy topological space (X,T), it is easy to see that $1 - cl(\lambda) = int(1 - \lambda)$ and $1 - int(\lambda) = cl(1 - \lambda)$ **[1].**

Definition 2.3 : Let(X ,T)and (Y,S) be any two fuzzy topological spaces. Let f be a function from the fuzzy topological space (X ,T) to the fuzzy topological space(Y,S). Let λ be a fuzzy set in (Y,S). The inverse image of λ under**f**written as$f^{-1}(\lambda)$, is the fuzzy set in (X ,T) defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$, for all $x\in X$. Also the image of μ in (X,T) under **f** written as f (μ) is the fuzzy set in (Y,S) defined by

$$f(\mu)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \text{ is non} - empty; \\ 0 & otherwise. \end{cases}$$ for each $y \in Y$.

Lemma 2.1 [3] : Let $f : (X ,T) \to (Y, S)$ be a mapping. Forfuzzy sets λ and μ of (X ,T) and (Y, S) respectively, the following statements hold :

(1) $f f^{-1}(\mu) \leq \mu$;
(2) $f^{-1} f(\lambda) \geq \lambda$;
(3) $f(1 - \lambda) \geq 1 - f(\lambda)$;
(4) $f^{-1}(1 - \mu) = 1 - f^{-1}(\mu)$;
(5) If f is one - to - one , then $f^{-1} f(\lambda) = \lambda$;
(6) If f is onto , then $f f^{-1}(\mu) = \mu$;
(7) If f is one - to -one and onto, then $f(1 - \lambda) = 1 - f(\lambda)$.

Definition 2.4 [9] : A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$.

Definition 2.5 [9] : A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non - zero fuzzy open set μ in (X,T) such that μ < cl (λ). That is, int cl (λ) = 0.

Definition 2.6[10] : A fuzzy topological space(X,T) is called a fuzzy open hereditarily irresolvable space if int cl (λ) ≠0, then int (λ) ≠ 0 for any non - zero fuzzy set in (X,T).

Definition 2.7 [2]:A fuzzy topological space (X,T) is called a fuzzy submaximal space, if cl (λ) = 1 for any non-zero fuzzy set λ in (X,T), then $\lambda \in T$.

Definition 2.8 [9]: A fuzzy topological space (X,T) is called fuzzy first category if $1 = \bigvee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy nowhere dense sets in (X,T). A topological space which is not of fuzzy first category,is said to be of fuzzy second category.

Lemma 2.2 [1] : For a family of $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space (X,T), $\vee$ cl $(\lambda_\alpha) \leq$ cl $(\vee\lambda_\alpha)$. In case A is a finite set,$\vee$ cl (λ_α)= cl $(\vee\lambda_\alpha)$. Also $\vee$ int $(\lambda_\alpha) \leq$ int $(\vee\lambda_\alpha)$.

Definition 2.9[10] : A fuzzy topological space (X,T) is called a fuzzy resolvable space if there exists a fuzzy dense set λ in (X,T) such that cl (1 –λ) = 1. Otherwise (X,T) is called a fuzzy irresolvable space.

DEFINITION 2.10[7] : Two fuzzy sets μ and νof a fuzzy topological space (X,T) is said to be disjoint if they do not intersect at any point of X. That is, μ(x) + ν(x) ≤1 and (μ∧ν) (x) = 0 for all x ∈ X.

DEFINITION 2.11[11]: Let (X,T) be a fuzzy topological space. Then(X,T) is called a fuzzy Baire Space if nt $(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$,where λ_i's fuzzy nowhere dense sets in (X,T).

3. Fuzzy Almost Resolvable Spaces

Motivated by the classical concept introduced in [8] we shall now define:

DEFINITION 3.1: A fuzzy topological space (X,T) is called a fuzzy almost resolvable space if $\bigvee_{i=1}^{\infty}(\lambda_i) = 1$, wherethe fuzzy sets λ_i' s in (X,T) are such that int (λ_i) =0. Otherwise (X,T) is called a fuzzy almost irresolvable space.

Example 3.1: Let X = {a, b, c}.The fuzzy sets λ, μ and υ are defined on X as follows:

λ : X → [0,1] is defined as λ (a) = 1; λ(b) = 0.3 ; λ (c) = 0.7

μ : X → [0,1] is defined as μ (a) = 0.4 ; μ (b) = 1; μ (c) = 0.6.

υ : X → [0,1 is defined as υ (a) = 0.5 ; υ (b) = 0.6 ; υ (c) = 1.

Then, clearly T = { 0, λ, μ, υ, λ ∨ μ , λ ∨υ, μ ∨υ, λ ∧ μ, λ ∧υ, μ ∧υ, λ ∨ (μ ∧υ), μ ∨ (λ ∧υ), υ∧(λ ∨ μ), 1} is a fuzzytopology on X. Now consider the following fuzzy sets defined on X as follows:

α : X → [0,1] is defined as α (a) = 1; α (b) = 0.2 ; α (c) = 0.3.

β : X → [0,1] is defined as β (a) = 0.2 ; β (b) = 1 ; β (c) = 0.7.

γ : X → [0,1] is defined as γ (a) = 0.3 ; γ (b) = 0.4 ; γ (c) = 1.

η: X → [0,1] is defined as η(a) = 1; η(b) = 0.1 ; η (c) = 0.5.

Then int (α) = 0 ; int (β) = 0; int (γ) = 0 and int (η) = 0 and { (α) ∨ (β) ∨ (γ) ∨ (η) } = 1.

Hence (X,T) is a fuzzy almost resolvable space.

Example 3.2: Let λ, μ, υand η be fuzzy sets of **I** = [0,1] as follows :

λ (x) =1 – x ; 0 ≤ x ≤ 1,

$$\mu(x) = \begin{cases} 1 - x, 0 \leq x \leq \frac{1}{2}; \\ x, \frac{1}{2} \leq x \leq 1. \end{cases}$$

$$\upsilon(x) = \begin{cases} 1 - 2x, 0 \leq x \leq \frac{1}{2}; \\ 0, \frac{1}{2} \leq x \leq 1. \end{cases}$$

$$\eta(x) = \begin{cases} 1, 0 \leq x \leq \frac{1}{2}; \\ 0, \frac{1}{2} \leq x \leq 1. \end{cases}$$

Clearly T = { 0, υ, η , υ∨η, υ∧η, 1} is a fuzzy topology on I. Now int (1 –λ) = 0, int (1 –μ) = 0, int (1 –υ) = 0,int (1 – η) = 0,int (1– [υ∨η]) = 0 and int (1– [υ∧η]) = 0.

Now { (1 –λ) ∨ (1 –μ) ∨ (1 –υ) ∨ (1 – η) ∨ (1 – [υ∨η]) ∨ (1 – [υ∧η]) } ≠ 1 and hence(I,T) is a fuzzy almost irresolvable space.

Proposition 3.1: A fuzzy topological space (X,T) is a fuzzy resolvable space if $\vee_{i=1}^{N}(\lambda_i) = 1$, wherethe fuzzy sets λ_i' s in(X,T) are such that int $(\lambda_i) = 0$.

Proof: Now $\vee_{i=1}^{N}(\lambda_i) = 1$implies that$1 - \vee_{i=1}^{N}(\lambda_i) = 0$. Then$\wedge_{i=1}^{N}(1 - \lambda_i) = 0$.

Hence there must be atleast two non - zero disjoint fuzzy sets $1 - \lambda_i$ and $1 - \lambda_j$ in (X,T). Then $(1 - \lambda_i) + (1 - \lambda_j) \leq 1$, which implies that $(1 - \lambda_i) \leq \lambda_j$ and hence cl$(1 - \lambda_i) \leq cl\,(\lambda_j)$ .Now int $(\lambda_i) = 0$ implies that 1–int (λ_i) =1 and hencecl$(1 - \lambda_i)$ =1.

Then $1 \leq cl\,(\lambda_j)$.That is, $cl\,(\lambda_j) = 1$. Also int $(\lambda_j) = 0$ implies that cl$(1 - \lambda_j) = 1$. Hence the fuzzy topological space (X,T) has a dense fuzzy set λ_j such that cl$(1 - \lambda_j) = 1$.Hence (X,T) is a fuzzy resolvable space. It is clear from the definition that every fuzzy resolvable spaceis a fuzzy almost resolvable space.

Proposition 3.2: If thefuzzy topological space (X,T) is a fuzzy first category space, then (X,T) is a fuzzy almost resolvable space.

Proof: Since the fuzzy topological space (X,T) is of fuzzy first category, we have $1 = \vee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy nowhere dense sets in (X,T). Now $\lambda_i$is a fuzzy nowhere dense set implies that int cl (λ_i) = 0**.**Sinceint (λ_i) ≤int cl (λ_i) implies that int (λ_i) = 0. Hence $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where int ($\lambda_i$) = 0 and therefore (X,T) is a fuzzy almost resolvable space.

Remarks: If thefuzzy topological space (X,T) is a fuzzy almost resolvable space, Then (X,T) need not be a fuzzy firstcategory space. For, consider the following example:

Example 3.3: Let X = {a, b, c}.The fuzzy sets λ, μ and υ are defined on X as follows:

λ : X → [0,1] is defined as λ (a) = 1; λ(b) = 0.2 ; λ (c) = 0.7

μ : X → [0,1] is defined as μ (a) = 0.3; μ (b)= 1; μ (c) = 0.2.

υ : X → [0,1] is defined as υ (a) = 0.7 ; υ (b) = 0.4 ; υ (c) = 1.

Then, clearly T = { 0, λ, μ, υ, λ ∨ μ , λ ∨υ, μ ∨υ, λ ∧ μ, λ ∧υ, μ ∧υ, λ ∨ (μ ∧υ),

μ ∨ (λ ∧υ), υ∧ (λ ∨ μ), 1} is a fuzzy topology on X. Now consider the following fuzzy sets defined on X as follows:

α : X → [0,1] is defined as α_n (a) = 1; α_n(b) = 0.1 ; α_n(c) = 0.n

β : X → [0,1] is defined as β_n(a) = 0.2 ; β_n(b) = 1 ; β_n(c) = 0.n.

γ : X → [0,1] is defined as γ_n (a) = 0.n ; γ_n(b) = 0.1 ; γ_n(c) = 1.

where n ∈ *I*. Then int (α_n) = 0 ; int (β_n) = 0 and int (γ_n) = 0 and (α_n) ∨ (β_n)∨ (γ_n) ∨ (η_n) = 1.

Hence (X,T) is a fuzzy almost resolvable space.The fuzzy nowhere dense sets in

(X,T) are[1–λ], [1–μ], [1 –υ], [1–(λ ∨ μ)] , [1–(λ ∨υ)], [1 – (μ ∨υ)], 1 – [λ ∨ (μ ∧υ)],

1 – [μ ∨ (λ ∧υ)], 1– [υ∧ (λ ∨ μ)]and [1 –α_n], [1 –β_n], [1 –γ_n] and $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, where λ_i's are fuzzy nowhere dense sets in (X,T) and hence (X,T) is not a fuzzy first category space.

Theorem 3.1 [10]: Ifthe fuzzy topological space(X,T) is a fuzzyopen hereditarily Irresolvable space , then int (λ) = 0 for any non-zero fuzzy dense set λ in (X,T) implies that int cl (λ) = 0.

Proposition 3.3: In an fuzzy open hereditarily irresolvable space (X,T) every fuzzy almost resolvable spaceis a fuzzy first category space.

Proof: Letthe fuzzy topological space (X,T) be a fuzzy almost resolvable space.Then we have $\vee_{i=1}^{\infty}(\lambda_i) = 1$, whereint ($\lambda_i$) = 0 in (X,T). Since (X,T) isfuzzy openhereditarily irresolvable int (λ_i) = 0implies that int cl (λ_i) = 0 and hence λ_i's are fuzzy nowhere dense sets in (X,T).Then $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where λ_i's are fuzzy nowhere sets in (X,T) and therefore (X,T) is a fuzzy first category space.

Proposition 3.4: If thefuzzy topological space (X,T) is a fuzzy almost resolvable space, then $\wedge_{i=1}^{\infty}(\mu_i) = 0$, where $\mu_i's$ are fuzzy dense sets in (X,T).

Proof: Since the fuzzy topological space (X,T) is a fuzzy almost resolvable space, we have $\vee_{i=1}^{\infty}(\lambda_i) = 1$, whereint ($\lambda_i$) = 0 in (X,T).Now int (λ_i) = 0 implies that 1 –int (λ_i) = 1– 0 =1 . Then cl (1–λ_i) = 1. Also $1 - \vee_{i=1}^{\infty}(\lambda_i) = 0$ implies that$\wedge_{i=1}^{\infty}$(1–λ_i) = 0. Let 1 –λ_i = μ_i. Hence $\wedge_{i=1}^{\infty}(\mu_i) = 0$where $\mu_i's$ are fuzzy dense sets in (X,T).

Proposition 3.5: If thefuzzy topological space (X,T) is a fuzzy almost resolvable and fuzzy submaximal space, then $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where λ_i's are fuzzy closed sets in (X,T).

Proof: Since the fuzzy topological space (X,T) is a fuzzy almost resolvable space, By proposition 3.4, we have $\wedge_{i=1}^{\infty}(\mu_i) = 0$, where $\mu_i's$ are fuzzy dense sets in (X,T). Since (X,T) is a fuzzy submaximal space, cl (μ_i) = 1 implies that $\mu_i \in T$. Then $\wedge_{i=1}^{\infty}(\mu_i) = 0$ implies that $1 - \wedge_{i=1}^{\infty}(\mu_i) = 1$ and hence$\vee_{i=1}^{\infty}(1-\mu_i) = 1$, where$1-\mu_i$'s are fuzzy closed sets in (X,T).1 –μ_i = λ_i . Hence $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where λ_i's are fuzzy closed sets in (X,T).

Proposition 3.6: If thefuzzy topological space (X,T) is a fuzzy irresolvable space, Then(X,T) is not a fuzzy almost resolvable space.

Proof: Suppose thatthe fuzzy topological space (X,T) is a fuzzy almost resolvable space. Then $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where int ($\lambda_i$) = 0 in (X,T), implies that $\wedge_{i=1}^{\infty}$(1–λ_i) = 0. Hence there must be atleast two non - zero disjoint fuzzy sets 1 –λ_i and $1 - \lambda_j$ in (X,T). Then (1 –λ_i) + $(1 - \lambda_j) \leq 1$, which implies that (1 –λ_i)≤λ_j and hence cl(1 –λ_i)≤ $cl(\lambda_j)$ .Then 1–int(λ_i) ≤ $cl(\lambda_j)$ .Now int (λ_i) = 0 implies that 1 ≤cl(λ_j). That is, $cl(\lambda_j) = 1$. Also int (λ_j) = 0 implies that 1–int (λ_j) =1 and hence cl(1 –λ_j) = 1. Hence (X,T)has a dense fuzzy set $\lambda_j$such that cl(1 –λ_j)= 1,which means that (X,T) is a fuzzy resolvable space.But this is a contradiction to (X,T) being a fuzzy irresolvable space. Therefore (X,T) is not a fuzzyalmost resolvable space.

Proposition 3.7: If thefuzzy topological space (X,T) is a fuzzy Baire space, Then (X,T) is a fuzzy almost irresolvable space.

Proof: Since (X,T) is a fuzzy Baire space,int ($\bigvee_{i=1}^{\infty}$ (λ_i)) = 0, where $\lambda_i^{'}s$ are fuzzy nowhere dense sets in (X,T). Now λ_i is a fuzzy nowhere dense set implies that int cl (λ_i) = 0**.**Sinceint (λ_i) ≤int cl (λ_i), we have int (λ_i) = 0. Hence ,int ($\bigvee_{i=1}^{\infty}$ (λ_i)) = 0, whereint (λ_i) = 0. Suppose that (X,T) is a fuzzy almost resolvable space. Then $\bigvee_{i=1}^{\infty}(\lambda_i)$ = 1 ,where int (λ_i) = 0. Now int ($\bigvee_{\square=1}^{\infty}(\lambda_i)$) = int (1) = 1 ,which implies that 0 = 1, a contradiction. Hence we must have $\bigvee_{\square=1}^{\infty}(\lambda_i) \neq 1$, where int (λ_i)= 0. Therefore (X,T) is a fuzzyalmost irresolvable space.

4. Levels of Fuzzy Irresolvability

Theorem 4.1 [10]: If(X,T) is a fuzzy irresolvable space if and only if int (λ) ≠ 0 for all fuzzy dense sets λ in (X,T).

Proposition 4.1: For any fuzzy topological space (X,T) we have the following relations: Fuzzy submaximality fuzzy open hereditarily irresolvablity fuzzy irresolvablity fuzzy almost irresolvablity.

Proof: Let (X,T) be a fuzzy submaximal space. Then, cl(λ) = 1 implies that $\lambda \in$ T. Suppose that int (λ) = 0 for any non -zero fuzzy set λ in (X,T). Then 1 –int (λ) = 1 –0 = 1 implies that cl (1–λ) = 1. Since (X,T) is fuzzy submaximal, (1–λ) ∈ T. Then λ is a fuzzy closed set in (X,T). Hence λ = cl (λ) implies that int (λ) = int cl (λ). int (λ) = 0 implies that int cl (λ) = 0. Therefore (X,T) is a fuzzyopen hereditarily irresolvable space. Hence fuzzy sub maximality fuzzy open hereditarily irresolvablity. Now let (X,T) be a fuzzy open hereditarily irresolvable space. Let λ be a fuzzy dense set in (X,T). Then , cl (λ) = 1implies that int (1–λ) = 0. Since(X,T) is fuzzyopen here ditarily irresolvable,int (1 –λ) = 0 implies that int cl (1–λ) = 0. Now we claim that cl (1–λ)≠ 1. Suppose cl (1–λ) = 1.Then, int cl (1–λ) = int (1) = 1 implies that 0 = 1, a contradiction. Hence we must have cl (1–λ)≠ 1. Therefore cl (λ) = 1 implies that cl (1–λ)≠ 1 which means that (X,T) is a fuzzy irresolvable space. Hence fuzzy open hereditarily irresolvablity□fuzzy irresolvablity. Now let (X,T) be a fuzzy irresolvable space. Then cl (λ_i) = 1 implies that cl (1− λ_i)≠ 1. Then int (λ_i)≠ 0 for all i ∈ I.Now cl (λ_i) = 1 implies that int (1− λ_i) = 0. We claim that $\bigvee_{\square=1}^{\infty}(1-\lambda_i) \neq 1$.Suppose that$\bigvee_{\square=1}^{\infty}(1-\lambda_i) = 1$.Then, 1 $-\wedge_{\square=1}^{\infty}(\lambda_i)$ = 1 implies that $\wedge_{\square=1}^{\infty}(\lambda_i) = 0$. Hence there must be atleast two non – zerodisjoint fuzzy sets λ_i and$\lambda_\square$ in (X,T). Then, $\lambda_i + \lambda_\square \leq 1$, which implies that $\lambda_i \leq 1-\lambda_\square$. Hence cl($\lambda_i$)≤ □□ ($1-\lambda_\square$) .Then 1≤ □□ ($1-\lambda_\square$) . That is,cl ((1 − λ_j) = 1.Then,int (λ_j) =0, a contradiction toint (λ_i)≠ 0 , for all i ∈ I. Hence $\bigvee_{\square=1}^{\infty}$ (1 − λ_i)≠ 1, where int (1− λ_i) = 0. Therefore (X,T) is a fuzzyalmost irresolvable space. Hence fuzzy irresolvablity□ fuzzyalmost irresolvablity.

Remarks : Theimplications of theabove relations need not betrue.For,consider thefollowing examples :

Example 4.1 : Let μ_1, $\mu_2$andμ_3 be fuzzy sets of **I** = [0,1] as follows :

$$\mu_1(x) = \begin{cases} 1, 0 \leq \square \leq \frac{1}{2}; \\ 2-2\square, \frac{1}{2} \leq \square \leq 1. \end{cases}$$

$$\mu_2(x) = \begin{cases} 0, 0 \leq \square \leq \frac{1}{4} \\ 4\square - 1, \frac{1}{4} \leq \square \leq \frac{1}{2} \\ 1, \frac{1}{2} \leq \square \leq 1. \end{cases}$$

$$\mu_3(x) = \begin{cases} 1, 0 \leq \square \leq \frac{1}{4}; \\ \frac{4}{3}(1-\square), \frac{1}{4} \leq \square \leq 1. \end{cases}$$

Clearly T = { 0, μ_2,μ_3, ($\mu_2 \vee \mu_3$),($\mu_2 \wedge \mu_3$), 1} is a fuzzy topology onI. Now We have int (1 –μ_1) = 0 ; int (1 –μ_2) = 0 ; int (1 –μ_3) = 0; int (1 –[($\mu_2 \vee \mu_3$)]) = 0 and int (1 –[($\mu_2 \wedge \mu_3$)]) = 0. Also int cl (1 –μ_1) = 0 ; int cl (1 –μ_2) = 0 ; int cl (1 –μ_3) = 0; int cl (1 –[($\mu_2 \vee \mu_3$)]) = 0 and int cl (1 –[($\mu_2 \wedge \mu_3$)]) = 0.Hence the fuzzy topological space (I,T) is fuzzyopen hereditarily irresolvable.But (I,T) is not a fuzzy submaximal space, since cl (μ_1) = 1 and $\mu_1$isnot afuzzy open setin (I,T).

Example 4.2: Let X = {a, b }.The fuzzy sets λ, μ are defined on X as follows :

λ : X → [0,1] is defined as λ (a) = 0.2; λ(b) = 0.3.

μ : X → [0,1] is defined as μ (a) = 0.6; μ (b)= 0.9.

Clearly T = { 0, λ, 1} is a fuzzy topology on X. Now μ is a fuzzy dense set in (X,T) and cl(1–μ) = 1–λ≠ 1. Hence (X,T) is a fuzzy irresolvable space. But (X,T) is not a fuzzy open hereditarily irresolvable space,since int (1–μ) = 0 but int cl (1–μ) = λ≠1.

Example 4.3: Let μ_1, μ_2 and μ_3 be fuzzy sets of **I** = [0,1] as follows :

$$\mu_1(x) = \begin{cases} 1, 0 \leq \square \leq \frac{1}{2}; \\ 2-2\square, \frac{1}{2} \leq \square \leq 1. \end{cases}$$

$$\mu_2(x) = \begin{cases} 0, 0 \leq \square \leq \frac{1}{4} \\ 4\square - 1, \frac{1}{4} \leq \square \leq \frac{1}{2} \\ 1, \frac{1}{2} \leq \square \leq 1. \end{cases}$$

$$\mu_3(x) = \begin{cases} 1, 0 \le x \le \frac{1}{4}; \\ \frac{4}{3}(1 - x), \frac{1}{4} \le x \le 1. \end{cases}$$

Clearly T = { 0, μ_1, μ_2, ($\mu_1 \wedge \mu_2$), 1} is a fuzzy topology on I. Now by computations we have int $(1 - \mu_1) = 0$; int $(1 - \mu_2) = 0$; int $(1 - [\mu_1 \wedge \mu_2]) = 0$; int $(\mu_3) = 0$ int $(1 - \mu_3) = 0$; int $(\mu_2 \wedge \mu_3) = 0$; int $(1 - [(\mu_2 \vee \mu_3)]) = 0$ and int $(1 - [\mu_2 \wedge \mu_3]) = 0$. $\{(1 - \mu_1) \vee (1 - \mu_2) \vee (\mu_3) \vee (1 - \mu_3) \vee (\mu_2 \wedge \mu_3) \vee (1 - [(\mu_2 \vee \mu_3)]) \vee (1 - [\mu_2 \wedge \mu_3]) \vee (1 - [\mu_1 \wedge \mu_2])\} \neq 1$ .Hence the fuzzy topological space(I,T) is a fuzzy almost irresolvable space . But (I,T) is not a fuzzy irresolvable space,sincecl(μ_3) = 1
and int (μ_3) = 0.

5. Functions and Fuzzy Almost Resolvablespaces

Definition 5.1 [9]:A function f: (X,T) →(Y,S) from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is called somewhat fuzzy open if $\lambda \in T$ and $\lambda \neq 0$ implies that there exists a fuzzy open set μ in (Y,S) such that$\mu \neq 0$ and $\mu \le f(\lambda)$.

Theorem 5.1 [10]: Let (X,T) and (Y,S) be any two fuzzy topological spaces. If the function f :(X,T) →(Y,S)is somewhat fuzzy open function and if int (λ) = 0 for any non -zero fuzzy set λ in (Y,S), int $f^{-1}(\lambda) = 0$ in (X,T).

Proposition 5.1: If f :(X,T) →(Y,S)is a somewhat fuzzy open function from a fuzzy topological space (X,T)onto a fuzzy almost resolvable space (Y,S) , then(X,T)is a fuzzy almost resolvable space.

Proof: Since(Y,S) is a fuzzy almost resolvable space,$\left(\vee_{i=1}^{\infty}(\lambda_i)\right) = 1$ where the λ_i's arefuzzy setsin(Y,S)suchthat int $(\lambda_i) = 0$.Then $f^{-1}\left(\vee_{i=1}^{\infty}(\lambda_i)\right) = f^{-1}(1) = 1$ implies that $\vee_{i=1}^{\infty} f^{-1}(\lambda_i) = 1$. Now since f is somewhat fuzzy open and int $(\lambda_i) = 0$, by theorem 5.1, we have int $f^{-1}(\lambda_i) = 0$.Hence $\vee_{i=1}^{\infty} f^{-1}(\lambda_i) = 1$,where int $f^{-1}(\lambda_i) = 0$ in(X,T).Therefore (X,T)is a fuzzy almost resolvable space.

Definition 5.2 [10]: Let (X,T) and (Y,S) be any two fuzzy topological spaces. If the function f :(X,T) →(Y,S)is a somewhat fuzzy continuous and one - to - one and if int (λ) = 0 for any non - zero fuzzy set λ in (X,T), int $[f(\lambda)] = 0$ in (Y,S).

Proposition 5.2: If f :(X,T) →(Y,S)is a somewhat fuzzy continuous function from a fuzzy almost resolvable space (X,T)onto a fuzzy topological space (Y,S) , then(Y,S) is a fuzzy almost resolvable space.

Proof: Since(X,T) is a fuzzy almost resolvable space,$\left(\vee_{i=1}^{\infty}(\lambda_i)\right) = 1$ where the λ_i's are fuzzy sets in(X,T) such that int $(\lambda_i) = 0$. Then f $\left(\vee_{i=1}^{\infty}(\lambda_i)\right)$ = f (1) = 1 .Then $f(\vee_{i=1}^{\infty}(\lambda_i)) \le \vee_{i=1}^{\infty} f(\lambda_i)$ implies that $1 \le \vee_{i=1}^{\infty} f(\lambda_i)$ .That is, $\vee_{i=1}^{\infty} f(\lambda_i) = 1$ Now since the function f is somewhat fuzzy continuous and int $(\lambda_i) = 0$, by theorem 5.2, we haveint $[f(\lambda_i)] = 0$.Hence $\vee_{i=1}^{\infty} f(\lambda_i) = 1$, where int $[f(\lambda_i)] = 0$ in (Y,S).Therefore (Y,S) is a fuzzy almost resolvable space.

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